

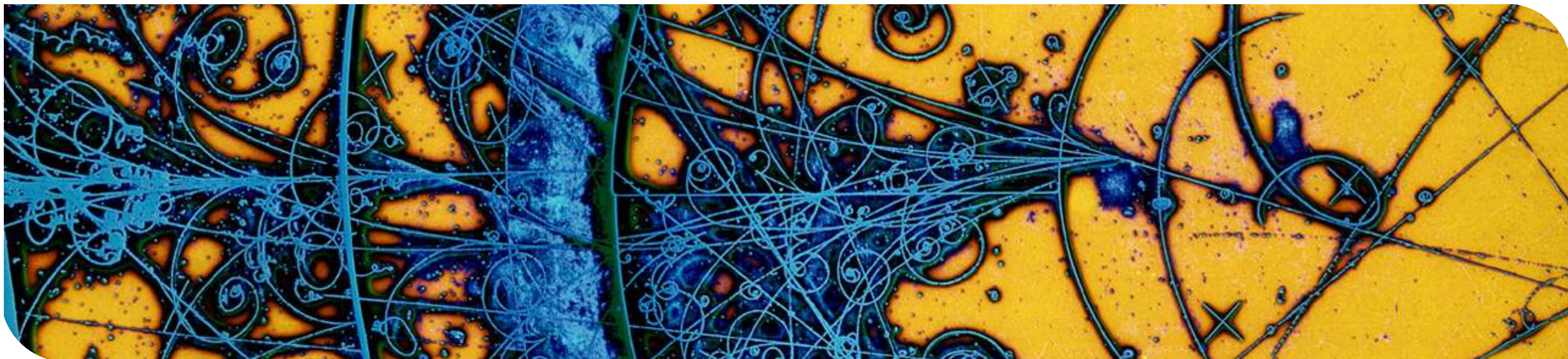
# Particle Physics 1

## Lecture 5: Symmetries & QCD (Theory)

Prof. Dr. Markus Klute ([markus.klute@kit.edu](mailto:markus.klute@kit.edu)), Dr. Pablo Goldenzweig ([pablo.goldenzweig@kit.edu](mailto:pablo.goldenzweig@kit.edu))

Institute of Experimental Particle Physics (ETP)

Winter 2024/2025

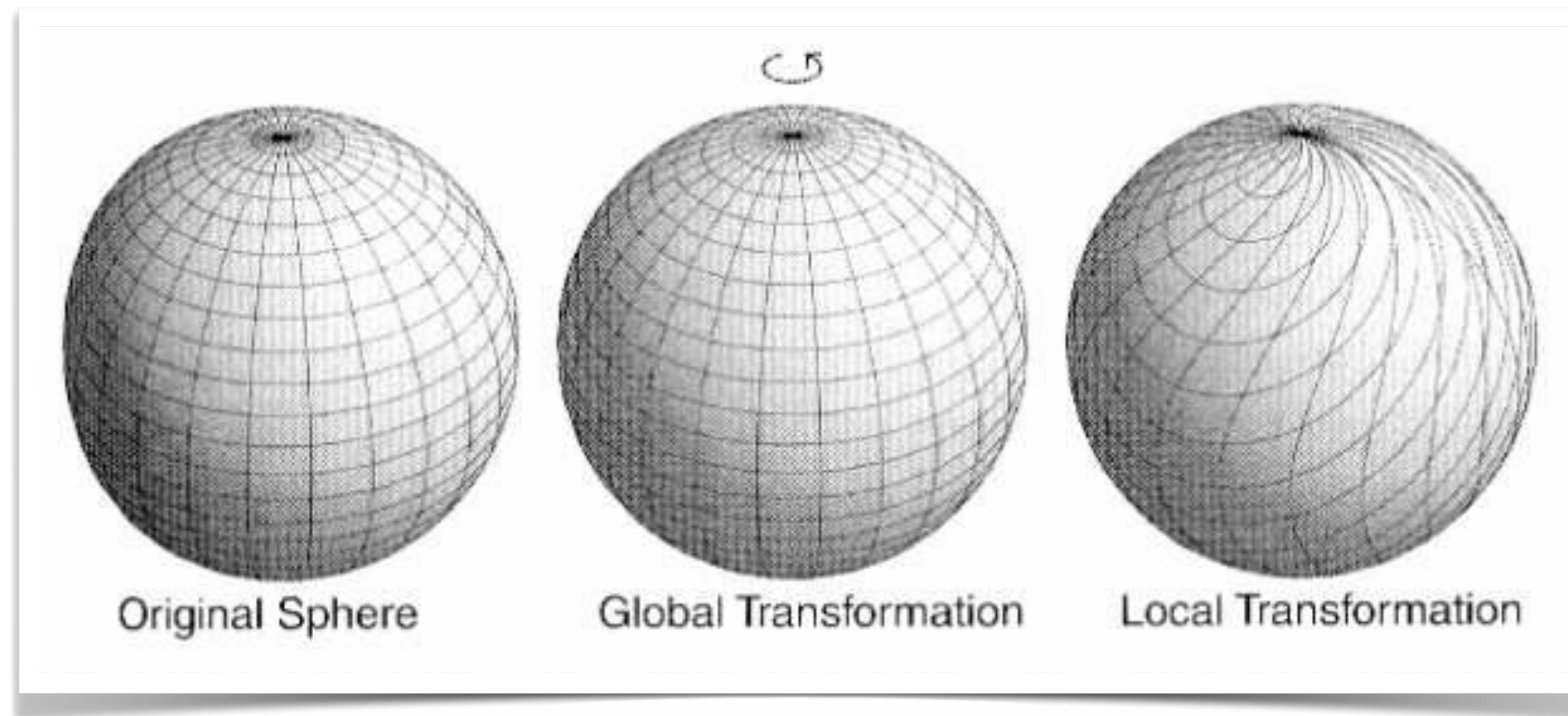


Credit: CERN

# Questions from past lectures

# Symmetries

- Symmetries are operations performed on a system that leaves it invariant
  - **Global** symmetries are the **same** at all space-time points
  - **Local** symmetries are **different** at different space-time points



Source: <https://universe-review.ca/l15-04-gauge2.jpg>

# Symmetries

- Requires that all physical predictions are invariant under the wavefunction transformation  $\psi \rightarrow \psi' = \hat{U}\psi$
- For physical predictions to be unchanged, wavefunction normalizations must remain unchanged

$$\langle \psi | \psi \rangle = \langle \psi' | \psi' \rangle = \langle \hat{U}\psi | \hat{U}\psi \rangle = \langle \psi | \hat{U}^\dagger \hat{U} | \psi \rangle$$

$$\hat{U}^\dagger \hat{U} = \mathbb{I} \quad \therefore \hat{U} \text{ is unitary}$$

