Particle Physics 1

Lecture 4: QED

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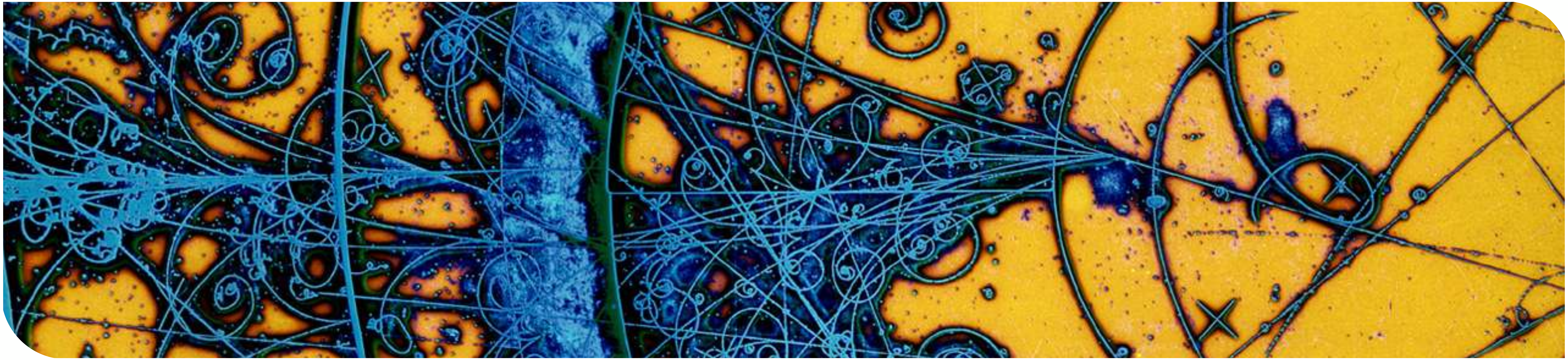


Questions from past lectures

Colloquium

Recent results on
$K^+ \to \pi^+ \nu \bar{\nu}$ *from NA62*

Radoslav Marchevski (EPFL)

Perturbation theory

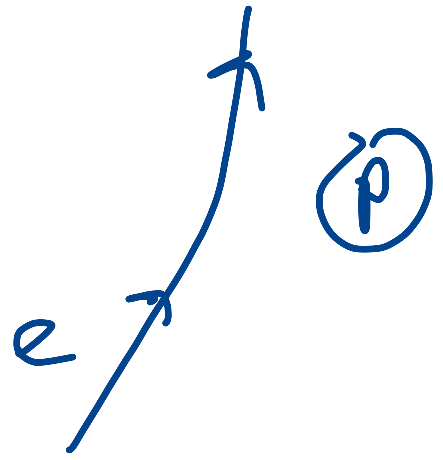
- Perturbation ≡

From Oxford Languages

  - A deviation of a system, moving object, or process from its regular or normal state or path, caused by an outside influence.
  - Anxiety; mental uneasiness.

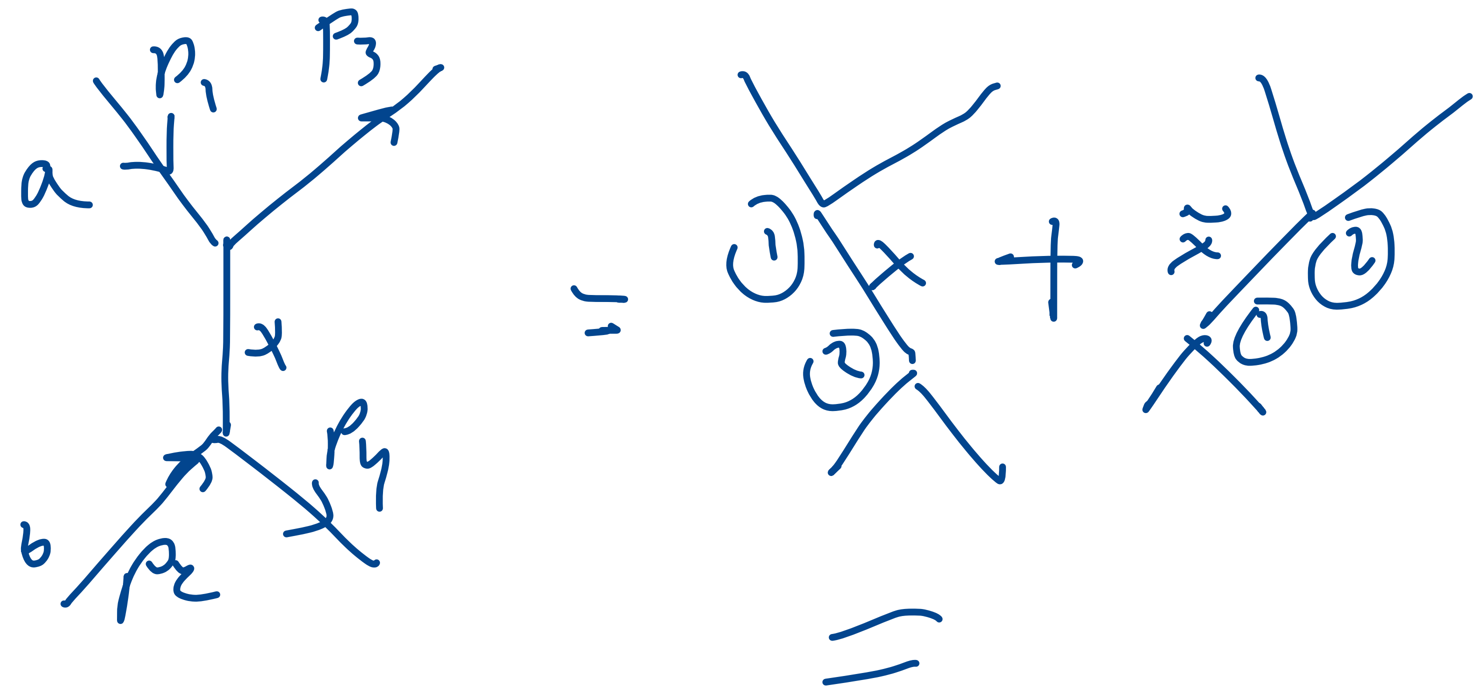
$\Gamma_{fi} = 2\pi |T_{fi}| \rho(E_f)$

$$T_{fi} = \langle f|V|i\rangle + \sum_{j\neq i} \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j} + \cdots$$



$\langle f|V(\vec{r})|i\rangle$

$\int \psi^* V(\vec{r}) \psi \, d^3\vec{r}$



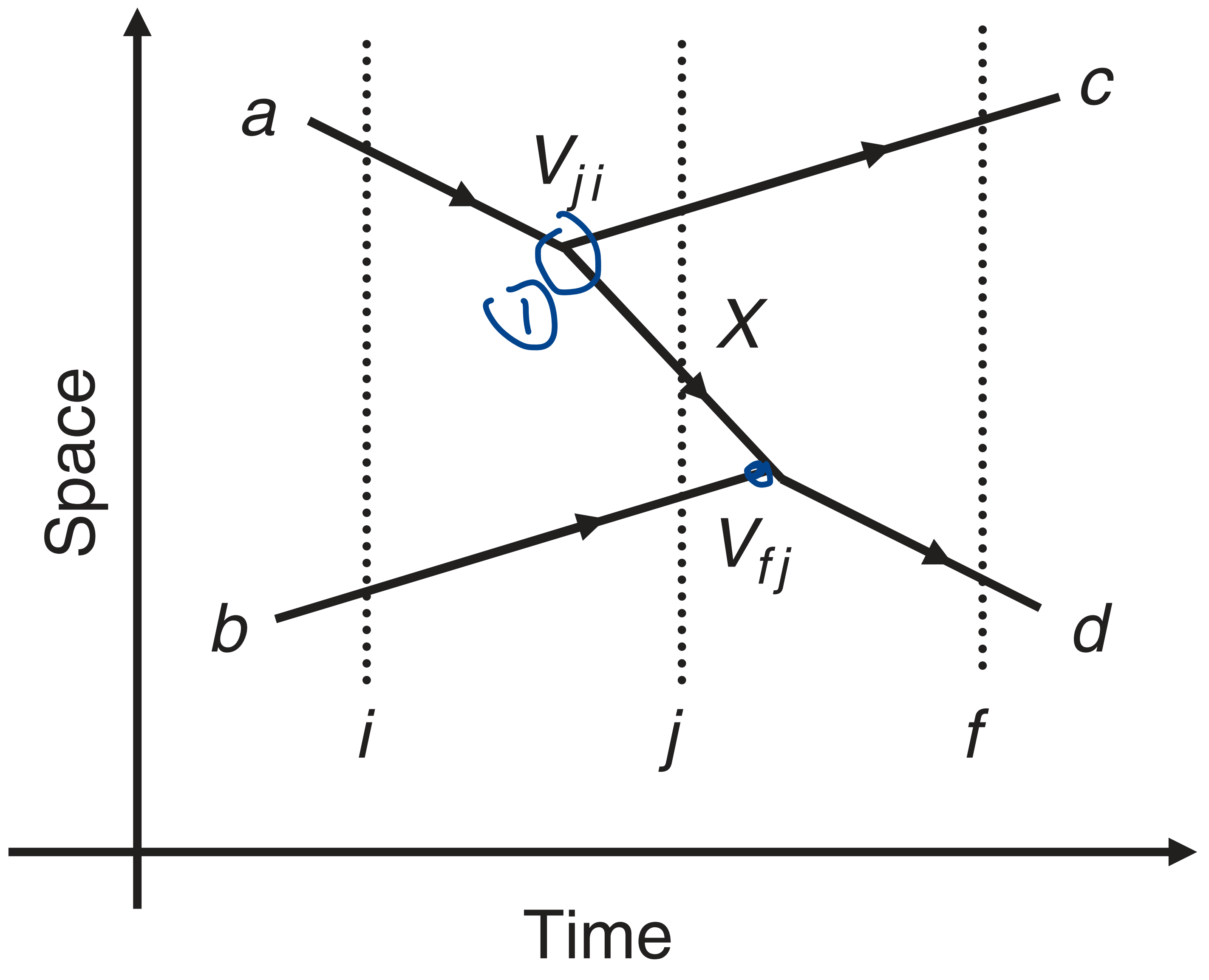
Perturbation theory (time-ordered)

- Focus on one of the two possible time-ordered diagrams

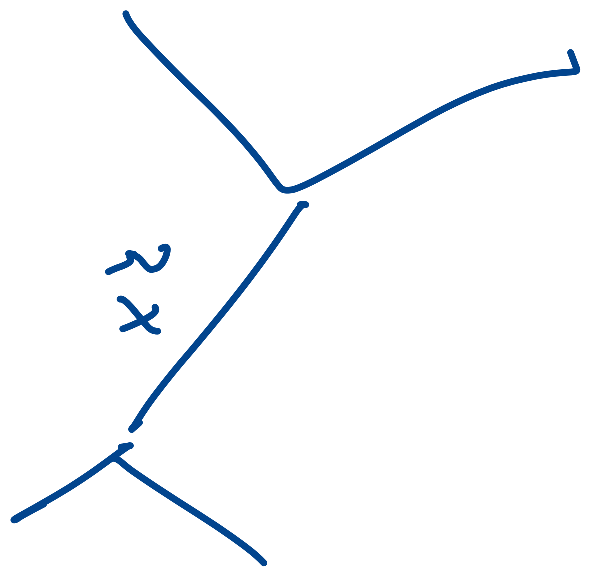
$$T_{fi} = \langle f|V|i\rangle + \sum_{j\neq i}\frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j} + \cdots$$

not LI

$$T^{ab}_{fi} = \frac{\langle d|V|X+b\rangle\langle c+X|V|a\rangle}{(E_a+E_b)-(E_c+E_X+E_b)}$$



