Karlsruhe Institute of Technology

Particle Physics 1
Lecture 5: Symmetries & QCD (Theory)

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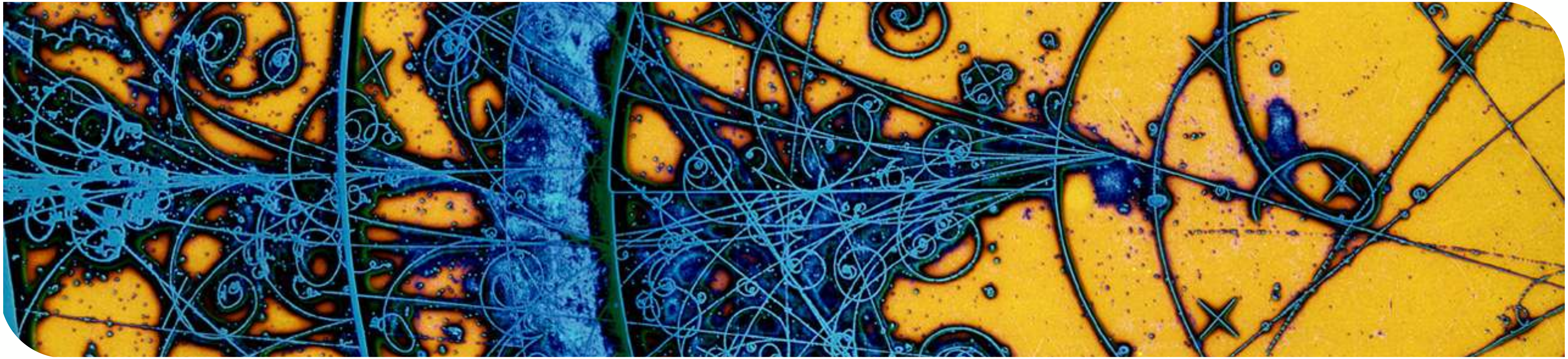
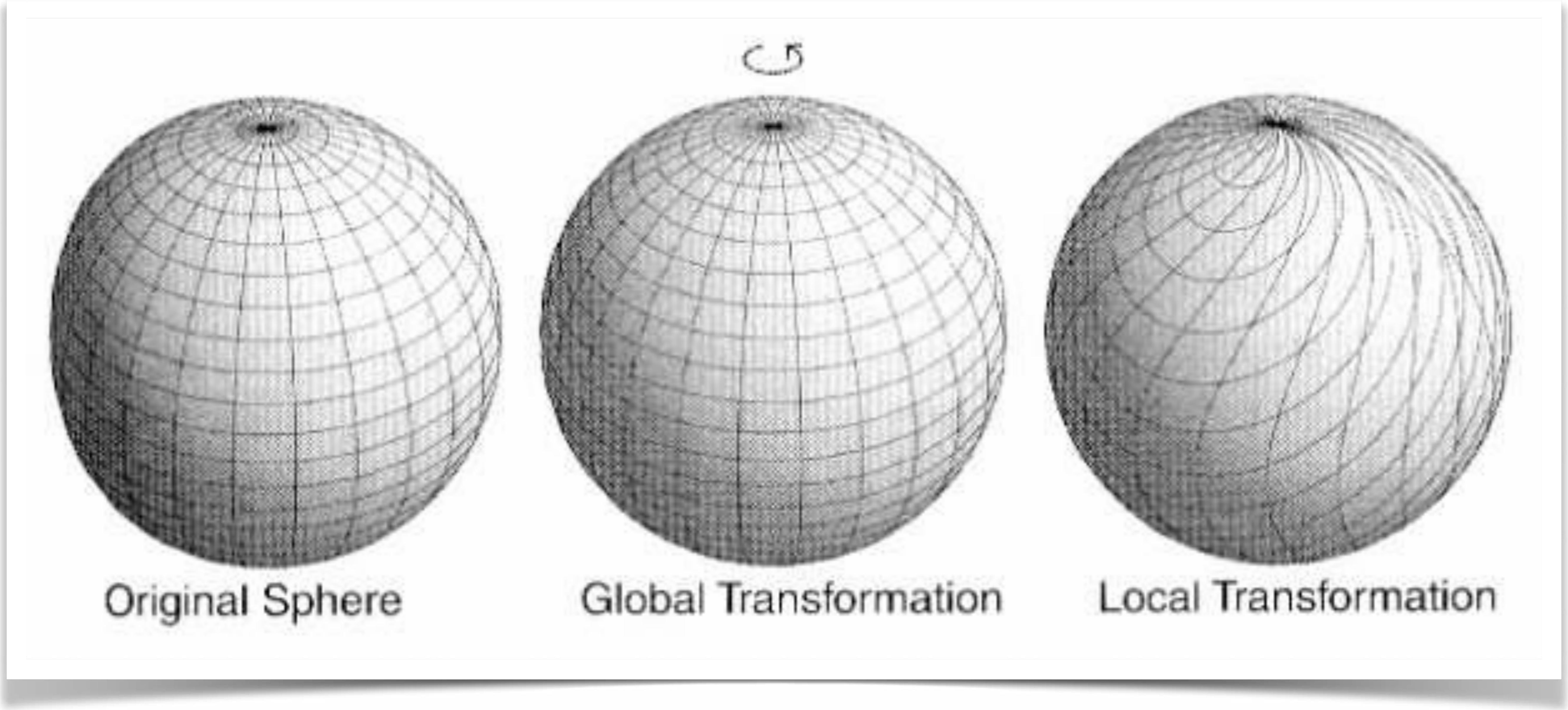
Winter 2024/2025