- The eigenstates of the system must also remain unchanged

$$\hat{H} \rightarrow \hat{H}' = \hat{H}$$

$\hat{H}$  must possess the symmetry in question & eigenstates must satisfy

$$\hat{H}\psi_i = E_i \psi_i \quad \Rightarrow \quad \hat{H}'\psi'_i = E_i \psi'_i$$

Energies of transformed eigenstates  $\psi'_i$  will be unchanged

$$\begin{aligned} \hat{H}'\hat{U}\psi_i &= E_i \hat{U}\psi_i = \hat{U}E_i\psi_i \\ &= \hat{U}\hat{H}\psi_i \end{aligned}$$

$$\therefore \hat{H}'\hat{U}\psi_i = \hat{U}\hat{H}\psi_i$$

$$[\hat{H}', \hat{U}] = \hat{H}'\hat{U} - \hat{U}\hat{H}' = 0$$

# Symmetries

- A finite continuous symmetry operation can be built up from a series of infinitesimal transformations  $\hat{U}(\epsilon) = I + i\epsilon\hat{G}$

$\hat{U}(\epsilon)$   $\nearrow$  unitary     
 $\epsilon$   $\nearrow$  infinitesimal parameter     
 $\hat{G}$   $\nearrow$  Generator of the transformation

$$\hat{U}(\epsilon)\hat{U}^\dagger(\epsilon) = (I + i\epsilon\hat{G})(I - i\epsilon\hat{G}) = I + i\epsilon(\underbrace{\hat{G} - \hat{G}^\dagger}_{=0}) + \cancel{\mathcal{O}(\epsilon^2)}$$

$\Rightarrow \boxed{\hat{G} = \hat{G}^\dagger}$  Hermitian

- For each symmetry of  $\hat{H}$ , there is a corresponding unitary symmetry operator  $\hat{U}$ , with an associated Hermitian generator  $\hat{G}$
- The eigenstates of a Hermitian operator are real  $\therefore \hat{G}$  is associated w/ an observable quantity  $G$ .
- $[\hat{H}, I - i\epsilon\hat{G}] = 0 \Rightarrow [\hat{H}, \hat{G}] = 0$

Time evolution of the expectation value  $\frac{d}{dt}\langle \hat{G} \rangle = i\langle [\hat{H}, \hat{G}] \rangle = 0$

$\Rightarrow$  for each sym. of  $\hat{H}$ , there is an observable conserved quantity

# A simple example

- Translational invariance in 1 dimension  $x \rightarrow x + \epsilon$

↑ infinitesimal distance  $x_0$

$$\psi'(x) = \psi(x + \epsilon) = \psi(x) + \frac{\partial \psi}{\partial x} \epsilon + \mathcal{O}(\epsilon^2)$$

$$\psi'(x) = \left(1 + \epsilon \frac{\partial}{\partial x}\right) \psi$$

$$\Downarrow \hat{p}_x = -i \frac{\partial}{\partial x}$$

$$\psi'(x) = (1 + i \epsilon \hat{p}_x) \psi(x)$$

↑ generator of the sym.  
transformation  $x \rightarrow x + \epsilon$

$$\vec{G} = \{G_i\}$$

$$\vec{U}(\vec{\epsilon}) = 1 + i \vec{\epsilon} \cdot \vec{G}$$

$$\vec{x} \rightarrow \vec{x} + \vec{\epsilon}$$

3D spatial trans  $\Rightarrow \vec{p} = (\hat{p}_x, \hat{p}_y, \hat{p}_z)$   
associated w/ the generators

Translational invariance of  $\hat{H}$  implies momentum conservation

# Finite transformations

- Any finite symmetry transformation can be expressed as a series of infinitesimal transformations

$$\hat{U}(\alpha) = \lim_{n \rightarrow \infty} \left( 1 + i \frac{1}{n} \alpha \cdot \hat{\mathbf{G}} \right)^n = \exp(i\alpha \cdot \mathbf{G})$$

$\underline{E_x}: x \rightarrow x + \epsilon$

$$\hat{U}(x_0) = e^{i x_0 \hat{p}_x} = e^{x_0 \frac{\partial}{\partial x}}$$

$$\psi'(x) = \hat{U} \psi(x) = \exp\left[x_0 \frac{\partial}{\partial x}\right] \psi(x) = 1 + x_0 \frac{\partial}{\partial x} + \frac{x_0^2}{2!} \frac{\partial^2}{\partial x^2} + \dots$$

$$\psi(x) \rightarrow \psi'(x) = \hat{U}(x_0) \psi = \psi(x + x_0)$$

$$e^x = \lim_{n \rightarrow \infty} \left( 1 + \frac{x}{n} \right)^n$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

taylor series expansion

# Flavor symmetries

- Extend the idea of isospin to the up- and down-quarks.

①  $u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$   $d = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$  , isospin doublet

$$\hat{U} = 1 + i\epsilon \hat{T}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} e^{i\phi}$$

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = \hat{U} \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

4 complex #s  $\Rightarrow$  8 real numbers

Select the pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\vec{T} = \frac{1}{2} \vec{\sigma}$$

$$\hat{U} = e^{i\vec{\alpha} \cdot \vec{T}}$$

$$U^\dagger U = I \Rightarrow 4 \text{ parameters}$$

= 4 linearly independent  $2 \times 2$  matrices

represent the generators of the trans.

$$\hat{U} = e^{i\alpha_i \hat{T}_i}$$

# A little group theory

- A **group** is a set of objects  $G$  on which some “multiplication” operation is defined, s.t.:
  - If  $a$  and  $b$  are in  $G$ ,  $a \cdot b$  is in  $G$
  - There is an identity element in  $G$  s.t.  $a \cdot i = a$  for any  $a$  in  $G$
  - For any  $a$  in  $G$ , there is an “inverse” element  $G$  s.t.  $a \cdot a^{-1} = i$
  - For  $a, b, c$  in  $G$ ,  $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
- A **Lie group** is a group of unitary operators in which the group elements are labeled by a set of continuous parameters
  - Example: the elements  $e^{i\alpha_a X_a}$  ( $a$  summed) under multiplication.
    - The  $\alpha_a$  are the set of continuous parameters, the  $X_a$  are operators.
    - The  $X_a$  are called **generators** because exponentiating them generates the group