Thomson Fig. 5.2



$$V_{ji} = M_{ji}\prod_k (2E)^{-1/2}$$

M^{ab}_{fi}

$$\langle c+X|V|a\rangle = \frac{\mathcal{M}_{a\to c+X}}{(2E_a 2E_c 2E_X)^{1/2}}$$

$$M_{fi} = M^{ab}_{fi} + M^{ba}_{fi}$$

$$= \frac{g_a g_b}{(p_a - p_c)^2 - M_X^2}$$

{coupling constants

4-momentum of X

q

Quantum Electrodynamics

- **QED $\equiv$ The QFT of the electromagnetic interaction**

$$\mathcal{M} = \langle \psi_c | V | \psi_a \rangle \frac{1}{q^2 - m_X^2} \langle \psi_d | V | \psi_b \rangle$$

LI matrix
element

- To obtain the QED matrix element for a scattering process:
  - Need the expression for the QED interaction vertex
  - Sum over the QM amplitudes of the possible polarization states of the spin-1 photon propagator

Interaction of electrons with EM field

- What is the form of the perturbation?
- Start as we did for finding the form of $\hat{C}$ (L03, S28-29)

$$\boxed{\hat{V}_D = q\gamma^0\gamma^\mu A_\mu}$$

$$A_\mu = (\phi, \vec{A})$$

$$\partial_\mu \to \partial_\mu + iqA_\mu$$

Free particle Dirac eqn: $\gamma^\mu\partial_\mu\psi + iq\gamma^\mu A_\mu\psi + im\psi = 0$

$i\gamma^0 \cdot (\quad)$

$$\boxed{i\frac{\partial\psi}{\partial t} + i\gamma^0\vec{\gamma}\cdot\vec{\nabla} - q\gamma^0\gamma^\mu A_\mu\psi - m\gamma^0\psi = 0}$$

$$\hat{H}\psi = i\frac{\partial\psi}{\partial t}$$

$$\hat{H} = (\underbrace{m\gamma^0}_{\text{mass term}} \underbrace{- i\gamma^0\vec{\gamma}\cdot\vec{\nabla}}_{\text{momentum term}}) + \underbrace{q\gamma^0\gamma^\mu A_\mu}_{\text{contribution to } H \text{ from the interaction}}$$

Additional energy due to the EM field A_μ

$$\vec{\gamma}\cdot\vec{\nabla} = \gamma^1\frac{\partial}{\partial x} + \gamma^2\frac{\partial}{\partial y} + \gamma^3\frac{\partial}{\partial z}$$

QED scattering $e^-\tau^- \to e^-\tau^-$

- Need to calculate the Lorentz invariant matrix element $\mathcal{M}$ for the ***t*-channel scattering** process

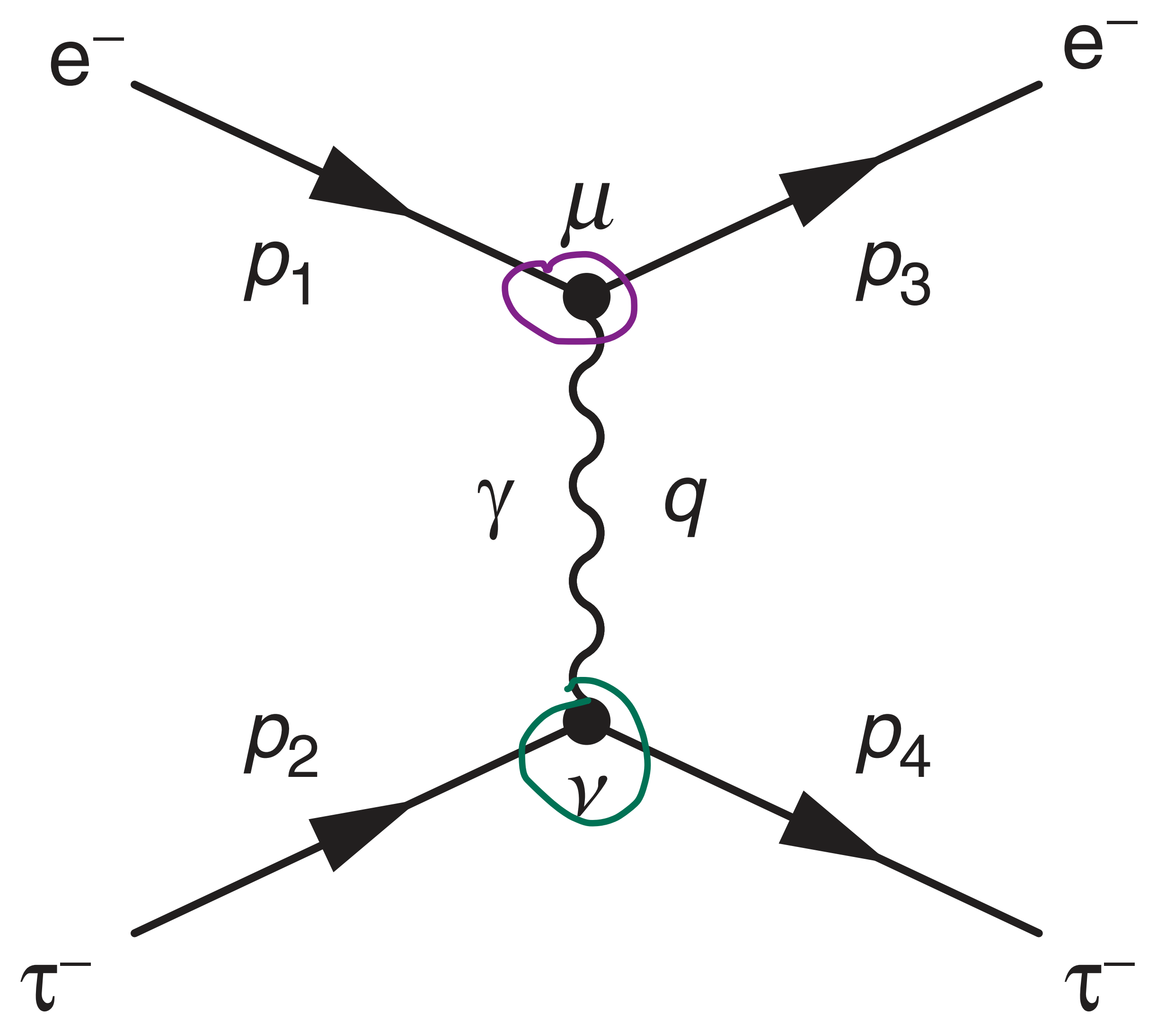
$A_\mu = \varepsilon_\mu^{(\lambda)} e^{i(\vec{p}\cdot\vec{x} - Et)}$

  - We now have the form of the perturbation: $\hat{V}_D = q\gamma^0\gamma^\mu A_\mu.$
  - Use this potential for the $e^-\gamma$ vertex interaction:

$$\langle\psi(p_3)|\hat{V}_D|\psi(p_1)\rangle \to u_e^\dagger(p_3)\, Q_e e\gamma^0\gamma^\mu\varepsilon_\mu^{(\lambda)}\, u_e(p_1)$$

  - And for the $\tau^-\gamma$ vertex interaction:

$$u_\tau^\dagger(p_4)\, Q_\tau e\gamma^0\gamma^\nu\varepsilon_\nu^{(\lambda)*}\, u_\tau(p_2)$$



Thomson Fig. 5.6

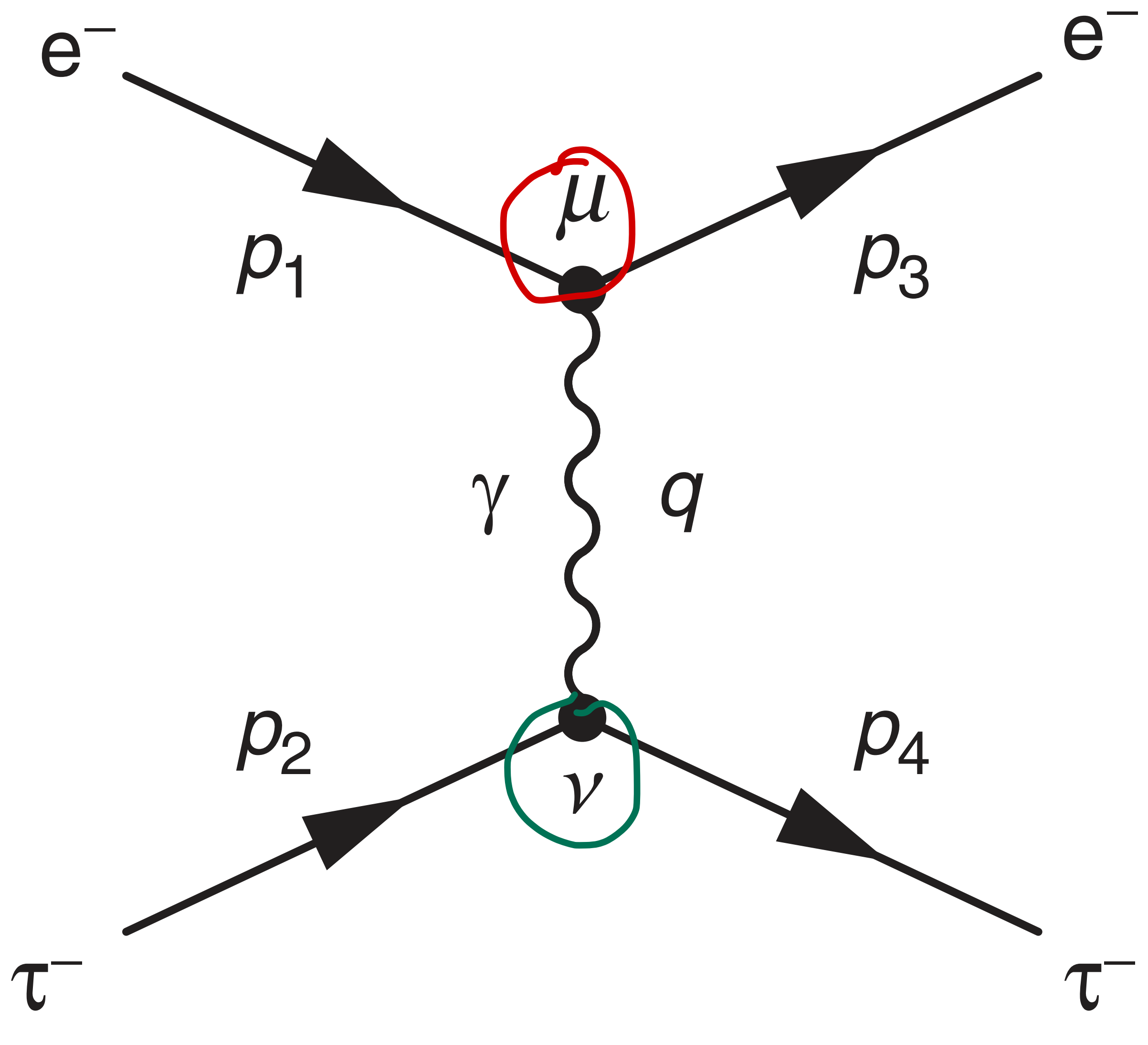
  - Sum over the 2 time-ordered diagrams (& include the photon polarization):

$$\mathcal{M} = \sum_\lambda \left[u_e^\dagger(p_3)Q_e e\gamma^0\gamma^\mu u_e(p_1)\right]\varepsilon_\mu^{(\lambda)}\frac{1}{q^2}\varepsilon_\nu^{(\lambda)*}\left[u_\tau^\dagger(p_4)Q_\tau e\gamma^0\gamma^\nu u_\tau(p_2)\right]$$

QED scattering $e^-\tau^- \to e^-\tau^-$

- Clean this up a bit

$$\mathcal{M} = \sum_\lambda \left[u_e^\dagger(p_3)Q_e e\gamma^0\gamma^\mu u_e(p_1)\right]\varepsilon_\mu^{(\lambda)}\frac{1}{q^2}\varepsilon_\nu^{(\lambda)*}\left[u_\tau^\dagger(p_4)Q_\tau e\gamma^0\gamma^\nu u_\tau(p_2)\right]$$



Thomson Fig. 5.6

  - Use this relation between the sum of over the polarization states of the virtual photon and the metric tensor:

$$\sum_\lambda \varepsilon_\mu^{(\lambda)}\varepsilon_\nu^{(\lambda)*} = -g_{\mu\nu}$$