Credit: CERN

Questions from past lectures

Symmetries

- Symmetries are operations performed on a system that leaves it invariant
  - **Global** symmetries are the **same** at all space-time points
  - **Local** symmetries are **different** at different space-time points



Source: https://universe-review.ca/I15-04-gauge2.jpg

Symmetries

- Requires that all physical predictions are invariant under the wavefunction transformation $\psi \to \psi' = \hat{U}\psi$

- For physical predictions to be unchanged, wavefunction normalizations must remain unchanged

- The eigenstates of the system must also remain unchanged

Symmetries

- A finite continuous symmetry operation can be built up from a series of infinitesimal transformations $\hat{U}(\epsilon) = I + i\epsilon\hat{G}$

A simple example

- Translational invariance in 1 dimension $x \to x + \epsilon$

Finite transformations

- Any finite symmetry transformation can be expressed as a series of infinitesimal transformations

$$\hat{U}(\alpha) = \lim_{n\to\infty}\left(1 + i\frac{1}{n}\alpha\cdot\hat{\mathbf{G}}\right)^n = \exp\left(i\alpha\cdot\mathbf{G}\right)$$

Flavor symmetries

- Extend the idea of isospin to the up- and down-quarks.

A little group theory

- A **group** is a set of objects G on which some “multiplication” operation is defined, s.t.:
  - If a and b are in G, $a \cdot b$ is in G
  - There is an identity element in G s.t. $a \cdot i = a$ for any a in G
  - For any a in G, there is an “inverse” element G s.t. $a \cdot a^{-1} = i$
  - For $a,\ b,\ c$ in G, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
- A **Lie group** is a group of unitary operators in which the group elements are labeled by a set of continuous parameters
  - Example: the elements $e^{i\alpha_a X_a}$ (a summed) under multiplication.
    - The α_a are the set of continuous parameters, the X_a are operators.
    - The X_a are called **generators** because exponentiating them generates the group

SU(3) flavor symmetry

- Extend the SU(2) flavor symmetry to include the s quark

$$\begin{pmatrix} \mathrm{u}' \\ \mathrm{d}' \\ \mathrm{s}' \end{pmatrix} = \hat{U} \begin{pmatrix} \mathrm{u} \\ \mathrm{d} \\ \mathrm{s} \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} \mathrm{u} \\ \mathrm{d} \\ \mathrm{s} \end{pmatrix}$$

SU(3) flavor symmetry

- SU(3) uds flavor symmetry contains the subgroup of SU(2) $u \leftrightarrow d$ flavor symmetry

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \text{and} \quad \lambda_X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \text{and} \quad \lambda_Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Hypercharge

- For the case of SU(2), we defined 3 Hermitian generators, each of which corresponds to an observable quantity

- In SU(3) there is an analogue of total isospin:

$$\hat{T}^2 = \sum_{i=1}^{8} \hat{T}_i^2 = \tfrac{1}{4} \sum_{i=1}^{8} \lambda_i^2 = \frac{4}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Take 5

Local gauge principle

- Recall gauge invariance form electromagnetism

- In relativistic QM, the gauge invariance of EM can be related to a local gauge principle

Local gauge principle

- For this local U(1) phase transformation, the free-particle Dirac equation $i\gamma^\mu \partial_\mu \psi = m\psi$ becomes

Local gauge principle

- The requirement that physics is invariant under local U(1) phase transformations implies the existence of a gauge field with couples to Dirac particles in **exactly** the same way as the photon.

- All of QED can be derived by requiring the invariance of physics under local U(1) transformations of the form $\hat{U} = \exp[iq\chi(x)]$

Thomson, Sec. 10.1

QED →QCD

- QED corresponds to a local U(1) gauge symmetry of the universe
- The QFT of the strong interaction is invariance under SU(3) local phase transformations

$$\psi(x) \to \psi'(x) = \exp\left[ig_S\,\boldsymbol{\alpha}(x)\cdot\hat{\mathbf{T}}\right]\psi(x)$$

- Since the generators of SU(3) are represented by 3×3 matrices, the wavefunction must now include 3 additional degrees of freedom represented by a 3 component vector

Dirac equation becomes

- As before, substitute the expression for ψ' into the free-particle Dirac equation $i\gamma^\mu \partial_\mu \psi = m\psi$ and you obtain:

$$i\gamma^\mu \left[\partial_\mu + ig_S(\partial_\mu \boldsymbol{\alpha}) \cdot \hat{\mathbf{T}}\right]\psi = m\psi.$$

- The **required** local gauge invariance can be asserted by introducing 8 new fields (just as we introduced the new A_μ photon field in U(1) QED)

Interactions with the new gauge field

$$i\gamma^{\mu}\left[\partial_{\mu} + ig_S(\partial_{\mu}\alpha)\cdot\hat{\mathbf{T}}\right]\psi = m\psi.$$