# SU(3) flavor symmetry

- Extend the SU(2) flavor symmetry to include the  $s$  quark

$$\begin{pmatrix} u' \\ d' \\ s' \end{pmatrix} = \hat{U} \begin{pmatrix} u \\ d \\ s \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$m_s - m_{u/d} \approx 100 \text{ MeV} \ll \text{typical binding energy of baryons (1 GeV)}$



16 real parameters

$U^\dagger U = 1 \Rightarrow 9 \text{ real parameters or } 9 \text{ linearly indep. } 3 \times 3 \text{ matrices}$

$\Rightarrow I e^{i\phi}$ , global phase between flavor states

The remaining 8 form an SU(3) group

$\Rightarrow$  expressed in terms of 8 indep Hermitian generators

$$U = e^{i\vec{\alpha} \cdot \vec{T}}$$

$$\vec{T} = \frac{1}{2} \vec{\lambda}$$

Action + the SU(3) representation of the  $u, d, s$  quarks

$$u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, d = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, s = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

# SU(3) flavor symmetry

- SU(3)  $uds$  flavor symmetry contains the subgroup of SU(2)  $u \leftrightarrow d$  flavor symmetry

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$u \leftrightarrow d$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \text{and} \quad \lambda_X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$su(2) \quad u \leftrightarrow s$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \text{and} \quad \lambda_Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$su(2) \quad d \leftrightarrow s$

since  $m_u \approx m_d$ , keep  $\lambda_3$  as one of the generators

$$\lambda_8 = \frac{1}{\sqrt{3}} \lambda_X + \frac{1}{\sqrt{3}} \lambda_Y$$

Treats  $u$  and  $d$  quarks symmetrically

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

# Hypercharge

- For the case of SU(2), we defined 3 Hermitian generators, each of which corresponds to an observable quantity

$$\hat{T} = \frac{1}{2} \vec{\sigma} \quad [T_i, T_j] \neq 0, \quad \left. \begin{aligned} \hat{T}^2 &= \hat{T}_1^2 + \hat{T}_2^2 + \hat{T}_3^2 \\ \hat{T}_3 \end{aligned} \right\} \text{SU(2) states defined in terms of their eigenstates.}$$

- In SU(3) there is an analogue of total isospin:

$$\hat{T}^2 = \sum_{i=1}^8 \hat{T}_i^2 = \frac{1}{4} \sum_{i=1}^8 \lambda_i^2 = \frac{4}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

only  $\hat{T}_3 = \frac{1}{2} \lambda_3$  &  $\hat{Y} = \frac{1}{2} \lambda_8$  commute  
 $\therefore$  describe compatible observable quantities

Corresponding quantum #s  
 $3^{\text{rd}}$  component of isospin  $\hat{T}_3 = \frac{1}{2} \lambda_3$   
 $\hat{Y} = \frac{1}{\sqrt{3}} \lambda_8$  } additive quantum #s which specify the flavor of the state

# Take 5



# Local gauge principle

- Recall gauge invariance form electromagnetism

$$\left. \begin{array}{l} \text{scalar } \phi \rightarrow \phi' : \phi - \frac{\partial \mathcal{K}}{\partial t} \\ \text{vector } \vec{A} \rightarrow \vec{A}' : \vec{A} + \vec{\nabla} \mathcal{K} \end{array} \right\} A_\mu \rightarrow A'_\mu : A_\mu - \partial_\mu \mathcal{K}$$

$\Rightarrow$  Fields are invariant under this gauge transformation

- In relativistic QM, the gauge invariance of EM can be related to a local gauge principle

$$U(1) \text{ global phase trans. } \psi \rightarrow \psi' = e^{i\phi} \psi$$

$$\text{Local: } \psi(x) \rightarrow \boxed{\psi'(x)} = \hat{U} \psi(x) = \boxed{e^{i g \mathcal{K}(x)} \psi(x)}$$

# Local gauge principle

- For this local U(1) phase transformation, the free-particle Dirac equation  $i\gamma^\mu \partial_\mu \psi = m\psi$  becomes

$$\Downarrow$$

$$i\gamma^\mu \partial_\mu (e^{iq\chi(x)} \psi) = m e^{iq\chi(x)} \psi$$

$$\Downarrow$$

$$i\gamma^\mu [\partial_\mu + \underbrace{iq \partial_\mu \chi(x)}_{\text{now has an extra term which must be "cancelled"}}] \psi = m \psi$$

now has an extra term  
which must be "cancelled"

The only way is to add a new degree of freedom  $A_\mu$

$$i\gamma^\mu (\partial_\mu + iq A_\mu) \psi = m \psi$$

$\Downarrow$   
invariant under  $\psi(x) \rightarrow \psi'(x) = e^{iq\chi(x)} \psi(x)$  ✓  
U(1)

$A_\mu$ : field which corresponds to a  
massless gauge boson.

$$i\gamma^\mu iq A_\mu = -g \gamma^\mu A_\mu$$

form of the  
perturbation  
from min. sub.

# Local gauge principle

- The requirement that physics is invariant under local U(1) phase transformations implies the existence of a gauge field which couples to Dirac particles in **exactly** the same way as the photon.
- All of QED can be derived by requiring the invariance of physics under local U(1) transformations of the form  $\hat{U} = \exp[iq\chi(x)]$

Thomson, Sec. 10.1

# QED → QCD

- QED corresponds to a local U(1) gauge symmetry of the universe
- The QFT of the strong interaction is invariance under SU(3) local phase transformations

$$\psi(x) \rightarrow \psi'(x) = \exp \left[ ig_S \alpha(x) \cdot \hat{\mathbf{T}} \right] \psi(x)$$

↑  
Sm. of space  
time coords.

↑  
generators

↑  $\hat{\mathbf{T}} = \frac{1}{2} \vec{\lambda}$   
 $\hat{\mathbf{T}} = \{ T^a \}$  The 8 generators of  
SU(3) sym group.