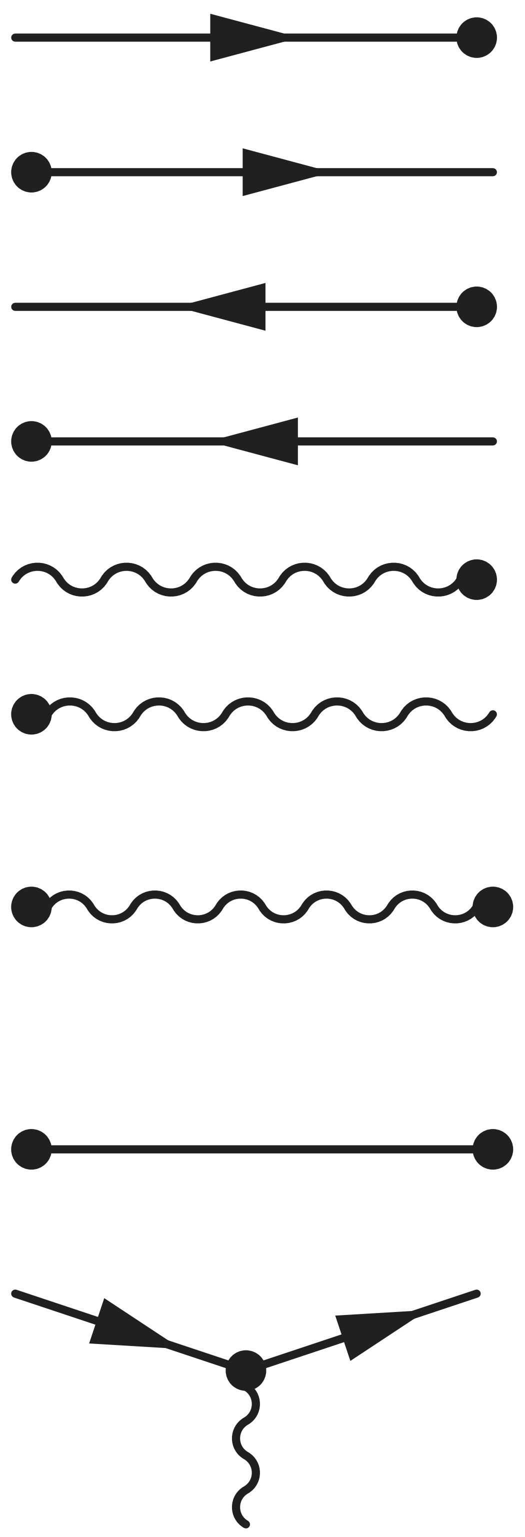
  - And the adjoint spinor $\bar{\psi} = \psi^\dagger\gamma^0$

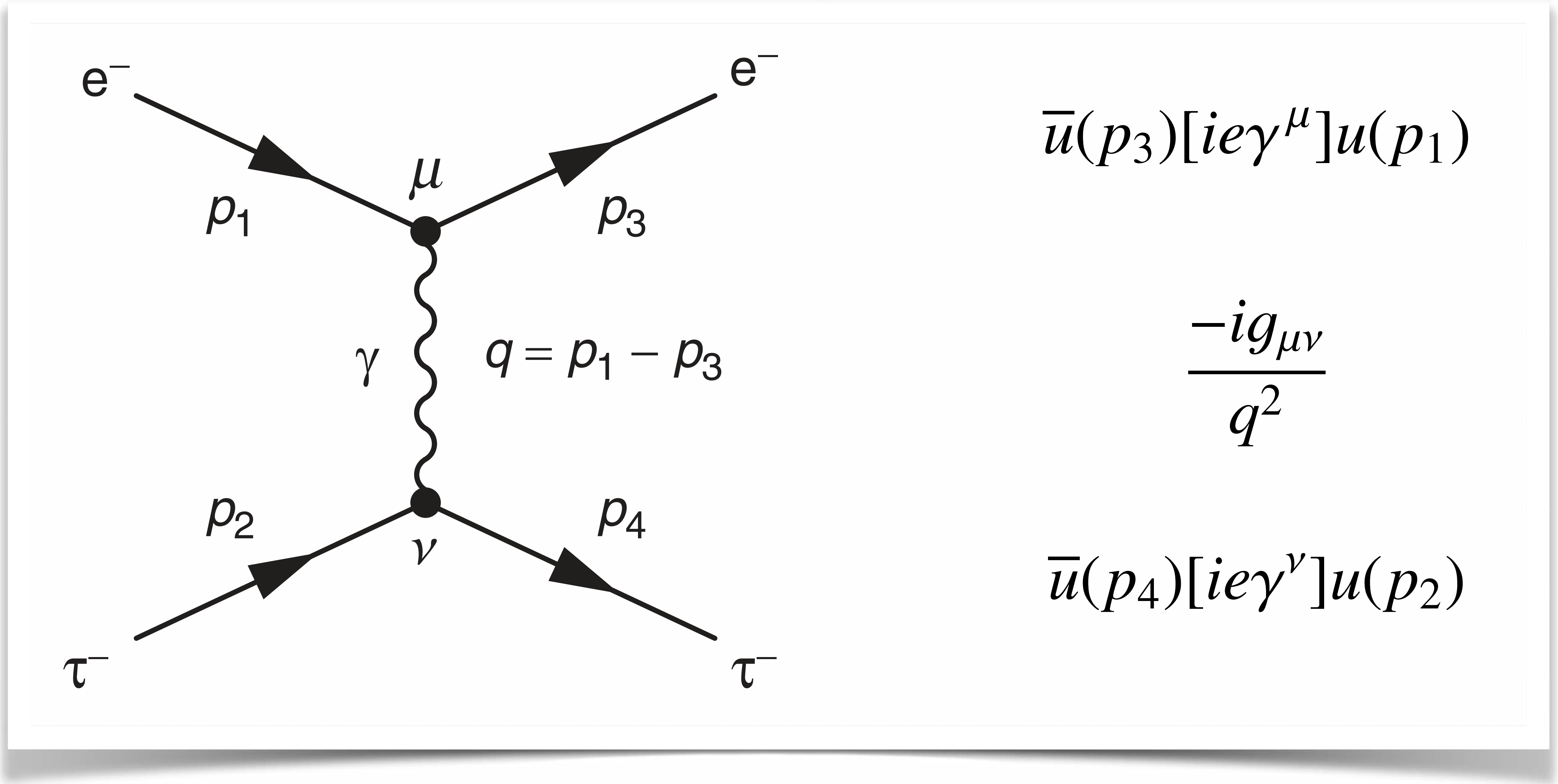
$$\mathcal{M} = -\underbrace{[Q_e e\,\bar{u}_e(p_3)\gamma^\mu u_e(p_1)]}_{j_e^\mu}\frac{g_{\mu\nu}}{q^2}\underbrace{[Q_\tau e\,\bar{u}_\tau(p_4)\gamma^\nu u_\tau(p_2)]}_{j_\tau^\nu}.$$

$$\mathcal{M} = -Q_e Q_\tau \frac{j_e \cdot j_\tau}{q^2}$$

Feynman rules for QED

initial-state particle:	$u(p)$	
final-state particle:	$\overline{u}(p)$	
initial-state antiparticle:	$\overline{v}(p)$	
final-state antiparticle:	$v(p)$	
initial-state photon:	$\varepsilon_\mu(p)$	
final-state photon:	$\varepsilon^*_\mu(p)$	
photon propagator:	$-\dfrac{ig_{\mu\nu}}{q^2}$	
fermion propagator:	$-\dfrac{i(\gamma^\mu q_\mu + m)}{q^2 - m^2}$	
QED vertex:	$-iQe\gamma^\mu$	