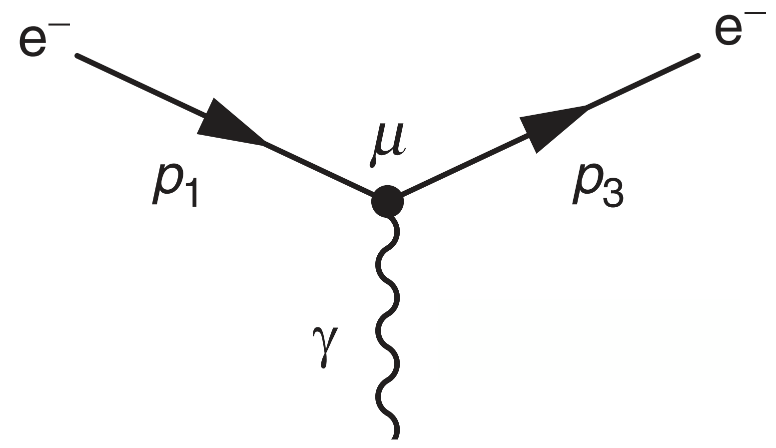
$$i\gamma^{\mu}\left[\partial_{\mu} + ig_S G_{\mu}^{a} T^{a}\right]\psi - m\psi = 0,$$

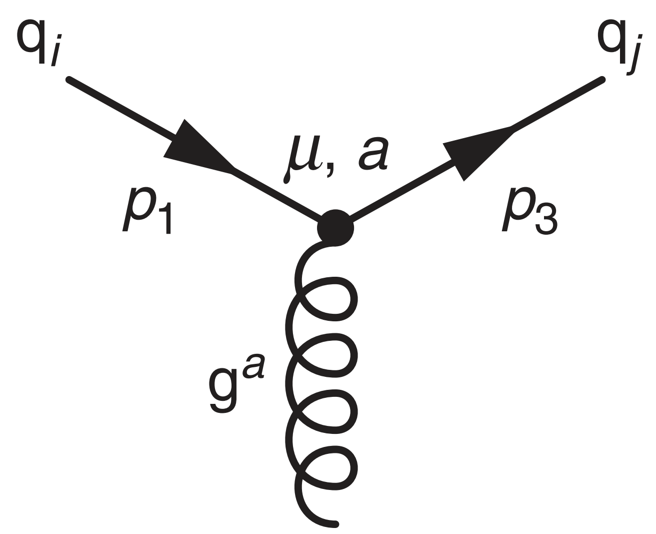
$$G_{\mu}^{k} \rightarrow G_{\mu}^{k\prime} = G_{\mu}^{k} - \partial_{\mu}\alpha_k - g_S f_{ijk}\alpha_i G_{\mu}^{j}.$$

Summary (so far)

	QED	QCD
	U(1)	SU(3)
Symmetry of the universe that requires the invariance of physics under local phase transformations	$\psi(x) \to \psi'(x) = \hat{U}(x)\psi(x) = e^{iq\chi(x)}\psi(x).$	$\psi(x) \to \psi'(x) = \exp\left[ig_S \boldsymbol{\alpha}(x) \cdot \hat{\mathbf{T}}\right]\psi(x)$
Substitute ψ' into the free particle Dirac eq. $i\gamma^\mu \partial_\mu \psi = m\psi$	$i\gamma^\mu(\partial_\mu + iq\partial_\mu\chi)\psi = m\psi,$	$i\gamma^\mu\left[\partial_\mu + ig_S(\partial_\mu\boldsymbol{\alpha}) \cdot \hat{\mathbf{T}}\right]\psi = m\psi.$
Modify $i\gamma^\mu \partial_\mu \psi = m\psi$ to include a new degree of freedom **to re-establish the required invariance**	$i\gamma^\mu(\partial_\mu + iqA_\mu)\psi - m\psi = 0,$	$i\gamma^\mu\left[\partial_\mu + ig_S G_\mu^a T^a\right]\psi - m\psi = 0,$
These equations are invariant under the local phase transformations provided that the new field transforms as:	$A_\mu \to A'_\mu = A_\mu - \partial_\mu\chi,$	$G_\mu^k \to G_\mu^{k\prime} = G_\mu^k - \partial_\mu\alpha_k - g_S f_{ijk}\alpha_i G_\mu^j.$

	QED	QCD
	U(1)	SU(3)
Interaction term	$-iq\gamma^\mu A_\mu \psi$	$-ig_S \frac{1}{2}\lambda^a \gamma^\mu G^a_\mu \psi$
Vertex factor	$-iq\gamma^\mu$	$-ig_S \gamma^\mu \frac{1}{2}\lambda^a$
4-vector current	$j_e^\mu = \overline{u}_e(p_3)\gamma^\mu u_e(p_1)$	$j_q^\mu = \overline{u}(p_3)c_j^\dagger \left\{-\tfrac{1}{2}ig_S \lambda^a \gamma^\mu\right\} c_i u(p_1),$