- Since the generators of SU(3) are represented by  $3 \times 3$  matrices, the wavefunction must now include 3 additional degrees of freedom represented by a 3 component vector

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

new degree of freedom  $\Rightarrow$  color

# Dirac equation becomes

- As before, substitute the expression for  $\psi'$  into the free-particle Dirac equation  $i\gamma^\mu \partial_\mu \psi = m\psi$  and you obtain:

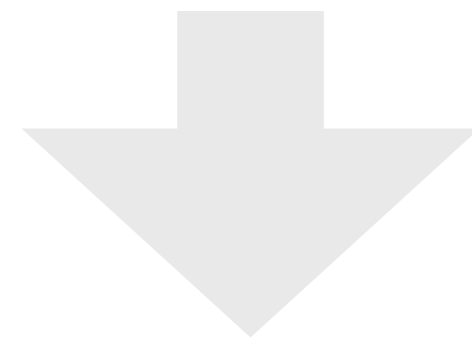
$$i\gamma^\mu \left[ \partial_\mu + ig_S (\overset{\rightarrow \alpha(x)}{\partial_\mu \alpha}) \cdot \hat{\mathbf{T}} \right] \psi = m\psi.$$

- The **required** local gauge invariance can be asserted by introducing 8 new fields (just as we introduced the new  $A_\mu$  photon field in U(1) QED)

$G_\mu^a(x)$ ,  $a=1,\dots,8$  each correspond to one of the 8 generators of  $su(3)$

# Interactions with the new gauge field

$$i\gamma^\mu \left[ \partial_\mu + ig_S (\partial_\mu \alpha) \cdot \hat{\mathbf{T}} \right] \psi = m\psi.$$



Dirac eqn including  
the interaction w/ the  
new gauge fields

$$i\gamma^\mu \left[ \partial_\mu + ig_S G_\mu^a T^a \right] \psi - m\psi = 0,$$

Invariant under  $SU(3)$  phase trans  
if the new fields transform as

$$G_\mu^k \rightarrow G_\mu^{k'} = G_\mu^k - \partial_\mu \alpha_k - \underbrace{g_S f_{ijk} \alpha_i G_\mu^j}_{\text{Since structure constants of } SU(3)}$$

Appears since the generators  
in  $SU(3)$  don't commute. Gives  
rise to gluon self interaction

# Summary (so far)

## QED

### U(1)

$$\psi(x) \rightarrow \psi'(x) = \hat{U}(x)\psi(x) = e^{iq\chi(x)}\psi(x).$$

Symmetry of the universe that requires the invariance of physics under local phase transformations

Substitute  $\psi'$  into the free particle Dirac eq.  
 $i\gamma^\mu \partial_\mu \psi = m\psi$

Modify  $i\gamma^\mu \partial_\mu \psi = m\psi$  to include a new degree of freedom **to re-establish the required invariance**

These equations are invariant under the local phase transformations provided that the new field transforms as:

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu \chi,$$

↑ mediated by massless photon  
 corresponding to the single generator  
 of the U(1) local gauge sym  
 single charge

antiparticles have opposite electrical charge

## QCD

### SU(3)

$$\psi(x) \rightarrow \psi'(x) = \exp \left[ ig_S \alpha(x) \cdot \hat{\mathbf{T}} \right] \psi(x)$$

Neither possess the hypothesized invariance under U(1) & SU(3) local phase trans.

$$i\gamma^\mu (\partial_\mu + iq\partial_\mu \chi) \psi = m\psi, \quad i\gamma^\mu \left[ \partial_\mu + ig_S (\partial_\mu \alpha) \cdot \hat{\mathbf{T}} \right] \psi = m\psi.$$

$$i\gamma^\mu (\partial_\mu + iqA_\mu) \psi - m\psi = 0,$$

$$i\gamma^\mu \left[ \partial_\mu + ig_S G_\mu^a T^a \right] \psi - m\psi = 0,$$

$$G_\mu^k \rightarrow G_\mu^{k'} = G_\mu^k - \partial_\mu \alpha_k - g_S f_{ijk} \alpha_i G_\mu^j.$$

8 massless gluons

3 conserved color charges

antiquarks have opposite color charge  $\bar{r}, \bar{g}, \bar{b}$

# QED

U(1)

Interaction term

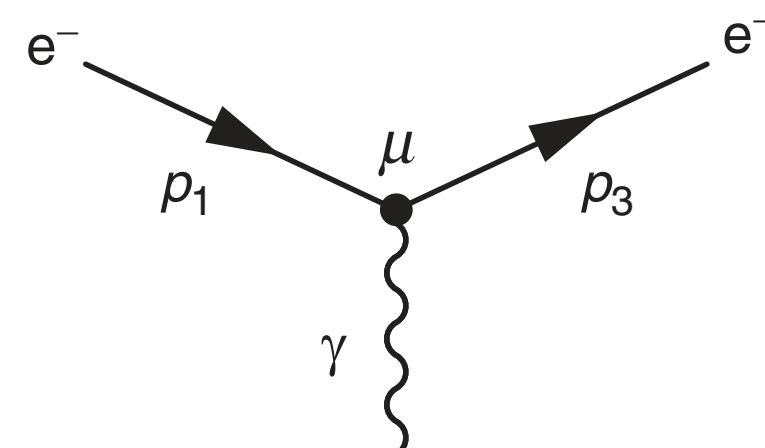
$$-iq\gamma^\mu A_\mu\psi$$

Vertex factor

$$-iq\gamma^\mu$$

4-vector current

$$j_e^\mu = \bar{u}_e(p_3)\gamma^\mu \underline{u_e(p_1)}$$



# QCD

SU(3)

$$-ig_s\frac{1}{2}\lambda^a\gamma^\mu G_\mu^a\psi$$

$$-ig_s\gamma^\mu\frac{1}{2}\lambda^a$$

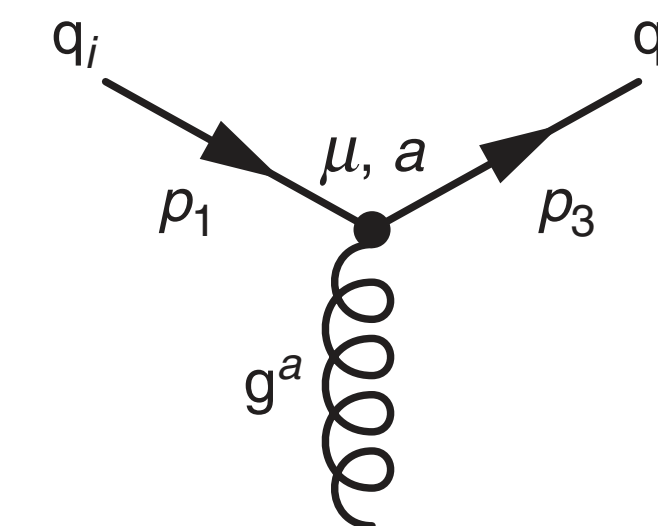
Handwritten annotations: A blue line points from the gamma matrix to the vertex factor. A purple line points from the lambda^a matrix to the vertex factor. A blue line points from the 1/2 factor to the vertex factor. A blue line points from the gamma matrix to the vertex factor.