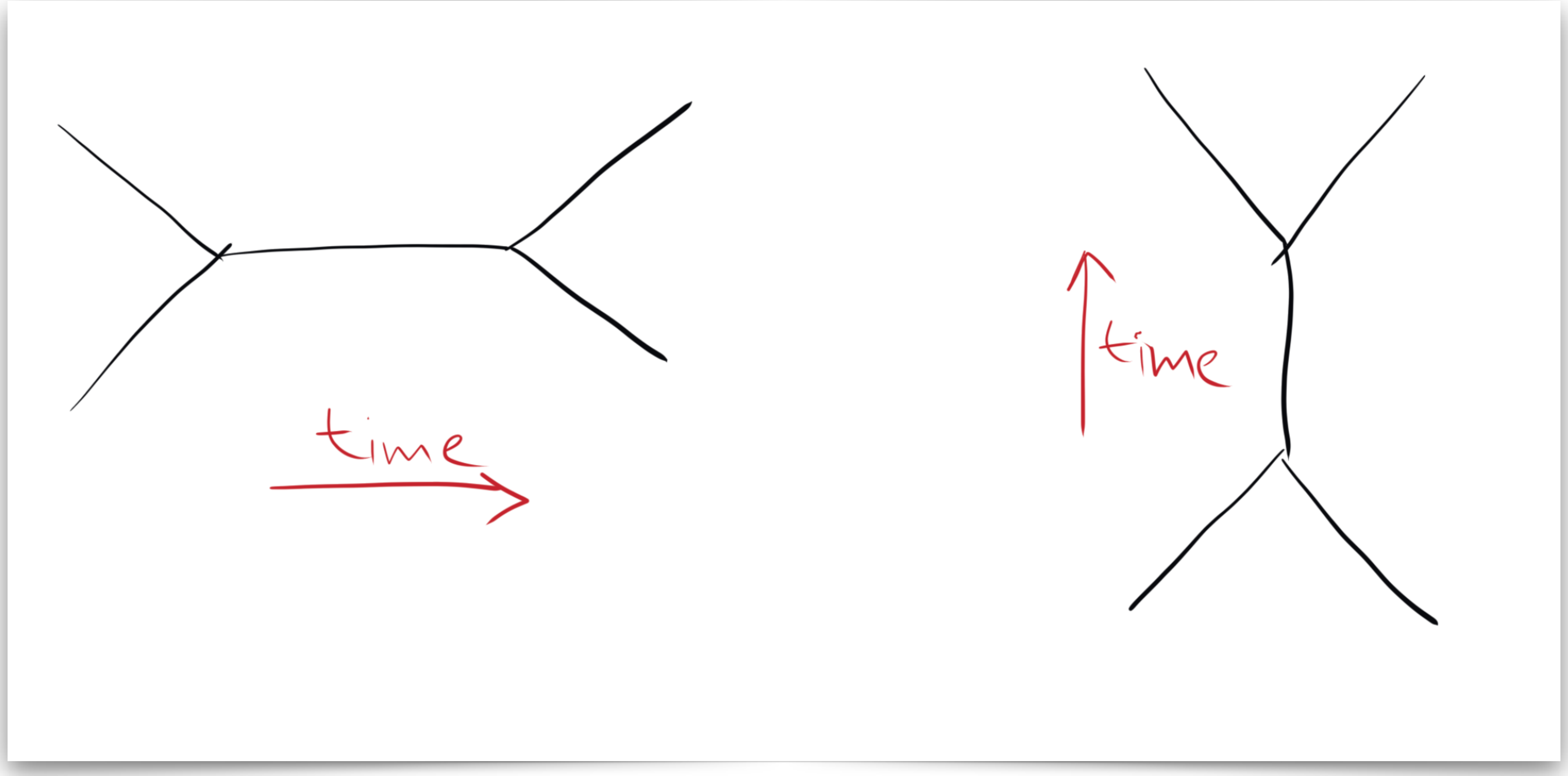




Thomson Fig. 5.7

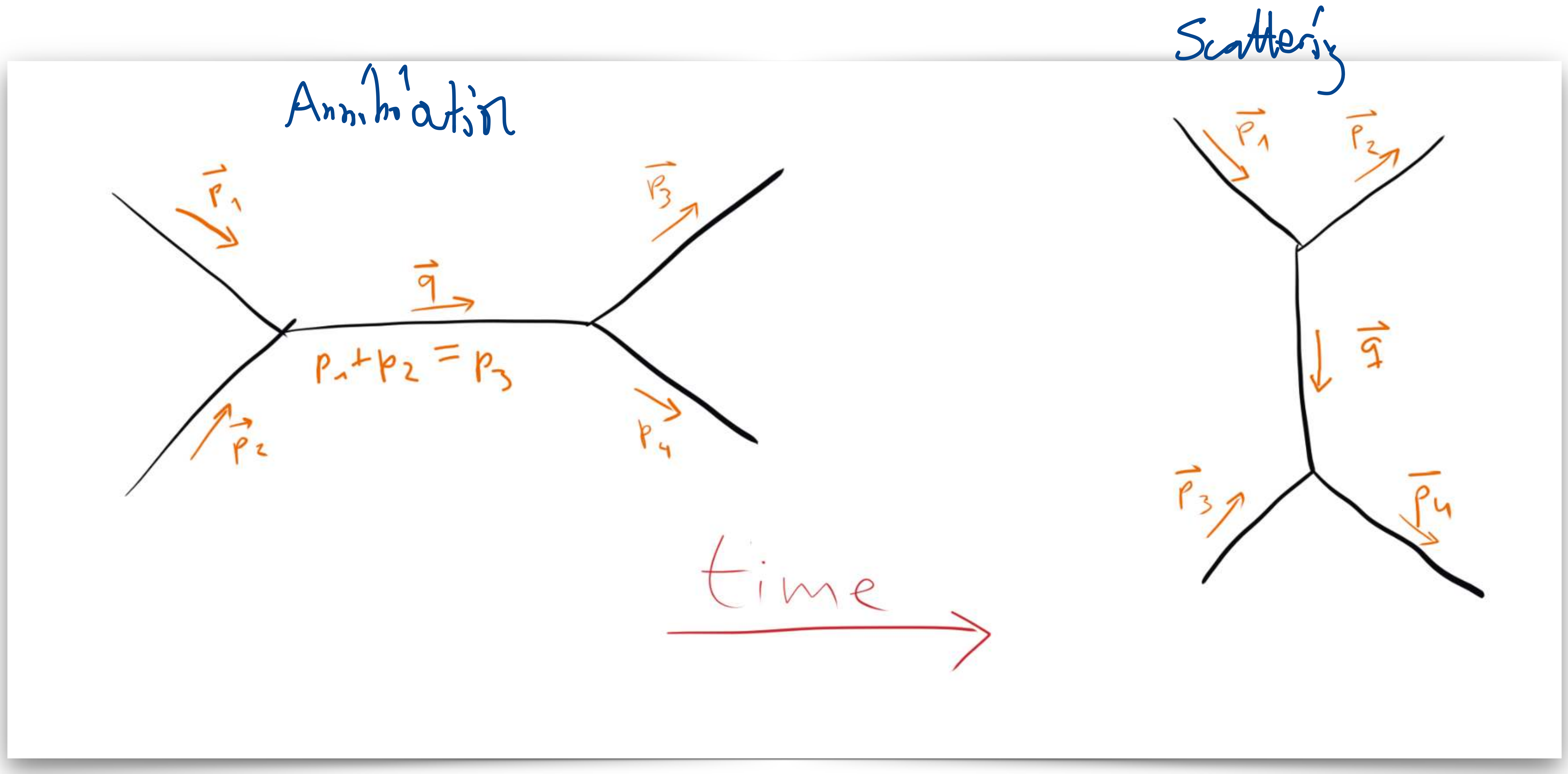
Feynman diagrams

- Direction of time is convention (I like left to right)



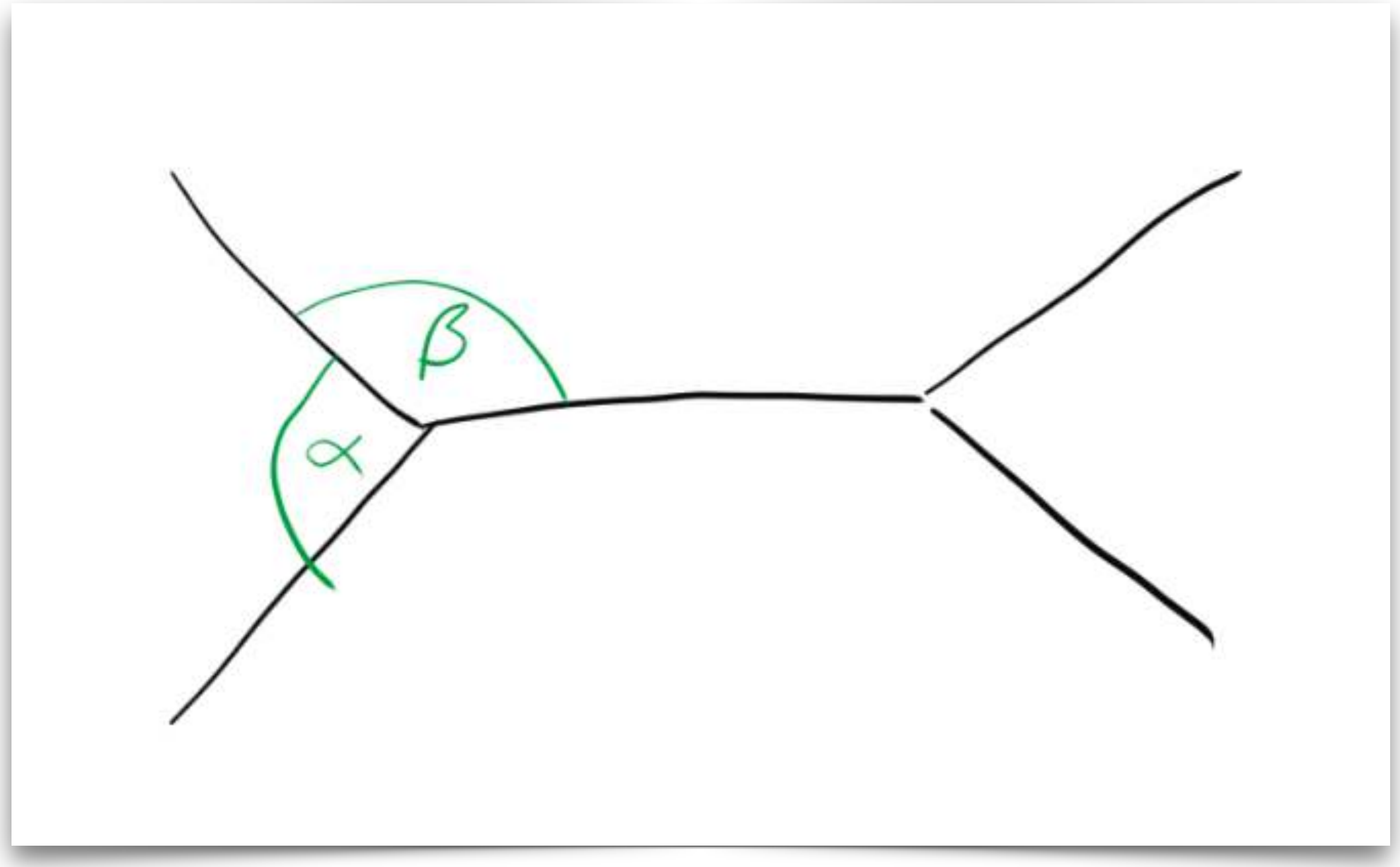
Feynman diagrams

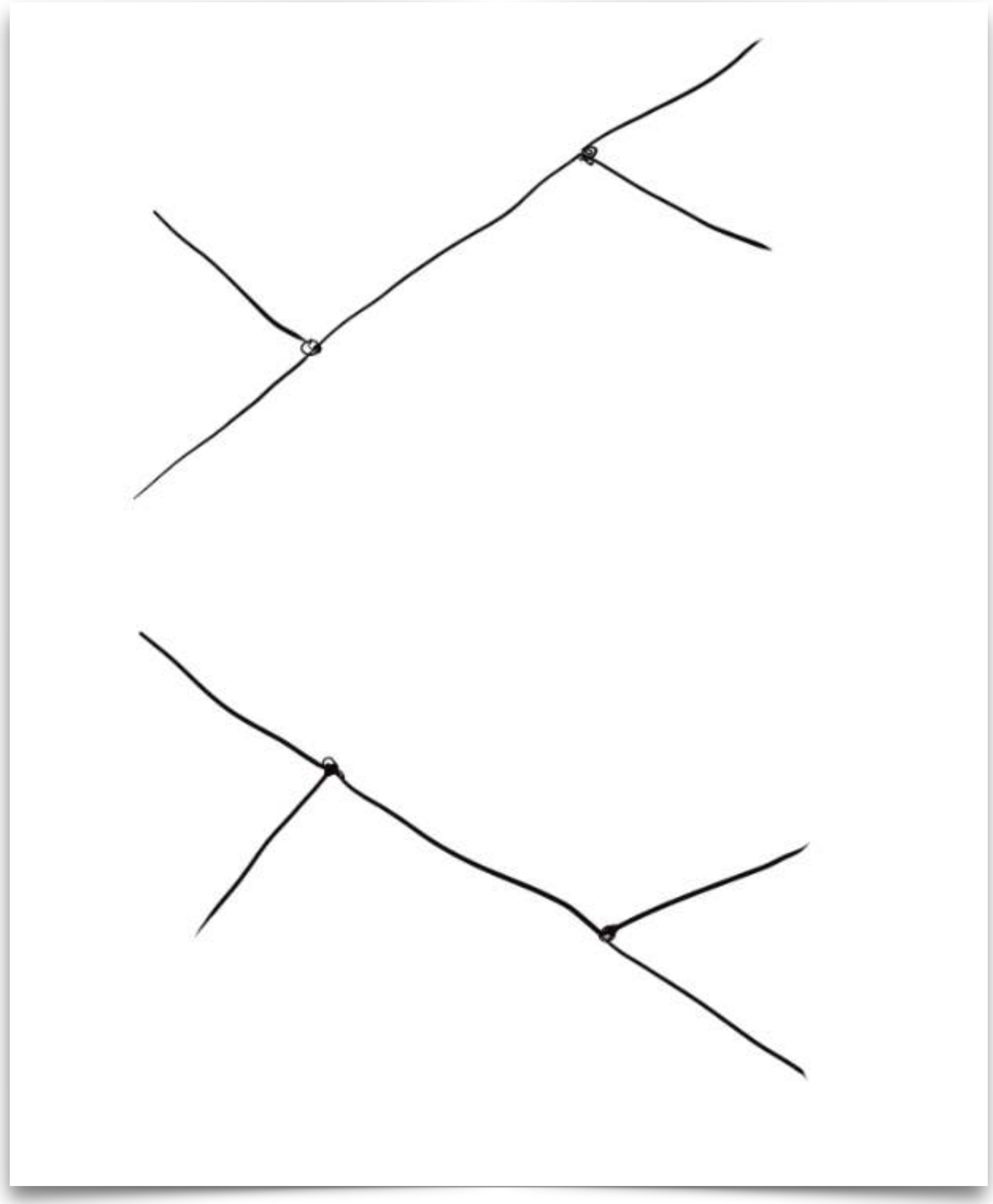
- Once you choose a convention, those diagrams describe different physics processes



Feynman diagrams

- The angles between the different lines have no spatial meaning
- The two diagrams on the right are considered identical