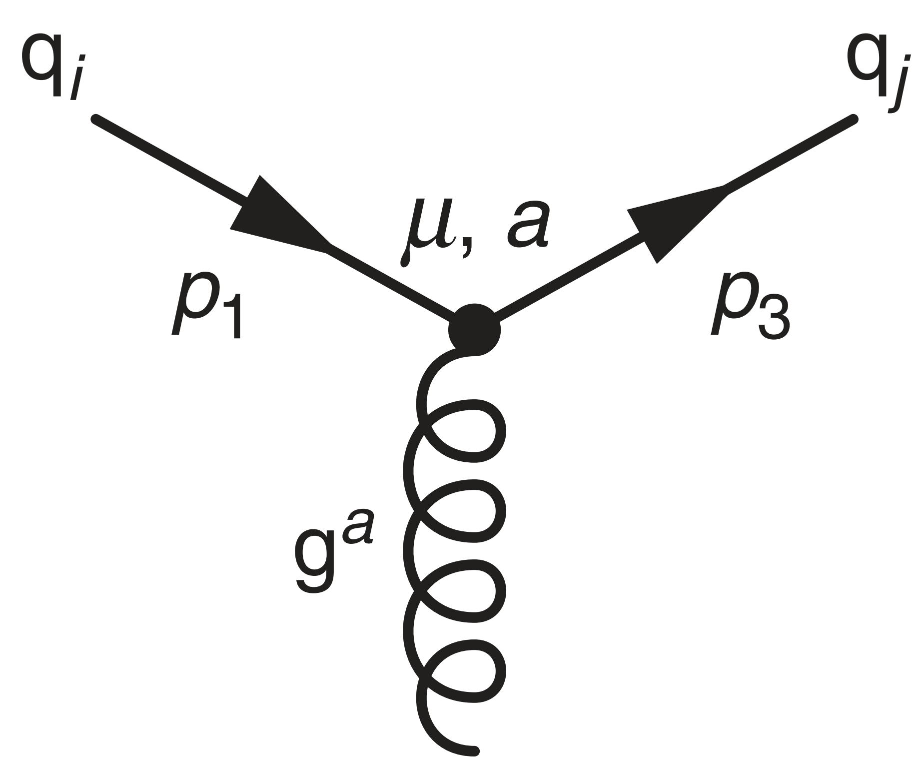




Factorize the qqg vertex

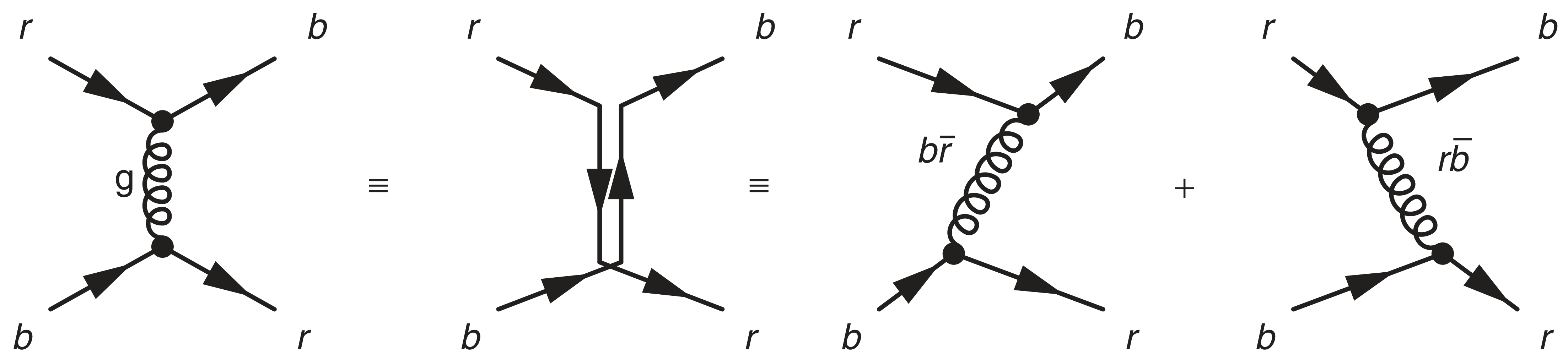
j_q^μ from last slide

$$\overline{u}(p_3)c_j^\dagger\{-\tfrac{1}{2}ig_s\lambda^a\gamma^\mu\}c_i u(p_1) = -\tfrac{1}{2}ig_s\left[c_j^\dagger\lambda^a c_i\right]\times\left[\overline{u}(p_3)\gamma^\mu u(p_1)\right]$$



Gluons

- QCD interaction vertex includes a factor of λ_{ij}, where i & j label the colors of the quarks
  - ⇒ *Gluons corresponding to the non-diagonal GM matrices connect quark states of diff. color*
  - ⇒ *Gluons **must** carry color charge (for color to be conserved)*



Gell-Mann matrices: The 8 matrices used to represent the **generators** of the SU(3) symmetry