$$j_q^\mu = \bar{u}(p_3)c_j^\dagger \left\{ -\frac{1}{2}ig_s\lambda^a\gamma^\mu \right\} c_i \underline{u(p_1)},$$

$$C_1 = r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$C_2 = g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

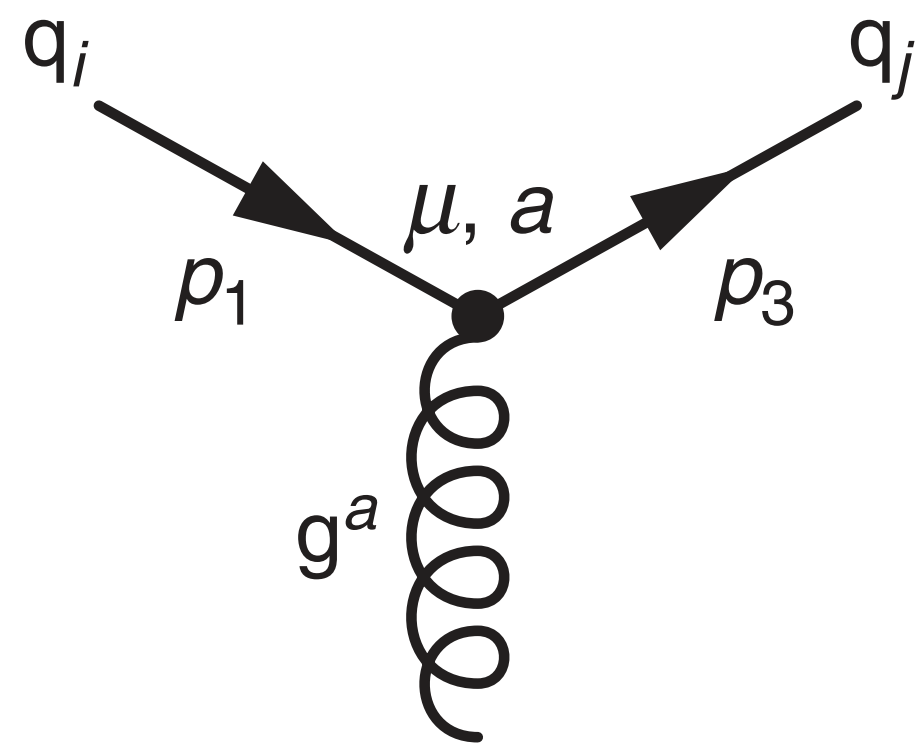
$$C_3 = b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



# Factorize the $qqg$ vertex

$j_q^\mu$  from last slide

$$\bar{u}(p_3)c_j^\dagger\{-\frac{1}{2}ig_s\lambda^a\gamma^\mu\}c_iu(p_1) = -\frac{1}{2}ig_s\left[c_j^\dagger\lambda^ac_i\right]\times[\bar{u}(p_3)\gamma^\mu u(p_1)]$$



color part

$$c_j^\dagger \begin{pmatrix} \lambda_{1i}^a \\ \lambda_{2i}^a \\ \lambda_{3i}^a \end{pmatrix} = \lambda_{ji}^a$$

This is a number.  
The  $ji^{\text{th}}$  element of  $\lambda^a$ .

# Gluons

- QCD interaction vertex includes a factor of  $\lambda_{ij}$ , where  $i$  &  $j$  label the colors of the quarks

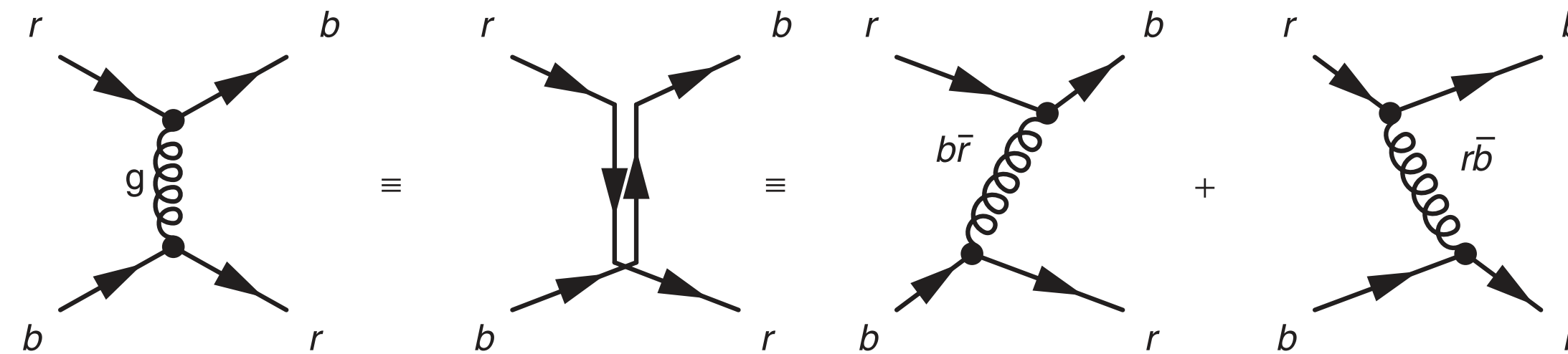
$\Rightarrow$  Gluons corresponding to the non-diagonal GM matrices connect quark states of diff. color

$\Rightarrow$  Gluons **must** carry color charge (for color to be conserved)

1 unit of color  
1 unit of anti-color

$T_3, T_8$

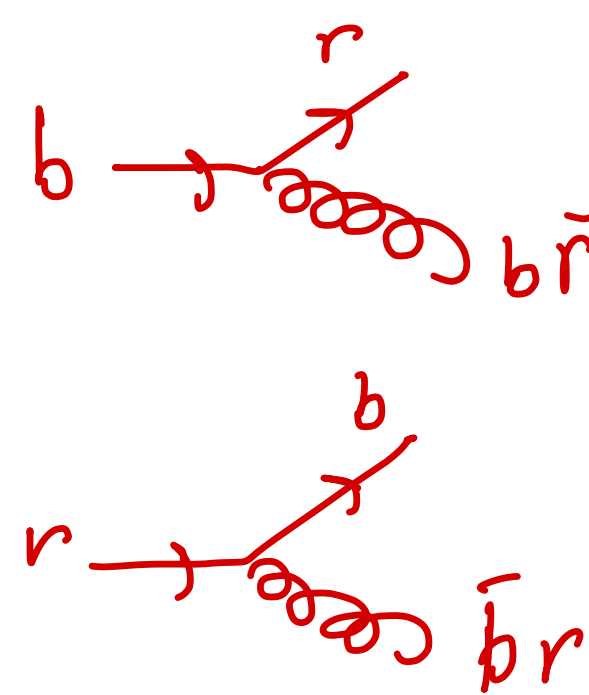
The 8 gluons correspond to these 8 linear combination of generators