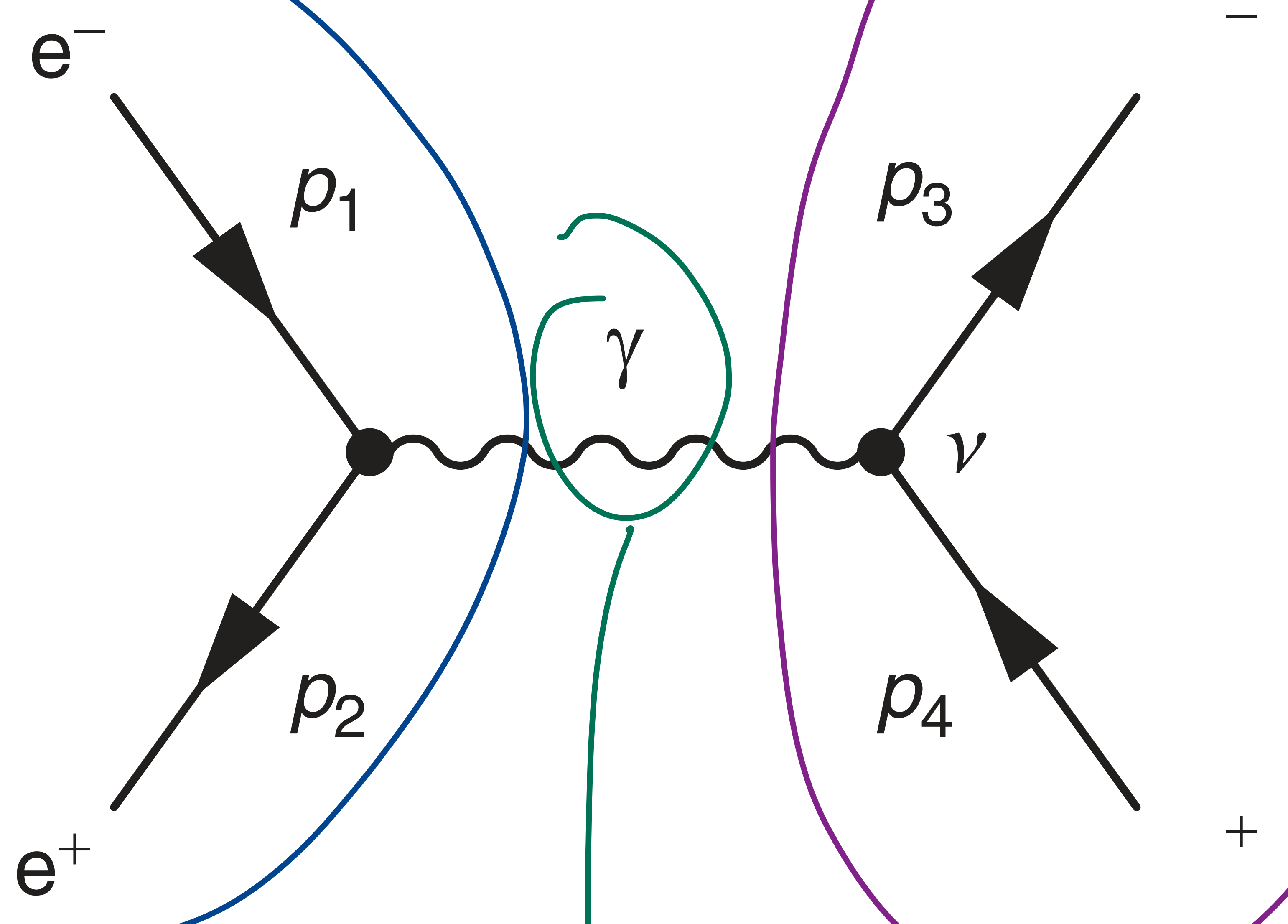


QED annihilation $e^+e^- \to \mu^+\mu^-$

- Use our rules to calculate $\mathcal{M}$ for the s**-channel annihilation** process

j_e^μ $\quad$ j_μ^ν

$$-i\mathcal{M} = \left[\bar{v}(p_2)\{ie\gamma^\mu\}u(p_1)\right]\left[\frac{-ig_{\mu\nu}}{q^2}\right]\left[\bar{u}(p_3)\{ie\gamma^\nu\}v(p_4)\right]$$



Thomson Fig. 5.8

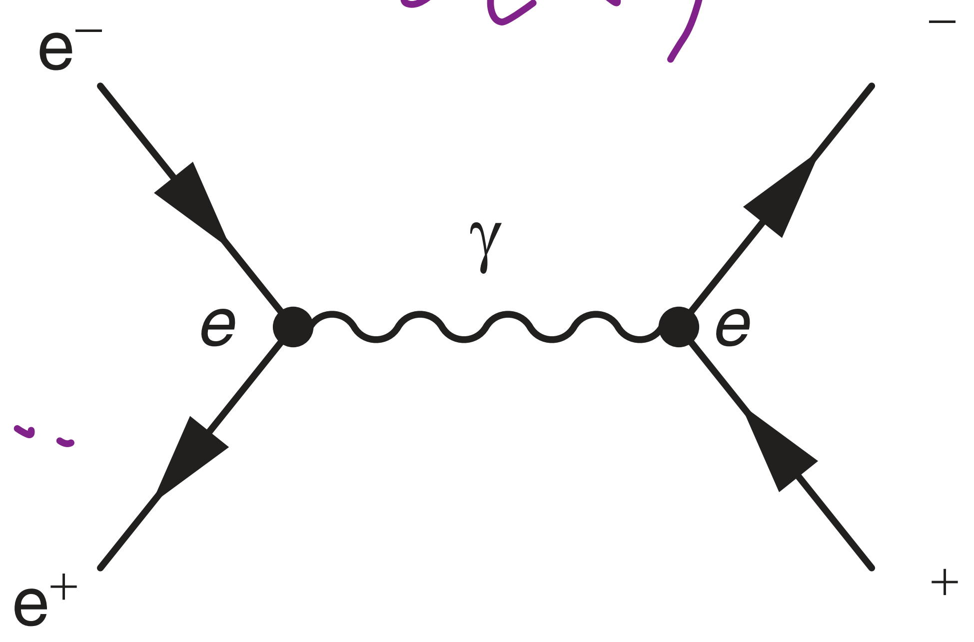
Calculations in perturbation theory

- Revisit the QED annihilation process $e^+e^- \rightarrow \mu^+\mu^-$
- We only looked at the lowest order Feynman Diagram:

$\mathcal{O}(\alpha^2)$

$M_{fi} = \alpha M_{LO} + \alpha^2 \sum M_{ij} + \dots$

$|M_{fi}|^2 = \alpha^2 |M_{LO}|^2 + \alpha^3 \sum (\quad) + \alpha^4 \dots$

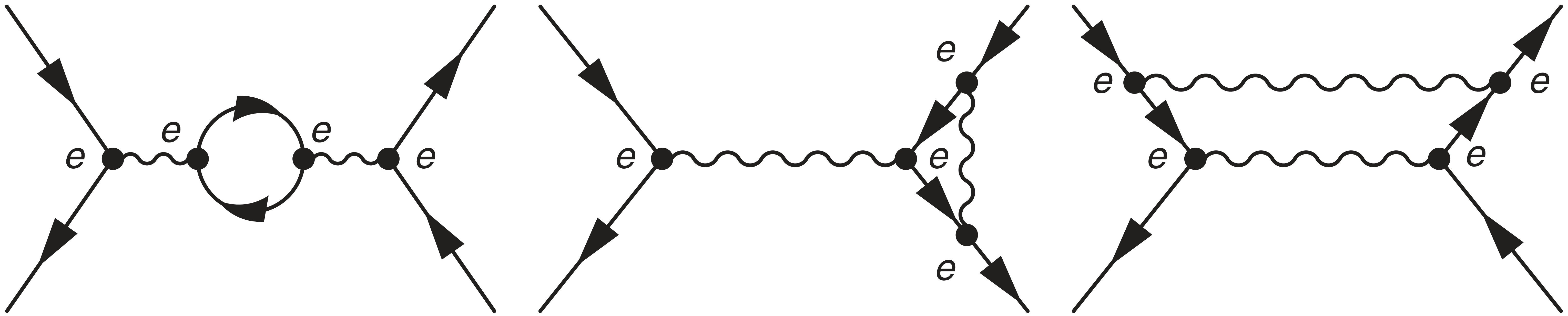


Thomson Fig. 6.1

$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$

- But there are an infinite number of higher order diagrams

$\mathcal{O}(\alpha^4)$



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Thomson Fig. 6.2

Including spin

- How many possible orthogonal helicity combinations?

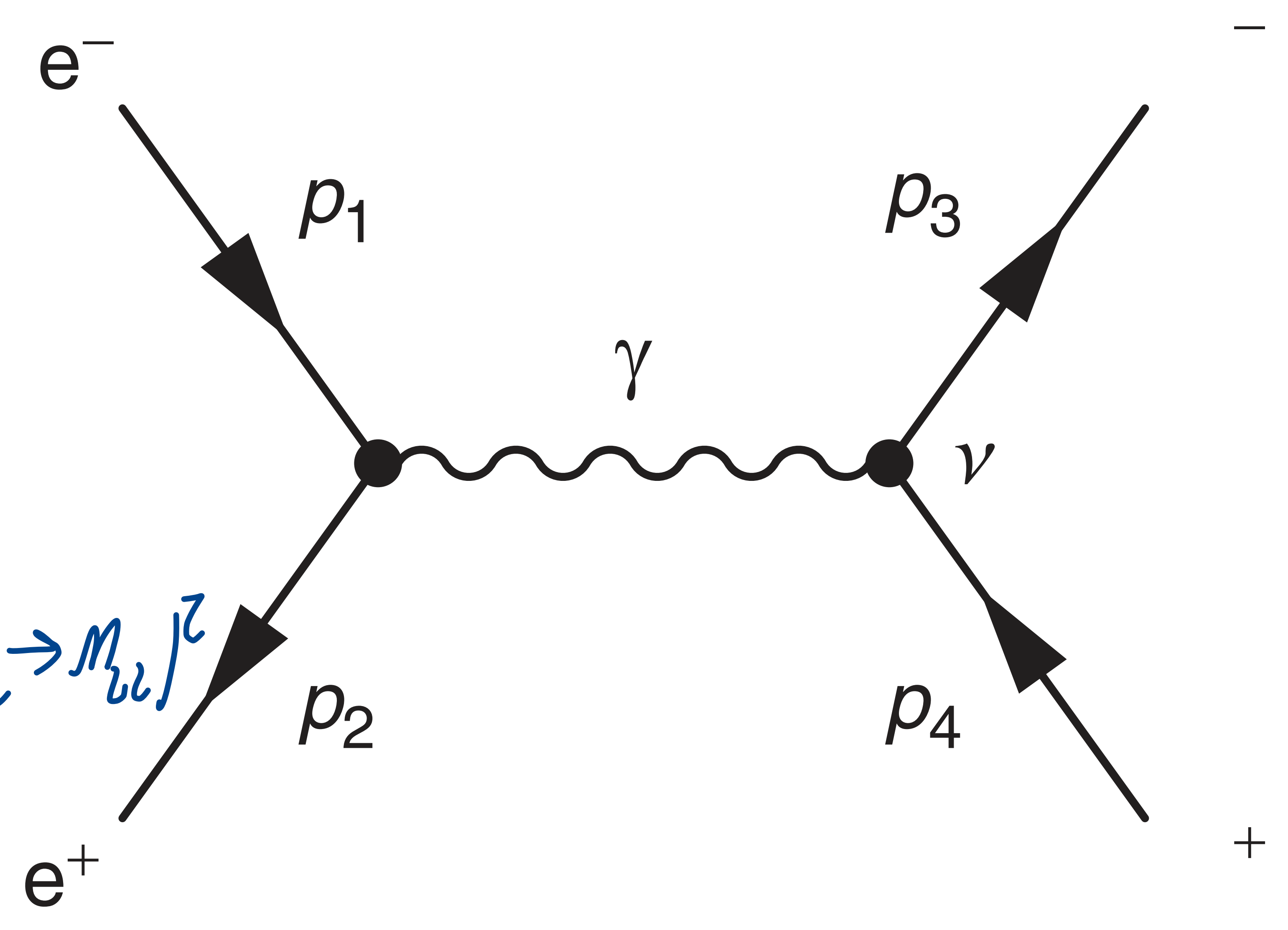
e^-	e^+		μ^+	μ^-	
$\rightarrow$	$\rightarrow$	R L	$\rightarrow$	$\rightarrow$	R L
$\rightarrow$	$\leftarrow$	R R	$\leftarrow$	$\rightarrow$	R R
$\leftarrow$	$\leftarrow$	L R	$\rightarrow$	$\leftarrow$	L L
$\leftarrow$	$\rightarrow$	L L	$\leftarrow$	$\leftarrow$	L R