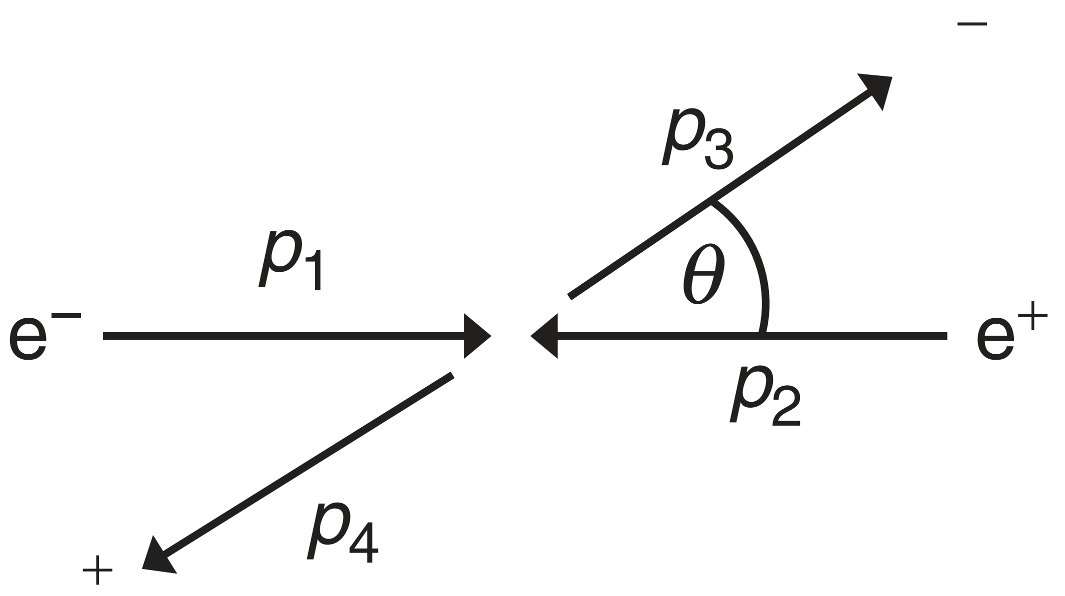
$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

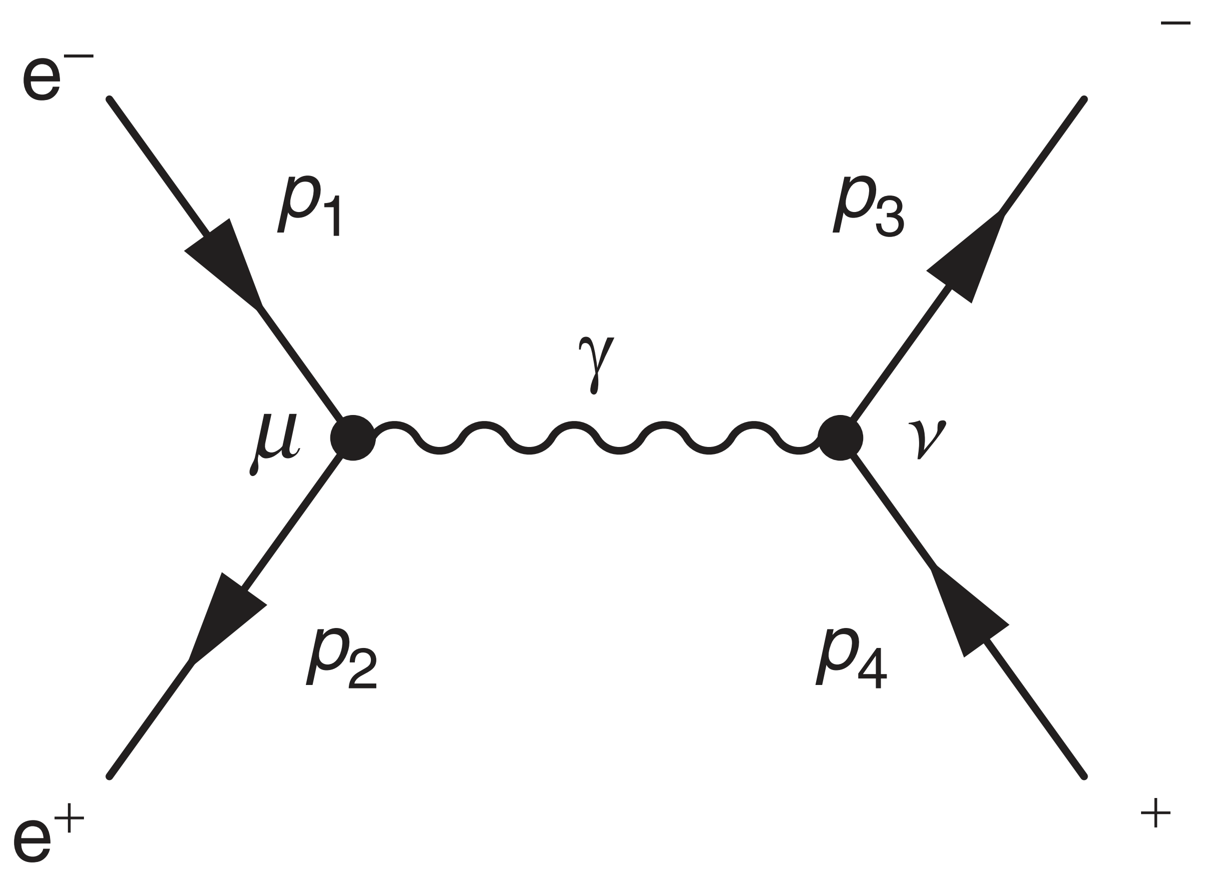
$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