$$T_4 \pm i T_5 = \frac{1}{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \pm \frac{i}{2} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

or

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$



Also have

$$T_1 \pm i T_2, T_3, T_8$$

$$T_6 \pm i T_7$$

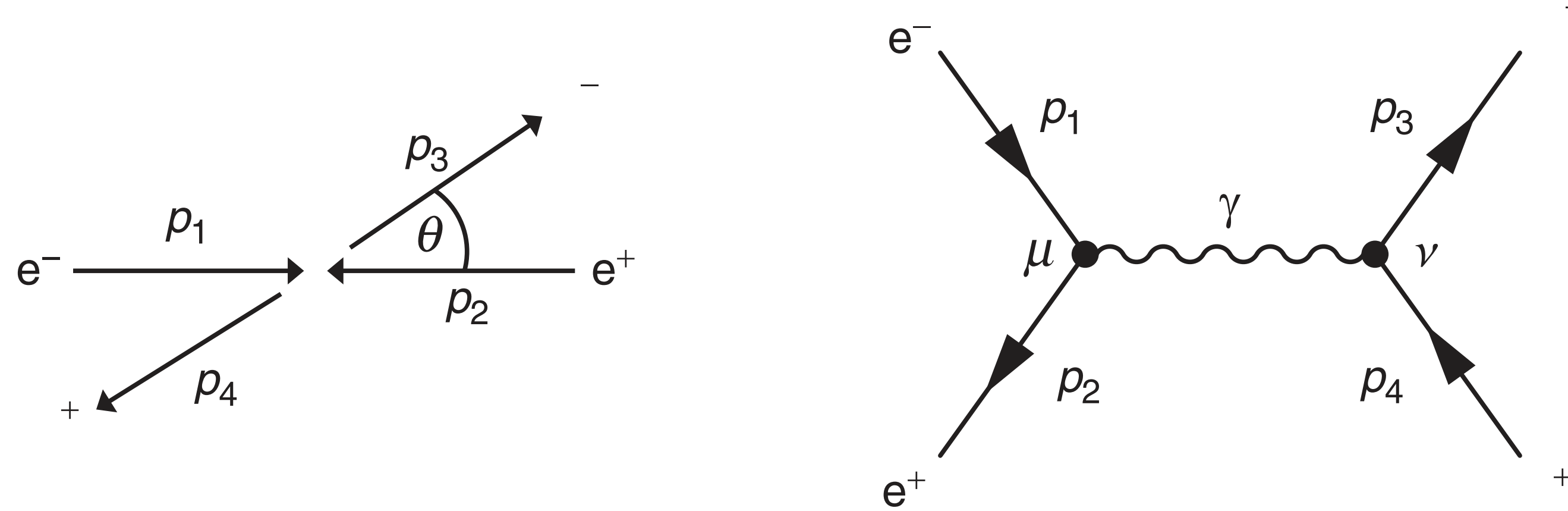
Gell-Mann matrices: The 8 matrices used to represent the **generators** of the SU(3) symmetry