$e^+_\uparrow e^-_\uparrow \to \mu^+_\uparrow \mu^-_\uparrow$

$$\sum |M_{RR}|^2 \,(e^+e^-) = |M_{RR\to RR}|^2 + |M_{RR\to RL}|^2 + |M_{RR\to LR}|^2 + |M_{RR\to LL}|^2$$

e^+e^- $\quad$ $\mu^+\mu^-$

$$\langle |M_{fi}|^2 \rangle = \frac{1}{4} \sum_{\text{spins}} |M|^2 = \frac{1}{4}\left(| \quad | + |M_{RL}|^2 + |M_{LR}|^2 + |M_{LL}|^2 \right)$$



Thomson Fig. 5.8

4-momenta in CM frame

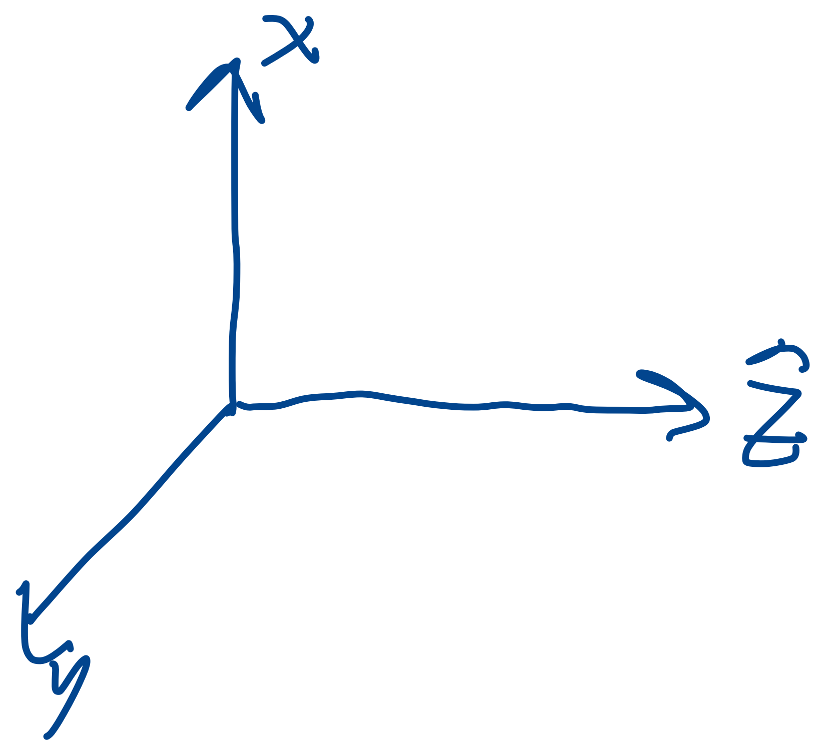
- Neglect the particle masses: $\sqrt{s} \gg m_\mu$
- $\mu^+\mu^-$ produced with $\phi = 0$ and $\phi = \pi$

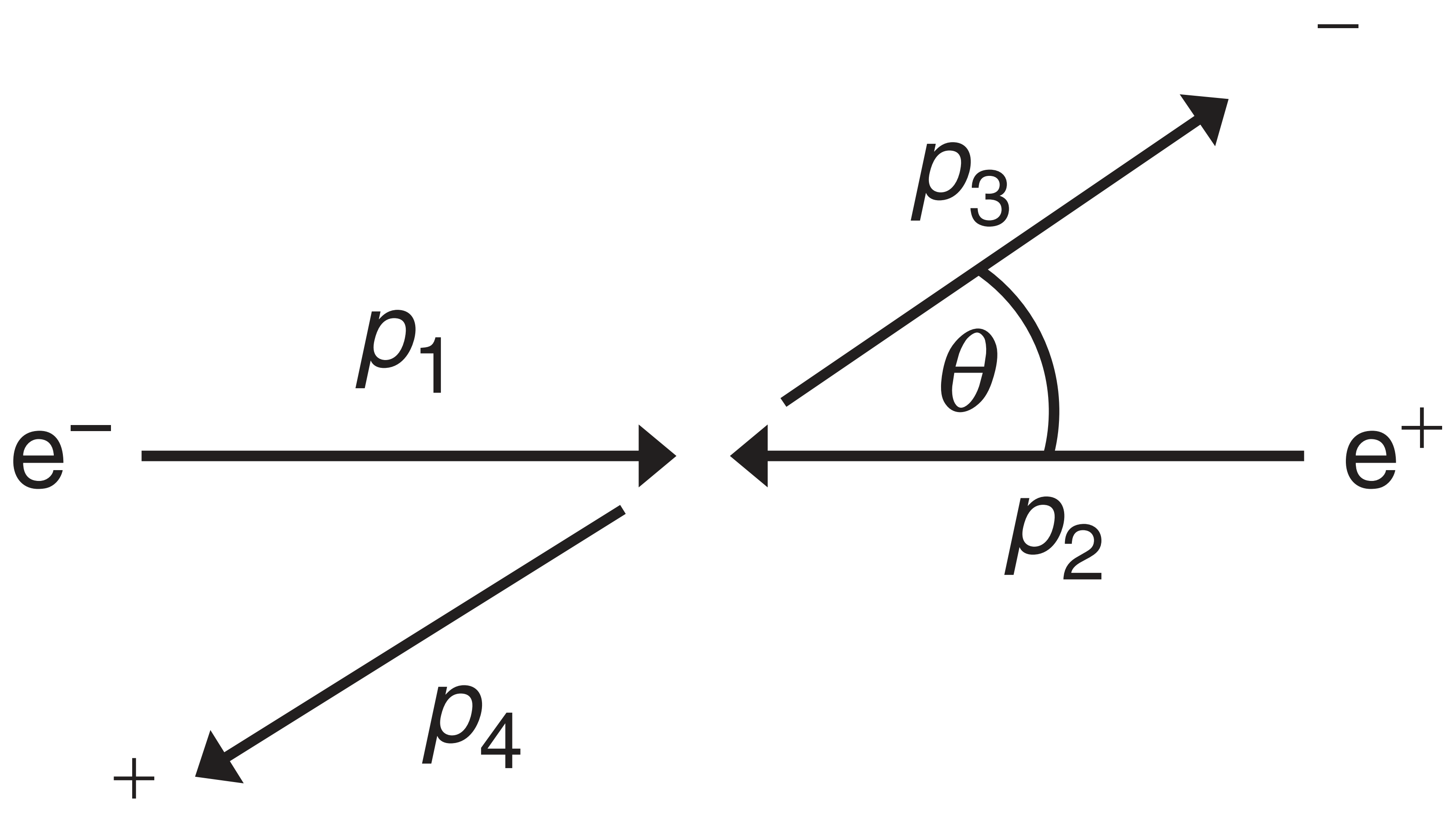
e^- $$p_1 = (E, 0, 0, E),$$

e^+ $$p_2 = (E, 0, 0, -E),$$

μ^- $$p_3 = (E, E\sin\theta, 0, E\cos\theta),$$

μ^+ $$p_4 = (E, -E\sin\theta, 0, -E\cos\theta),$$





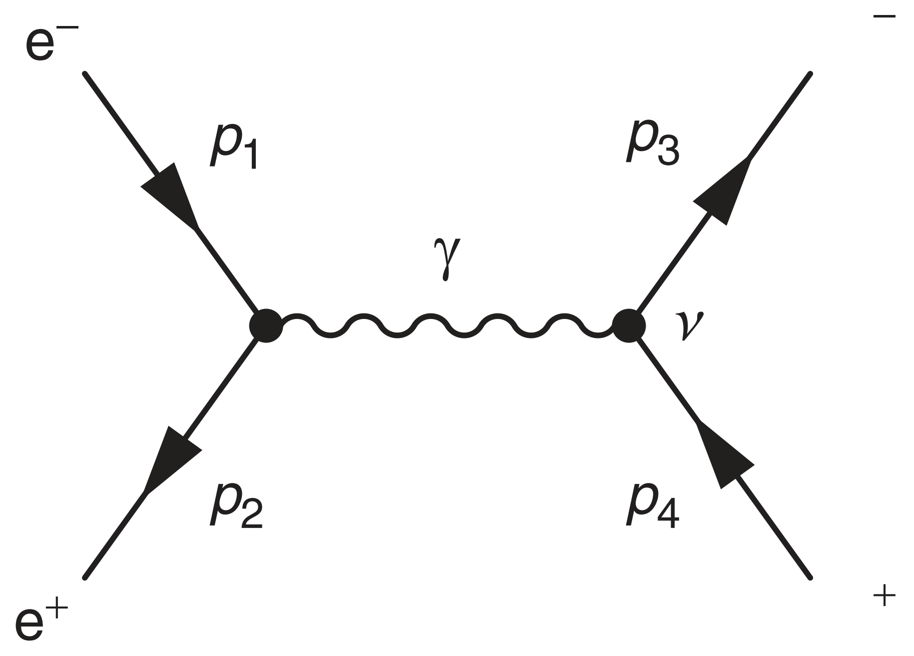
Thomson Fig. 6.4

Helicity amplitudes

- Recall the ME for the lowest-order diagram (S11) with 4-vector currents j

$$\mathcal{M} = -\frac{e^2}{q^2} g_{\mu\nu} \left[\overline{v}(p_2)\gamma^\mu u(p_1)\right]\left[\overline{u}(p_3)\gamma^\nu v(p_4)\right]$$

$$j_{\mathrm{e}}^{\mu} = \overline{v}(p_2)\gamma^\mu u(p_1) \quad \text{and} \quad j_{\mu}^{\nu} = \overline{u}(p_3)\gamma^\nu v(p_4).$$



- The spinors in these currents are the ultra-relativistic limit ($E \gg m$) of the helicity eigenstates (L03, S25)

$$u_\uparrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}, \quad u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix}, \quad v_\uparrow = \sqrt{E}\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix}, \quad v_\downarrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix},$$

$$s = \sin\frac{\theta}{2}$$

$$c = \cos\frac{\theta}{2}$$

Spinors for the e^+e^- initial state

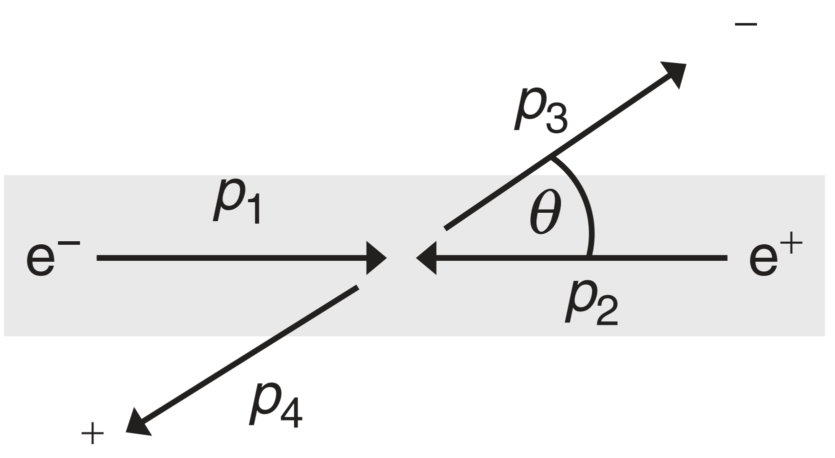
$$u_\uparrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix},\ u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix},\ v_\uparrow = \sqrt{E}\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix},\ v_\downarrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix},$$

Initial state **electron** with $(\theta = 0,\ \phi = 0)$

$$u_\uparrow(p_1) = \sqrt{E}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix},\ u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix},$$

Initial state **positron** with $(\theta = \pi,\ \phi = \pi)$

$$v_\uparrow(p_2) = \sqrt{E}\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix},\ v_\downarrow(p_2) = \sqrt{E}\begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix}.$$