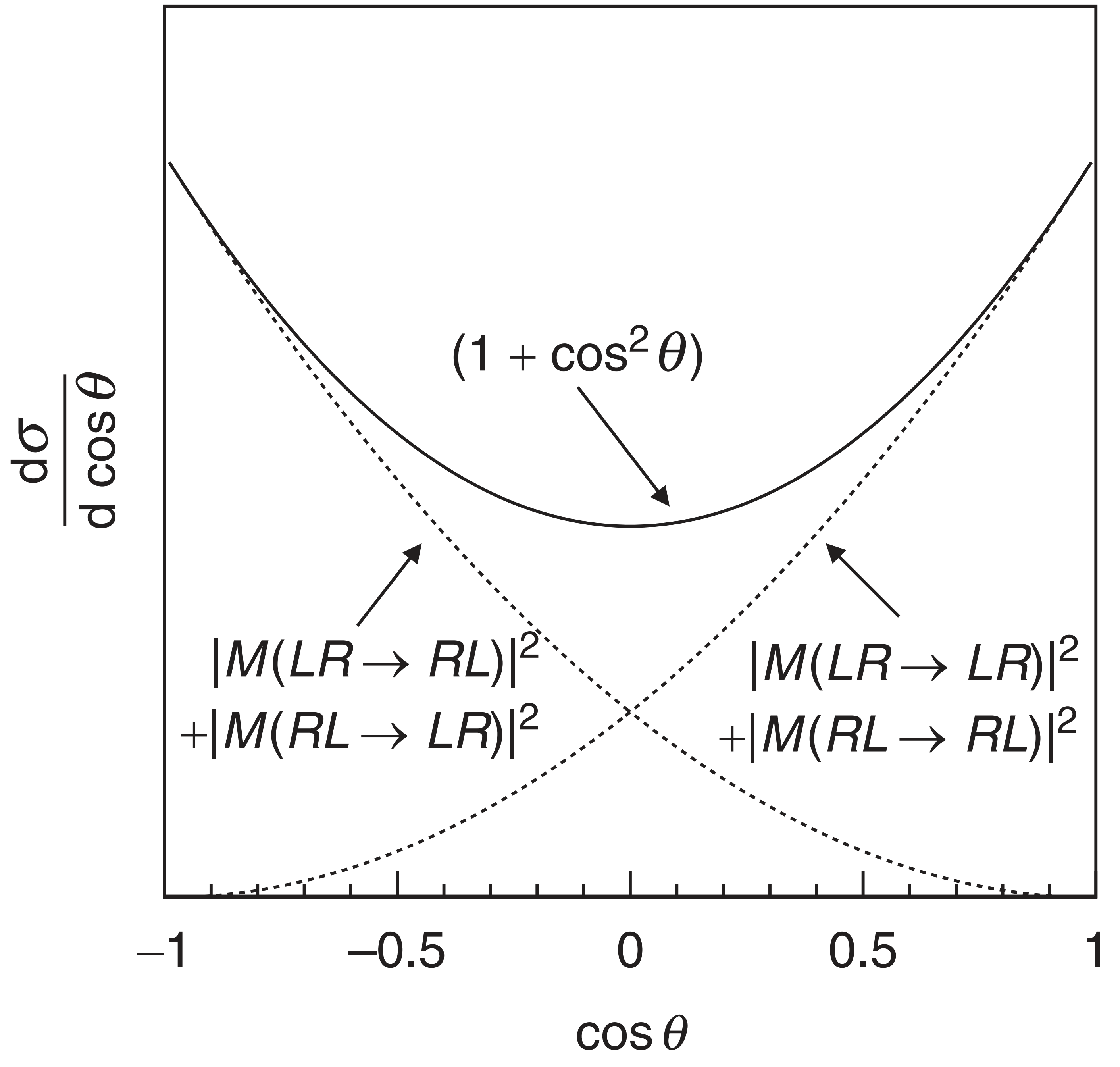
$e^+e^- \to \mu^+\mu^-$

Review





Thomson Figs. 6.4, 6.7



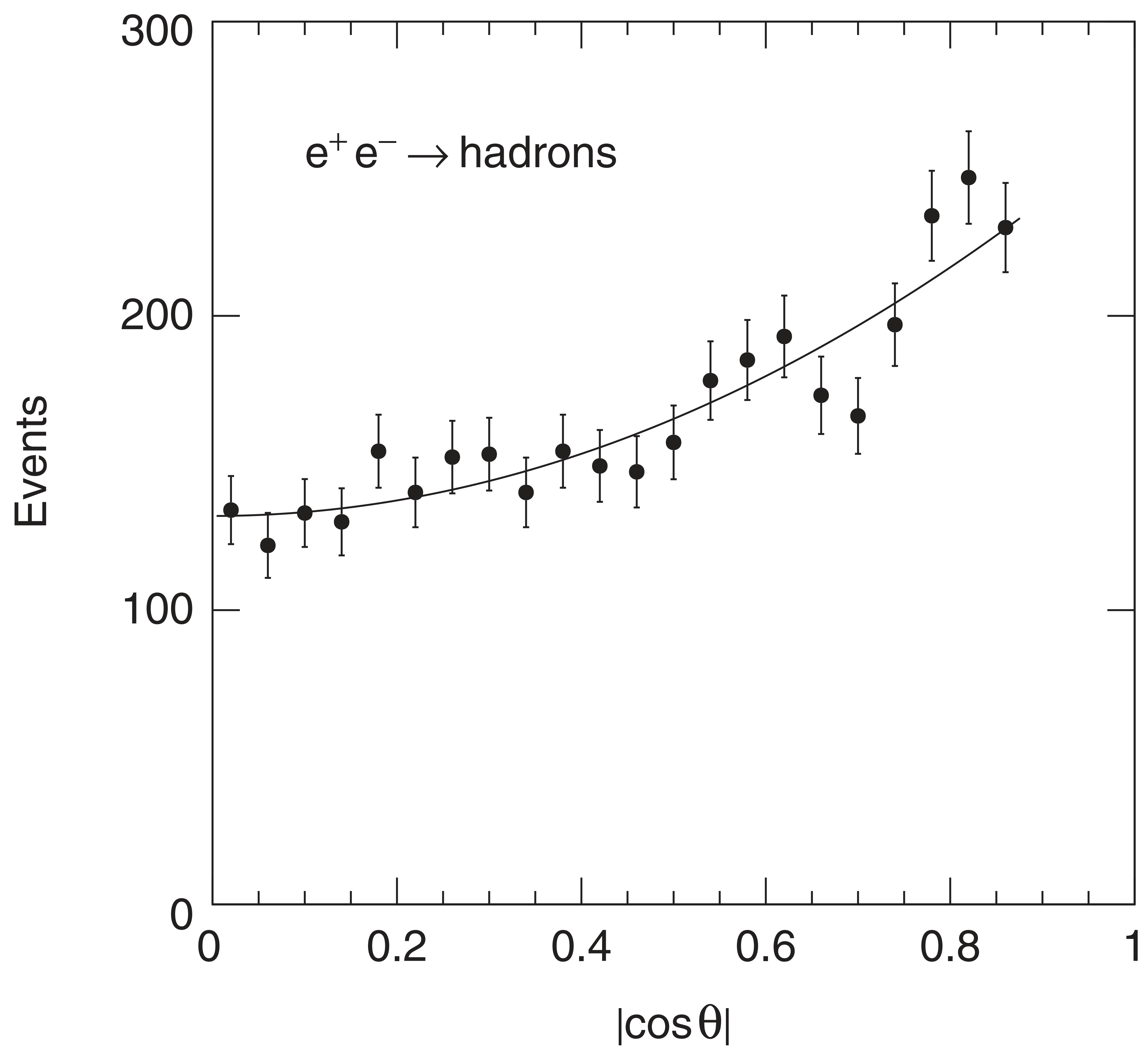
$$\langle|\mathcal{M}_{fi}|^2\rangle = \frac{1}{4} \times \left(|\mathcal{M}_{RL\to RL}|^2 + |\mathcal{M}_{RL\to LR}|^2 + |\mathcal{M}_{LR\to RL}|^2 + |\mathcal{M}^2_{LR\to LR}|\right)$$
$$= \frac{1}{4}e^4\left[2(1+\cos\theta)^2 + 2(1-\cos\theta)^2\right]$$
$$= e^4(1+\cos^2\theta).$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta).$$