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

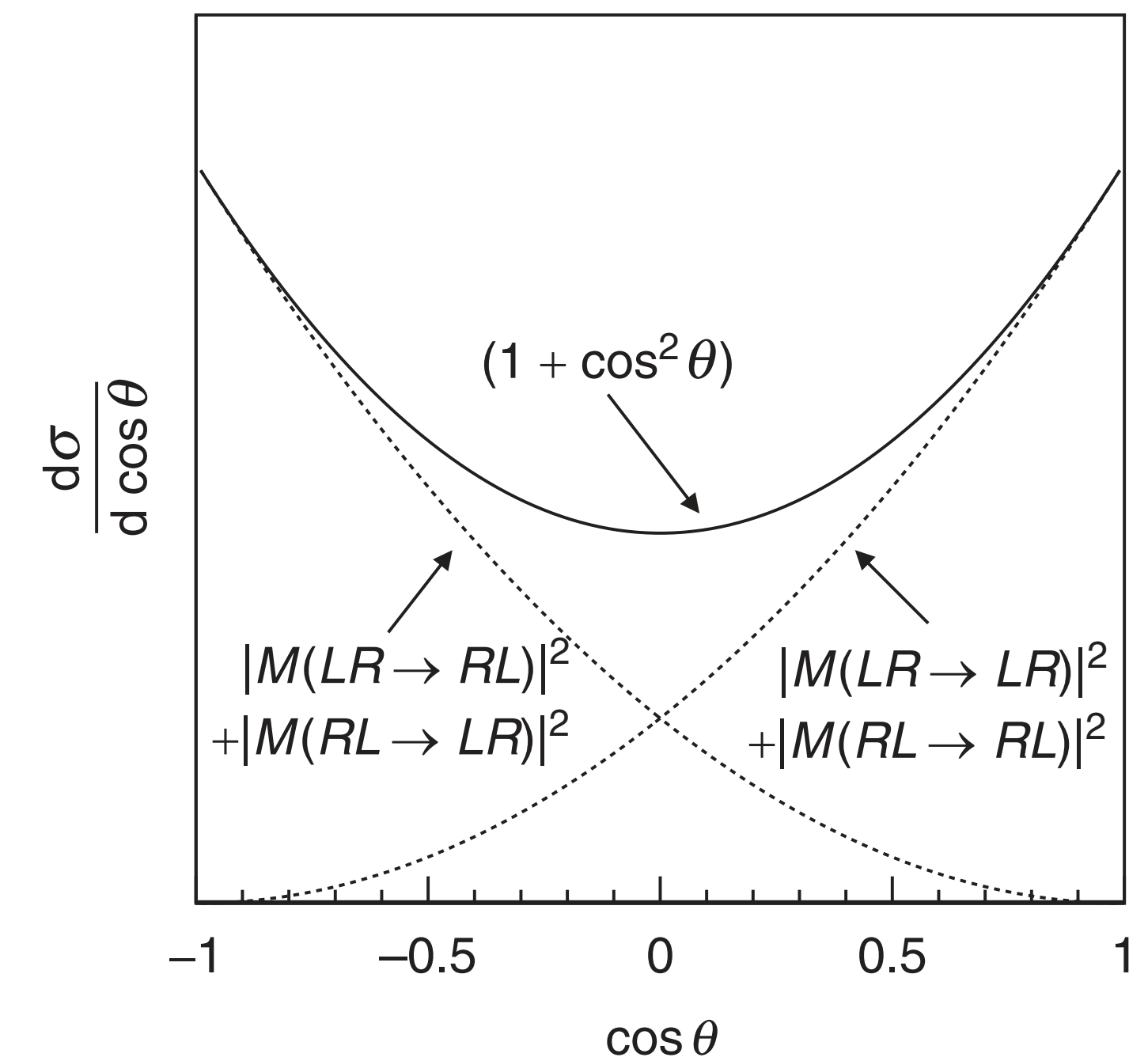
$$e^+e^- \rightarrow \mu^+\mu^-$$



$$\begin{aligned}\langle |\mathcal{M}_{fi}|^2 \rangle &= \frac{1}{4} \times (|\mathcal{M}_{RL \rightarrow RL}|^2 + |\mathcal{M}_{RL \rightarrow LR}|^2 + |\mathcal{M}_{LR \rightarrow RL}|^2 + |\mathcal{M}_{LR \rightarrow LR}|^2) \\ &= \frac{1}{4} e^4 [2(1 + \cos \theta)^2 + 2(1 - \cos \theta)^2] \\ &= e^4 (1 + \cos^2 \theta).\end{aligned}$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta).$$

Thomson Figs. 6.4, 6.7

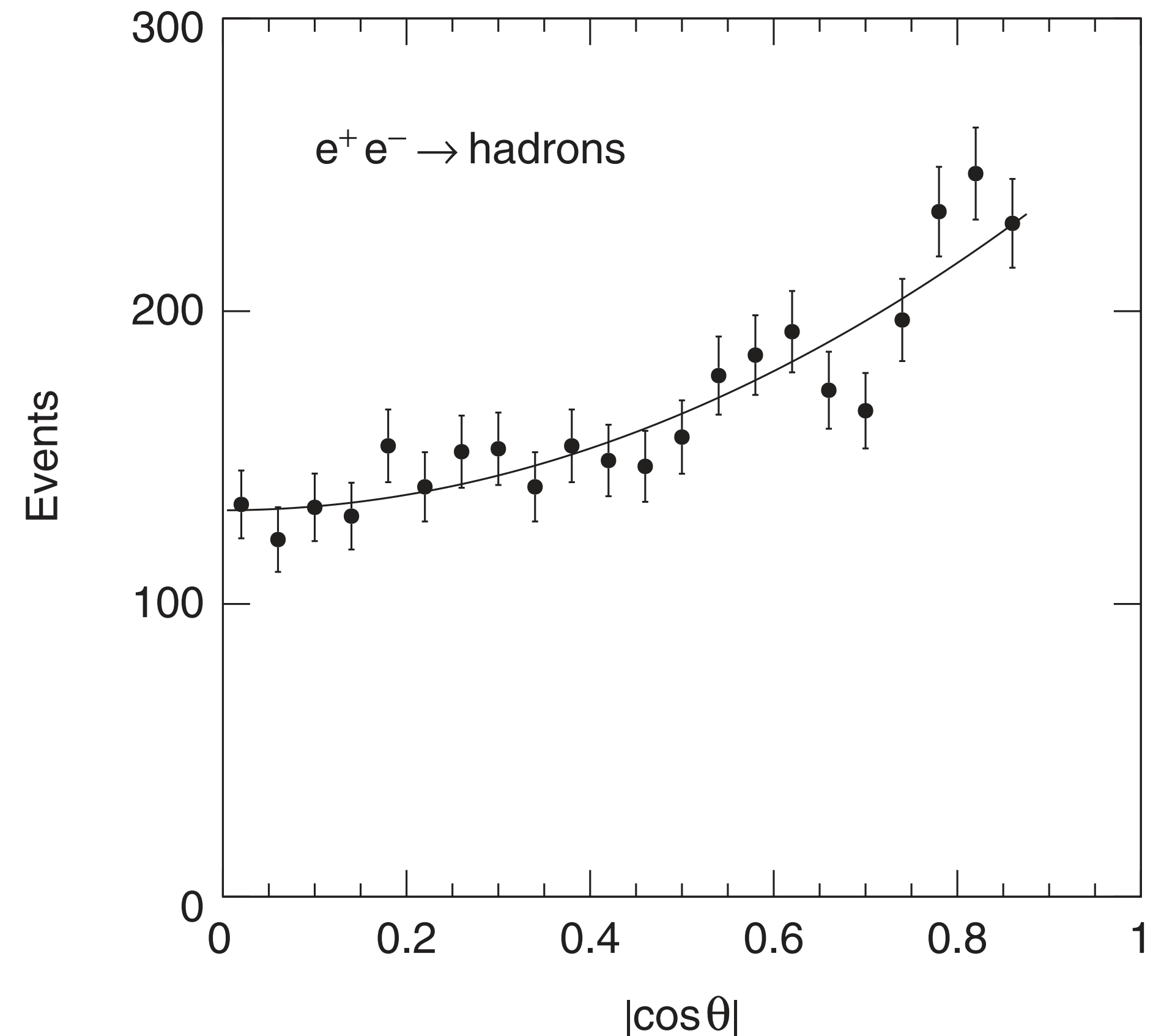


$$e^+e^- \rightarrow q\bar{q}$$

- Assuming quarks are spin-half particles, should have the same angular dependence