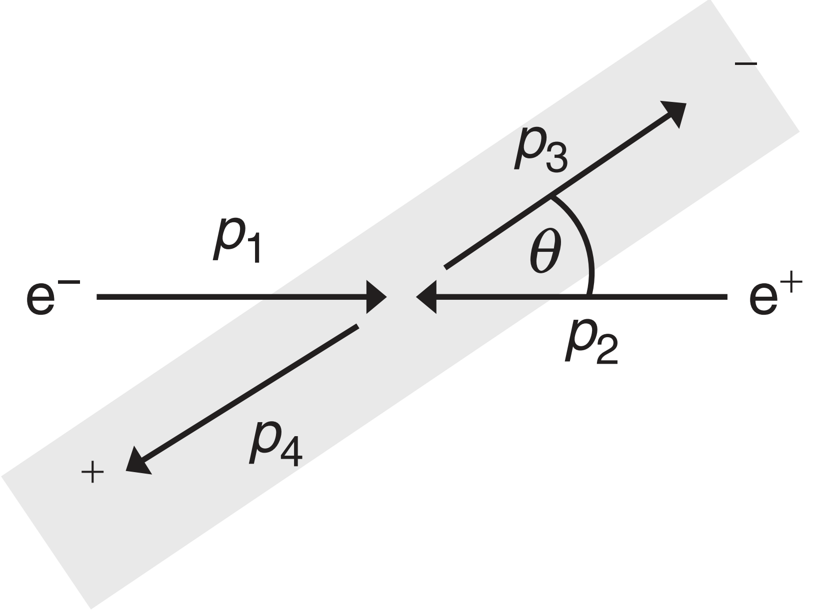
Spinors for the $\mu^+\mu^-$ final state

$$u_\uparrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}, \; u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix}, \; v_\uparrow = \sqrt{E}\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix}, \; v_\downarrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix},$$

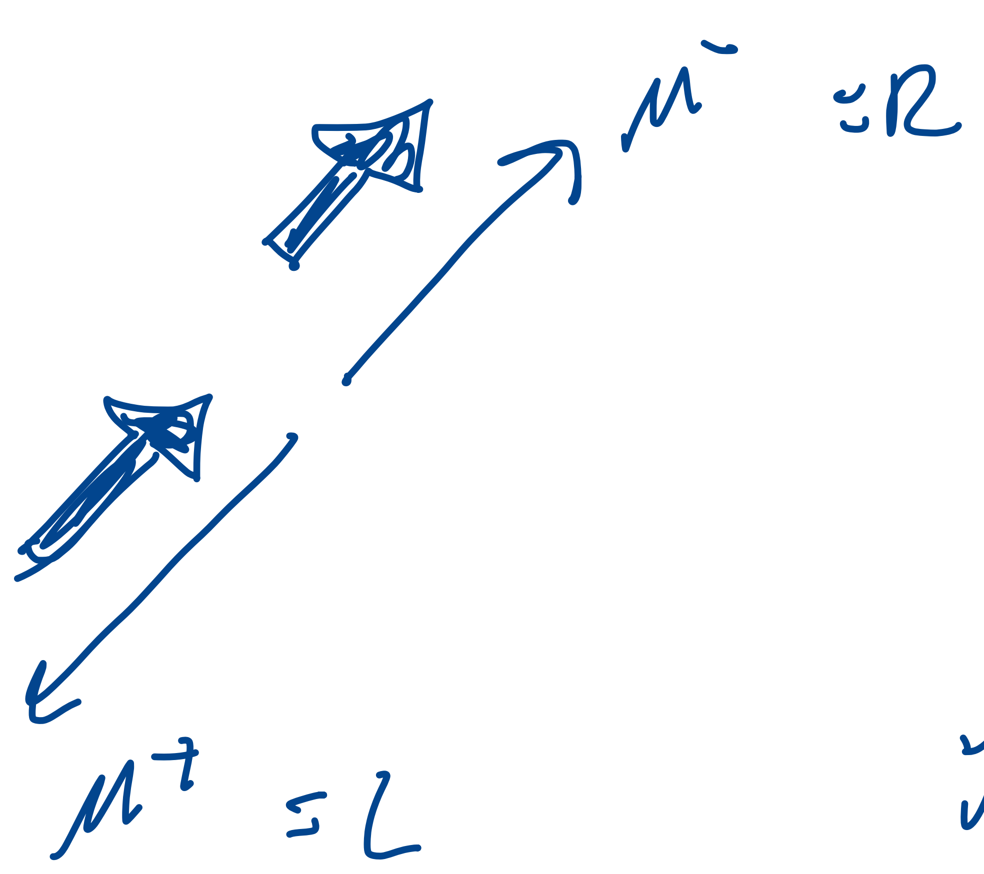
Final state $\boldsymbol{\mu}^-$ with $(\theta, 0)$

Final state $\boldsymbol{\mu}^+$ with $(\theta - \pi, \pi)$

$$u_\uparrow(p_3) = \sqrt{E}\begin{pmatrix} c \\ s \\ c \\ s \end{pmatrix}, \; u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix}, \; v_\uparrow(p_4) = \sqrt{E}\begin{pmatrix} c \\ s \\ -c \\ -s \end{pmatrix}, \; v_\downarrow(p_4) = \sqrt{E}\begin{pmatrix} s \\ -c \\ s \\ -c \end{pmatrix}.$$



Determine the muon current



$\mu^-\mu^+ = RL$

$$j^{\nu}_{\mu} = \bar{u}(p_3)\gamma^{\nu}v(p_4) \qquad \nu = 0,1,2,3$$

$$j^{0}_{\mu} = \bar{u}(p_3)\gamma^{0}v(p_4)$$

$$\bar{u}_{\uparrow}(p_3)\gamma^{0}v_{\downarrow}(p_4)$$

$$= \sqrt{E}\,(c,s,c,s)\begin{pmatrix}1&&&0\\&1&&\\&&-1&\\0&&&-1\end{pmatrix}\begin{pmatrix}s\\-c\\s\\-c\end{pmatrix} = E(cs-sc+cs-sc) = 0$$

$$j^{0}_{\mu} = \bar{u}_{\uparrow}(p_3)\gamma^{0}v_{\downarrow}(p_4) = E(cs - sc + cs - sc) = 0,$$
$$j^{1}_{\mu} = \bar{u}_{\uparrow}(p_3)\gamma^{1}v_{\downarrow}(p_4) = E(-c^2 + s^2 - c^2 + s^2) = 2E(s^2 - c^2) = -2E\cos\theta,$$
$$j^{2}_{\mu} = \bar{u}_{\uparrow}(p_3)\gamma^{2}v_{\downarrow}(p_4) = -iE(-c^2 - s^2 - c^2 - s^2) = 2iE,$$
$$j^{3}_{\mu} = \bar{u}_{\uparrow}(p_3)\gamma^{3}v_{\downarrow}(p_4) = E(cs + sc + cs + sc) = 4Esc = 2E\sin\theta.$$

In general

$$\overline{\psi}\gamma^0\phi = \psi^\dagger\gamma^0\gamma^0\phi = \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4,$$
$$\overline{\psi}\gamma^1\phi = \psi^\dagger\gamma^0\gamma^1\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1,$$
$$\overline{\psi}\gamma^2\phi = \psi^\dagger\gamma^0\gamma^2\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1),$$
$$\overline{\psi}\gamma^3\phi = \psi^\dagger\gamma^0\gamma^3\phi = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2.$$

Determine the muon current

- Repeat the calculation for the other $\mu^+\mu^-$ helicity combinations

$2E\,(0, -\cos\theta, -i, \sin\theta)$

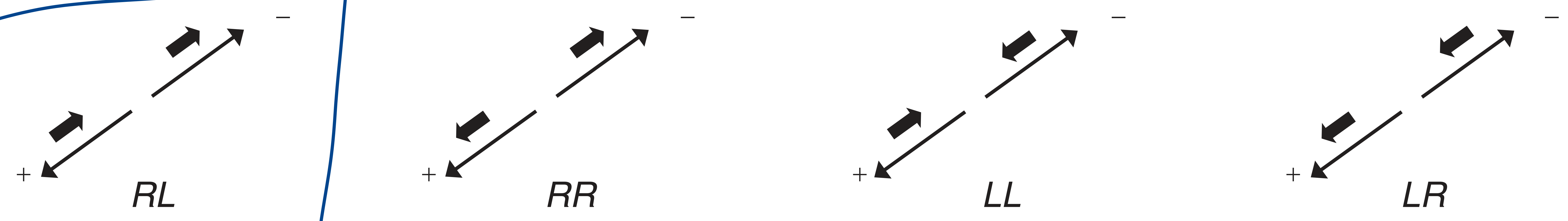
$$\begin{aligned}
j_{\mu,RL} &= \bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4) = 2E(0, -\cos\theta, i, \sin\theta), \\
j_{\mu,RR} &= \bar{u}_\uparrow(p_3)\gamma^\nu v_\uparrow(p_4) = (0,0,0,0), \qquad = 0 \\
j_{\mu,LL} &= \bar{u}_\downarrow(p_3)\gamma^\nu v_\downarrow(p_4) = (0,0,0,0), \qquad = 0 \\
j_{\mu,LR} &= \bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) = 2E(0, -\cos\theta, -i, \sin\theta).
\end{aligned}$$

$\hookrightarrow 2E\,(0, -\cos\theta, i, \sin\theta)$

What about the electron currents?

$$\left[\bar{u}(p_3)\gamma^\mu v(p_4)\right]^\dagger$$

$$= \left[\bar{v}(p_4)\gamma^\mu u(p_3)\right]$$



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Thomson Fig. 6.5

$\sigma(e^+e^- \to \mu^+\mu^-)$

- Calculate the spin-averaged ME

$$\langle|\mathcal{M}_{fi}|^2\rangle = \frac{1}{4}\times\left(|\mathcal{M}_{RL\to RL}|^2 + |\mathcal{M}_{RL\to LR}|^2 + |\mathcal{M}_{LR\to RL}|^2 + |\mathcal{M}^2_{LR\to LR}|\right)$$

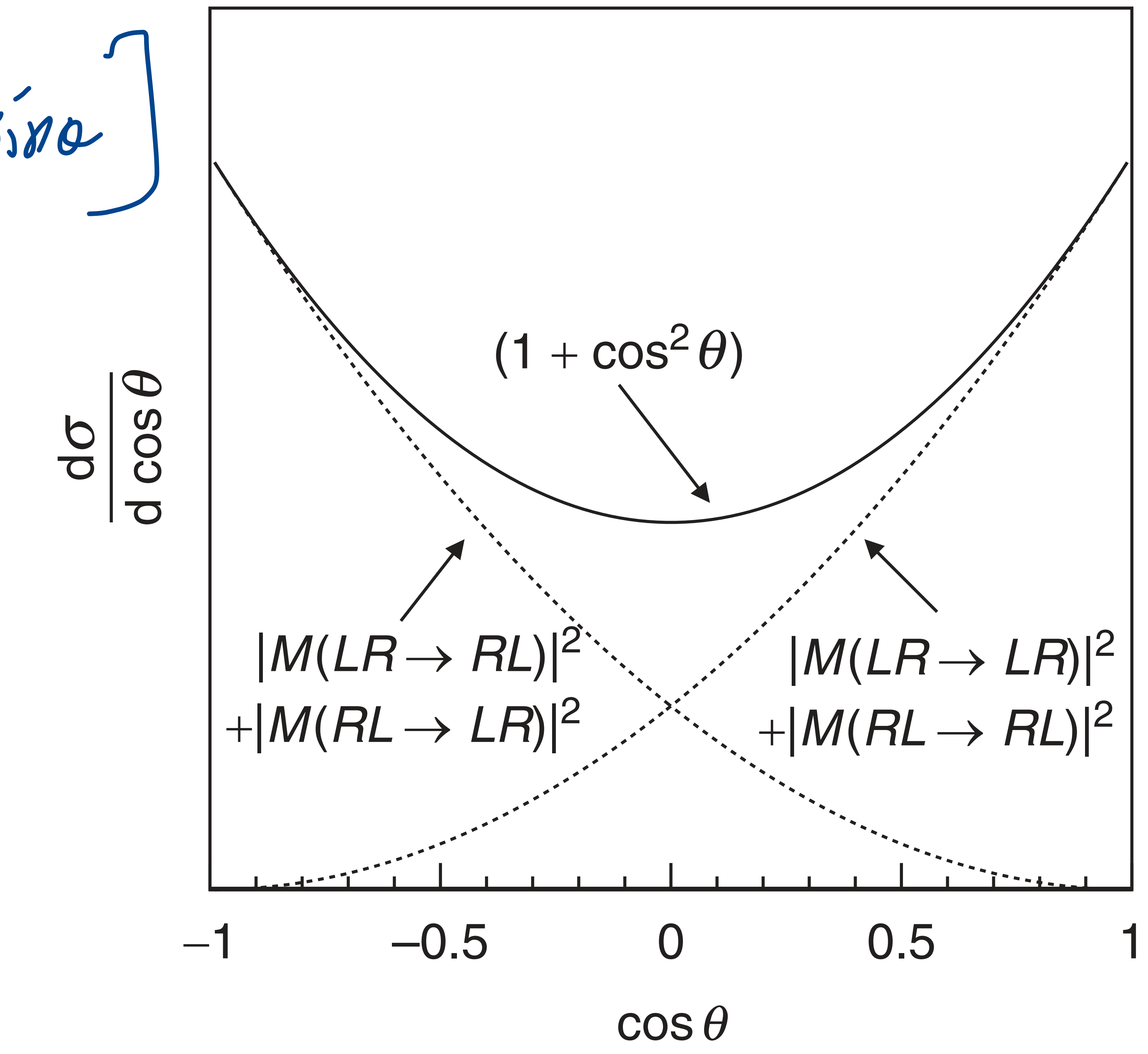
$$= -\frac{e^2}{s}\left[2E(0,-1,-i,0)\right]\cdot\left[2E(0,-\cos\theta, i, \sin\theta)\right]$$