$e^+e^- \to q\bar{q}$

- Assuming quarks are spin-half particles, should have the same angular dependence

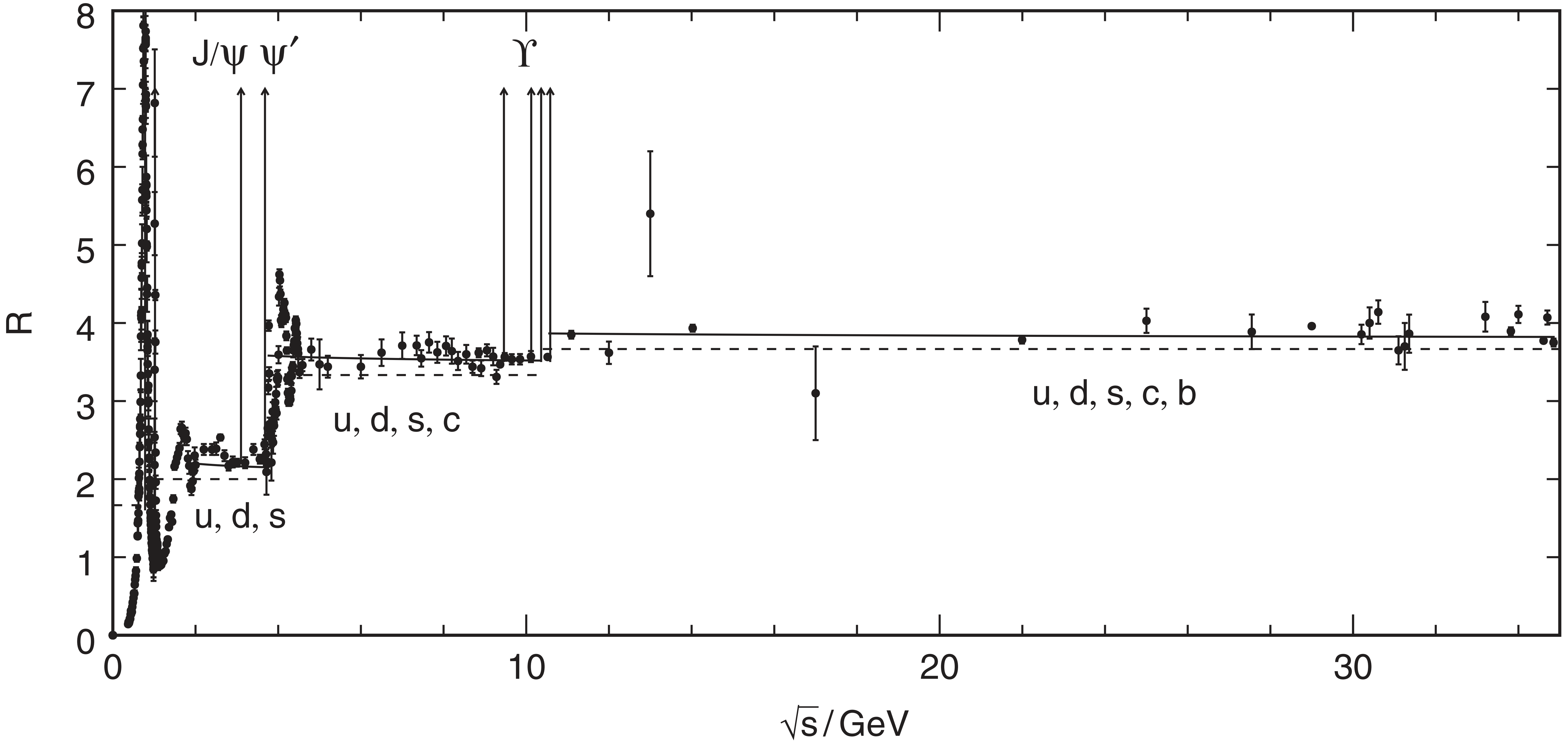
- However:
  - Generally not possible to ID which flavor of quark was produced, *so express as an inclusive sum over all hadrons* $e^+e^- \to$ *hadrons*
  - Usually not possible to ID which jet came from the quark and which from the antiquark, so quote in terms of $|\cos\theta|$