## However:

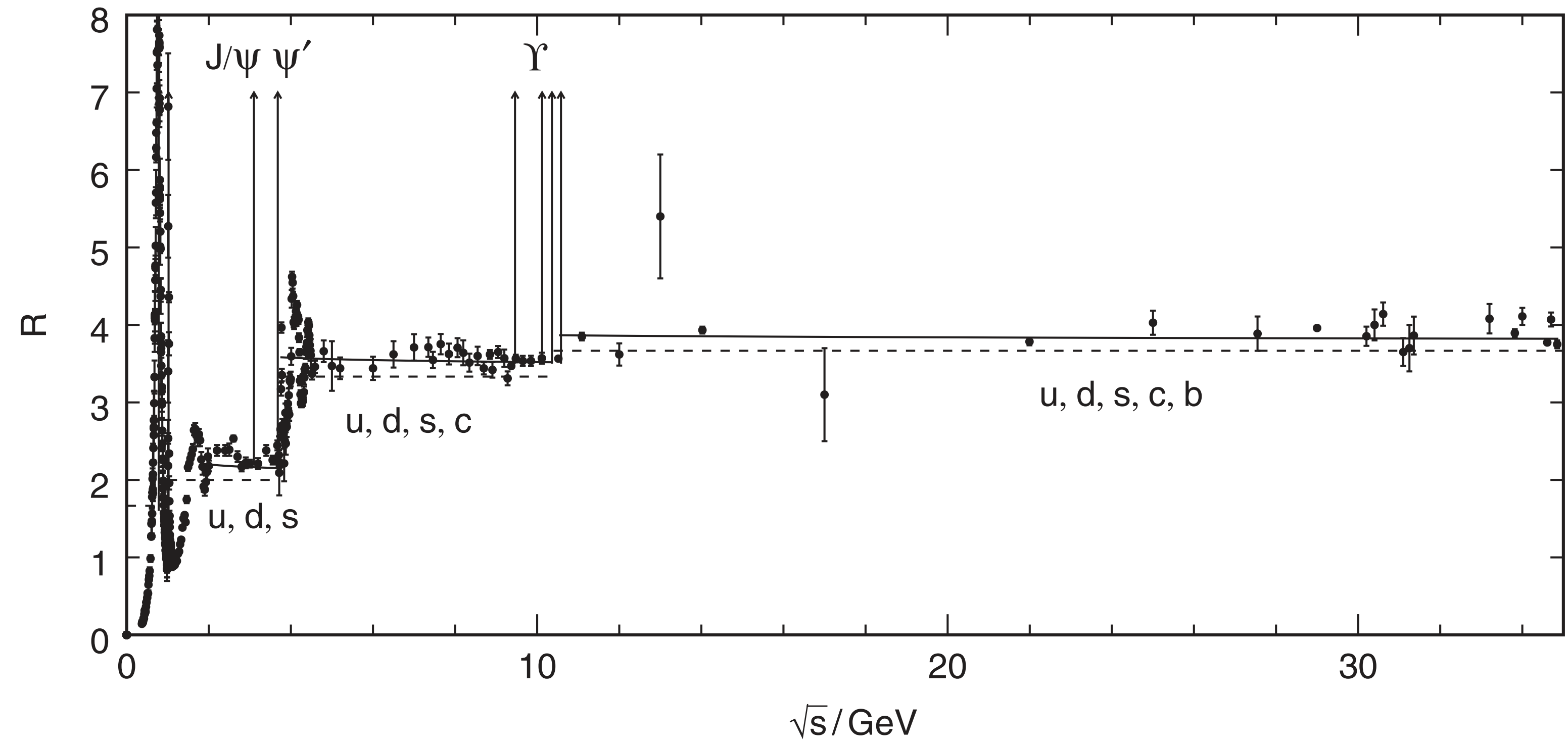
- Generally not possible to ID which flavor of quark was produced, so *express as an inclusive sum over all hadrons*  $e^+e^- \rightarrow \text{hadrons}$
- Usually not possible to ID which jet came from the quark and which from the antiquark, so quote in terms of  $|\cos \theta|$



# ***R*-ratio**

- Determine the number of colors from production and decay processes involving quarks. Compare production of hadrons and  $\mu^+\mu^-$  in  $e^+e^-$  scattering
- The total QED  $e^+e^- \rightarrow \mu^+\mu^-$  cross section
- For the inclusive QED production of hadrons

# $R$ -ratio





- PINGO:
  - Session: Particle Physics 1 (WS 24/25).
  - Accession number: 559016
  - Link: <https://pingo.coactum.de/events/559016>

- What happens to the  $R$ -ratio for energies above  $\sqrt{s} \gtrsim 350 \text{ GeV}$ ?
  - The  $R$ -ratio shows a sharp peak because  $t\bar{t}$  bound states appear like for all other quarks
  - The  $R$ -ratio shows a step and becomes  $R = 5$
  - Since there are no  $t\bar{t}$  bound states, there is neither a sharp peak nor a step

# Cooperative project seminar

## “Particle Physics meets Music Informatics”

KIT – Karlsruher Institut für Technologie, ETP – Institut für experimentelle Teilchenphysik / CERN, IMWI – Institut für Musikinformatik und Musikwissenschaft, Hochschule für Musik Karlsruhe

Prof. Dr. Markus Klute (KIT), Dr. Michael Hoch (CERN), Prof. Dr. Marc Bangert (IMWI), Prof. Dr. Christoph Seibert (IMWI), Daniel Höpfner (IMWI), Michele Samarotto (IMWI), Amir Teymuri (IMWI), Christophe Weis (IMWI)

Grundsätzliches Ziel dieses kooperativen Projektseminars ist es, einen Raum zur Begegnung und gegenseitigen Befruchtung wissenschaftlicher und künstlerischer Praktiken zu schaffen.

Um den Austausch zwischen den Beteiligten und (Teil-)Projekten zu fördern, sollen über das Semester hinweg regelmäßige Treffen, in Form eines Kolloquiums gesetzt werden.

Den Auftakt bilden drei Kick-Off-Veranstaltungen mit Impulsvorträgen und anschließendem Rahmen zur Diskussion. Die daraus resultierenden Ideen können dann in konkrete Projektausarbeitungen münden. Diese sollen in kleinen Arbeitsgruppen mit Beteiligten beider Institutionen unter Betreuung der jeweiligen Lehrenden (ggf. auch als aktiv Mitgestaltende/-entwickelnde) realisiert werden, die relativ autark Teilbereiche einer größeren Projektidee oder kleinere autonome Projekte realisieren können. Die Bandbreite möglicher Projekte kann dabei von freien Kunstwerken wie Elektroakustische Kompositionen oder interaktiven Installationen bis hin zu Forschungstools im Sinne eines Transmodal Data Display reichen.

14.11., HfM Karlsruhe MUT Seminarraum 206, 17 Uhr  
Impulsvorträge von Prof. Dr. Markus Klute (KIT) und Lehrenden des IMWI

21.11., HfM Karlsruhe MUT Seminarraum 206, 17 Uhr  
Impulsvorträge von Dr. Michael Hoch (CERN) und Lehrenden des IMWI

28.11., HfM Karlsruhe MUT Seminarraum 206, 17 Uhr  
Austausch und Ideenfindung

Die weiteren Termine werden dann sukzessive festgelegt.

Zusatztermin:

Di, 19.11., Staatliche Akademie der Bildenden Künste Karlsruhe, 17 Uhr  
Prof. Dr. Markus Klute, Vortrag über Raum und Zeit

# Reading assignment

- Modern particle physics (Mark Thomson)
  - Chap. 9 (complete)
  - Chap. 10
    - 10.1-10.3
    - 10.6-10.7