$$= 4\pi\alpha(1+\cos\theta)$$

$$\langle |M_{fi}|^2\rangle = e^4(1+\cos^2\theta)$$

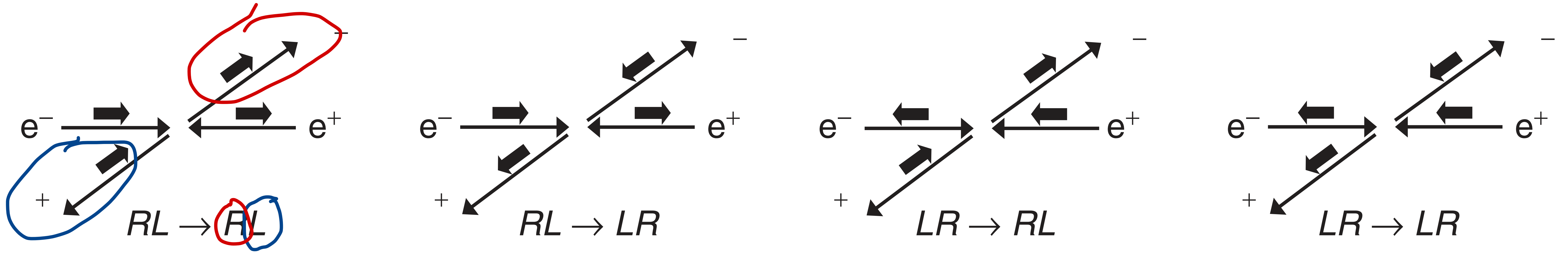
$$\frac{\partial\sigma}{\partial\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

Thomson Fig. 6.7



Chirality

- These are the 4 (out of 16) helicity combinations for $e^+e^- \to \mu^+\mu^-$ which give $\mathscr{M} \neq 0$



Thomson Fig. 6.6

- ***Not due to chance!***

- Reflects the chiral structure of QED.

$\cancel{u_R}$, $u_\uparrow$

$\cancel{u_L}$, $u_\downarrow$

The eigenstates of the γ^5- matrix are **defined** as left- and right-handed *chiral* states

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

Chirality

- Chiral **states** are denoted with subscripts $R,\ L$

$$\gamma^5 u_R = +\,u_R,\ \ \gamma^5 u_L = -\,u_L$$

$$\gamma^5 v_R = -\,v_R,\ \ \gamma^5 v_L = +\,u_L$$

With this convention, the **chiral** eigenstates = the **helicity** eigenstates when $E \gg m$

General solutions to the Dirac equation, which are also eigenstates of γ^5

$$u_R \equiv N\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix},\ u_L \equiv N\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix},\ v_R \equiv N\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix} \text{ and } v_L \equiv N\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix},$$

$N = \sqrt{E+m}$

Chiral projection operators

- Define chiral projection operators: $$P_R = \frac{1}{2}(1+\gamma^5),$$ $$P_L = \frac{1}{2}(1-\gamma^5).$$
- Used to decompose Dirac spinors into right- and left-handed chiral components
- P_R projects our *right-handed* chiral *particle* states and *left-handed* antiparticle *states*
  - $P_R u_R = u_R$, $P_R u_L = 0$, $P_R v_R = 0$, and $P_R v_L = v_L$
- P_L projects our *left-handed* chiral *particle* states and ~~*left*~~ right *-handed* antiparticle *states*
  - $P_L u_R = 0$, $P_L u_L = u_L$, $P_L v_R = v_R$, and $P_L v_L = 0$

$$\underset{\text{any spinor}}{u} = \underbrace{a_R u_R + a_L u_L}_{\text{coeffs complex}}$$

Helicity & chirality

- Helicity eigenstates **defined by** the projection of the spin of a particle onto its direction of motion
- Chiral states **defined as** the eigenstates of the γ^5 matrix
- What is the relationship between their eigenstates?

Start with the right-handed **helicity** spinor (L03, S25)

RH helicity particle spinor

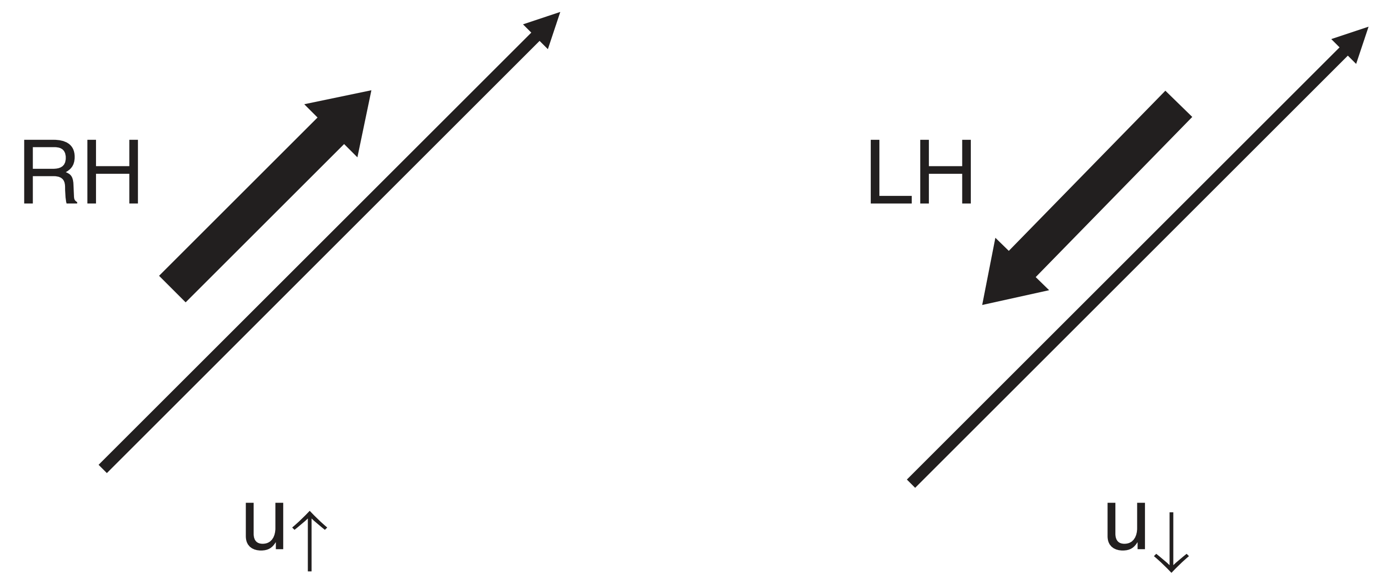
$$u_\uparrow = \sqrt{E+m}\begin{pmatrix} c \\ s e^{i\phi} \\ \frac{p}{E+m} c \\ \frac{p}{E+m} s e^{i\phi} \end{pmatrix}$$

LH helicity particle spinor

$$u_\downarrow = \sqrt{E+m}\begin{pmatrix} -s \\ c e^{i\phi} \\ \frac{p}{E+m} s \\ -\frac{p}{E+m} c e^{i\phi} \end{pmatrix},$$

$$\left.\begin{array}{l} N = \sqrt{E+m} \\ k = \dfrac{p}{E+m} \end{array}\right\} \quad u_\uparrow = N \begin{pmatrix} c \\ s e^{i\phi} \\ k c \\ k s e^{i\phi} \end{pmatrix}$$

Particles



Helicity & chirality

- Decompose the right-handed helicity spinor into left- and right-handed chiral components

$$P_R u_\uparrow = \frac{1}{2}(1+\gamma^5)u_\uparrow = \frac{1}{2}N\left(I + \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\right)u_\uparrow$$

$$= \frac{1}{2}N\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} c \\ se^{i\phi} \\ kc \\ kse^{i\phi} \end{pmatrix}$$

$$= \frac{1}{2}(1+k)N\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}$$

P_R: chirality, $u_\uparrow$: helicity

$$P_L u_\uparrow = \frac{1}{2}(1-k)N\begin{pmatrix} c \\ se^{i\phi} \\ -c \\ -se^{i\phi} \end{pmatrix}$$

$$\Rightarrow \underline{u_\uparrow}(p,\theta,\phi) = \frac{1}{2}(1+k)N\begin{pmatrix} \; \end{pmatrix} + \frac{1}{2}(1-k)N\begin{pmatrix} \; \end{pmatrix}$$

$$\propto \frac{1}{2}(1+k)u_R + \frac{1}{2}(1-k)u_L$$

only when $E \gg 1$ $(k \to 1)$, chiral Es = helicity Es.

Chiral nature of QED

- On S24, we said the fact that only 4 (out of 16) helicity combinations give $\mathscr{M} \neq 0$ was due to the chiral nature of QED

$\psi = u(E, \vec{p}) e^{i(\vec{p}\cdot\vec{x} - Et)}$

- How do we see this?

- Start with our 4-vector current and decompose it into contributions from left- and right-handed chiral states using the chiral projection operators (as we just did for $u_\uparrow$):

$$\begin{aligned}\overline{\psi}\gamma^\mu\phi &= (a_R^*\overline{\psi}_R + a_L^*\overline{\psi}_L)\gamma^\mu(b_R\phi_R + b_L\phi_L) \\ &= a_R^* b_R \overline{\psi}_R \gamma^\mu \phi_R + a_R^* b_L \overline{\psi}_R \gamma^\mu \phi_L + a_L^* b_R \overline{\psi}_L \gamma^\mu \phi_R + a_L^* b_L \overline{\psi}_L \gamma^\mu \phi_L\end{aligned}$$

- Work through it (Sec. 6.4.1) and you'll see that only certain combinations of chiral eigenstates give $\neq 0$

Chirality summary

- Similar to, but **more abstract** *(pro-tip: think of chirality always as an abstract tool!)* than helicity: determined by whether the particle transforms in a right- or left-handed representation of the Poincaré group (important property in electroweak interactions: chirality is conserved)
- Chirality projection operators: $P_L = \frac{1}{2}(1-\gamma^5)$ and $P_R = \frac{1}{2}(1+\gamma^5)$
- Any spinor can be decomposed into left- and right-handed components, u and v are chirality Eigenstates ($\gamma^5 u = \pm u$)
- **Lorentz-invariant** (but not a constant of motion)
- A particle with right-handed chirality can have left or right-handed helicity, depending on the reference frame

Reading assignment

- Modern particle physics (Mark Thomson)
  - Chap. 2
    - 2.3.6
  - Chap. 3 (complete). Should be mostly review from BSc.
  - Chap. 5 (complete)
  - Chap. 6
    - 6.1-6.4
    - 6.5* (extra reading, not covered in lecture yet)

What questions do you have?