R-ratio

- Determine the number of colors from production and decay processes involving quarks. Compare production of hadrons and $\mu^+\mu^-$ in e^+e^-scattering
- The total QED $e^+e^- \to \mu^+\mu^-$ cross section
- For the inclusive QED production of hadrons

R-ratio



PINGO: R-ratio

- PINGO:
  - Session: Particle Physics 1 (WS 24/25).
  - Accession number: 559016
  - Link: https://pingo.coactum.de/events/559016

PINGO: R-ratio

- What happens to the R-ratio for energies above $\sqrt{s} \gtrsim 350$ GeV?
  - The R-ratio shows a sharp peak because $t\bar{t}$ bound states appear like for all other quarks
  - The R-ratio shows a step and becomes $R = 5$
  - Since there are no $t\bar{t}$ bound states, there is neither a sharp peak nor a step

Cooperative project seminar
“Particle Physics meets Music Informatics"

KIT – Karlsruher Institut für Technologie, ETP – Institut für experimentelle Teilchenphysik / CERN, IMWI – Institut für Musikinformatik und Musikwissenschaft, Hochschule für Musik Karlsruhe

Prof. Dr. Markus Klute (KIT), Dr. Michael Hoch (CERN), Prof. Dr. Marc Bangert (IMWI), Prof. Dr. Christoph Seibert (IMWI), Daniel Höpfner (IMWI), Michele Samarotto (IMWI), Amir Teymuri (IMWI), Christophe Weis (IMWI)

Grundsätzliches Ziel dieses kooperativen Projektseminars ist es, einen Raum zur Begegnung und gegenseitigen Befruchtung wissenschaftlicher und künstlerischer Praktiken zu schaffen.
Um den Austausch zwischen den Beteiligten und (Teil-)Projekten zu fördern, sollen über das Semester hinweg regelmäßige Treffen, in Form eines Kolloquiums gesetzt werden.

Den Auftakt bilden drei Kick-Off-Veranstaltungen mit Impulsvorträgen und anschließendem Rahmen zur Diskussion. Die daraus resultierenden Ideen können dann in konkrete Projektausarbeitungen münden. Diese sollen in kleinen Arbeitsgruppen mit Beteiligten beider Institutionen unter Betreuung der jeweiligen Lehrenden (ggf. auch als aktiv Mitgestaltende/-entwickelnde) realisiert werden, die relativ autark Teilbereiche einer größeren Projektidee oder kleinere autonome Projekte realisieren können. Die Bandbreite möglicher Projekte kann dabei von freien Kunstwerken wie Elektroakustische Kompositionen oder interaktiven Installationen bis hin zu Forschungstools im Sinne eines Transmodal Data Display reichen.

14.11., HfM Karlsruhe MUT Seminarraum 206, 17 Uhr
Impulsvorträge von Prof. Dr. Markus Klute (KIT) und Lehrenden des IMWI

21.11., HfM Karlsruhe MUT Seminarraum 206, 17 Uhr
Impulsvorträge von Dr. Michale Hoch (CERN) und Lehrenden des IMWI

28.11., HfM Karlsruhe MUT Seminarraum 206, 17 Uhr
Austausch und Ideenfindung

Die weiteren Termine werden dann sukzessive festgelegt.

Zusatztermin:
Di, 19.11., Staatliche Akademie der Bildenden Künste Karlsruhe, 17 Uhr
Prof. Dr. Markus Klute, Vortrag über Raum und Zeit

Reading assignment

- Modern particle physics (Mark Thomson)
  - Chap. 9 (complete)
  - Chap. 10
    - 10.1-10.3
    - 10.6-10.7