# Mixing and $CP$ Violation in the Kaon System

Prof. Dr. Ulrich Nierste
Dr. Pablo Goldenzweig

**Flavor Physics Lectures**
**II / XII**

Karlsruhe Institute of Technology

Winter Semester 2020/2021
11. November, 2020

# Reading material and references

**Lecture material based on several textbooks and online lectures/notes.**
**Credits for material and figures include:**

## Literature

- Perkins, Donald H. (2000), *Introduction to High Energy Physics*.
- Griffiths, David J. (2nd edition), *Introduction to Elementary Particles*.
- Stone, Sheldon (2nd edition), $B$ *decays*.

## Online Resources

- Belle/BaBar Collaborations, *The Phyiscs of the* $B$*-Factories*.
  http://arxiv.org/abs/1406.6311
- Bona, Marcella (University of London), $CP$ *Violation Lecture Notes*,
  http://pprc.qmul.ac.uk/ bona/ulpg/cpv/
- Richman, Jeremy D. (UCSB), *Heavy Quark Phyiscs and* $CP$ *Violation*.
  http://physics.ucsd.edu/students/courses/winter2010/physics222/references/driver_houches12.pdf
- Thomson, Mark (Cambridge University), *Particle Physics Lecture Handouts*,
  http://www.hep.phy.cam.ac.uk/ thomson/partIIIparticles/welcome.html
- Grossman, Yuval (Cornell University), *Just a Taste. Lectures on Flavor Physics*,
  http://www.lepp.cornell.edu/ pt267/files/notes/FlavorNotes.pdf
- Kooijman, P. & Tuning, N., $CP$ *Violation*,
  https://www.nikhef.nl/ h71/Lectures/2015/ppII-cpviolation-29012015.pdf

# Recap & Outline

Last time we showed that:

- $CP$ violation is built into the SM as an irreducible complex phase in the CKM matrix.
- The amount of $CP$ violation predicted by the CKM matrix is several orders of magnitudes too small to account for the observed matter – anti-matter asymmetry in the universe.
- Unitarity of the CKM matrix allows one to construct "unitarity triangles" in the complex plane. The orthogonality of the first and third columns yields a UT with sides of the same order in $\lambda$.
- The CKM picture of $CP$ violation can be tested by over-constraining this UT and ensuring that it closes and is not flat.
- This MUST be done by experiment!

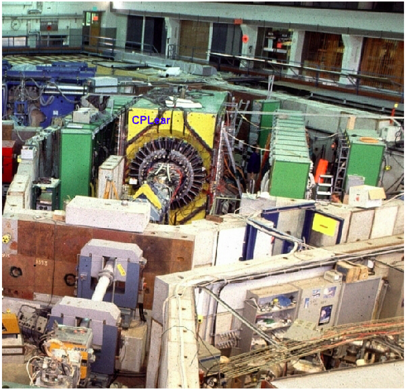
- To begin our study of $CP$ violation in specific processes, we'll start with the neutral kaon system, *where $CP$ violation was first discovered in 1964*.
- We'll need to develop several tools along the way:
  - Understand how to determine the $CP$ of a system of particles, to ensure conservation *and look for violation*.
  - We'll see that to do so properly, we'll need to develop the time-dependent formalism of particle – anti-particle mixing (aka oscillations).

# Production of neutral Kaons

The neutral kaon system is an ideal laboratory for which to study $CP$ violation, as neutral kaons are copiously produced in strong interaction processes into states of definite strangeness. why? $\Rightarrow$ *because strangness* ***is*** *conserved in strong interactions*

**The CPLEAR experiment: CERN 1990-1996** (Low energy anti-proton beam)

- Neutral kaons produced in reactions:
  $\overline{p}p \rightarrow K^-\pi^+K^0(d\overline{s})$
  $\overline{p}p \rightarrow K^+\pi^-\overline{K}^0(\overline{d}s)$
- Low energy, so particles produced almost at rest.
- Observe production and decay processes in the same detector.
- Through a technique called ***tagging*** we can unambiguosly determine the initial kaon.
- Provides a direct probe of *strangness oscillations*.



# $CP$ eigenstates in the neutral kaon system

How do the strong eigenstates transform under $\mathcal{P}$ and $\mathcal{C}$?

- The strong eigenstates $K^0(\bar{s}d)$ and $\overline{K}^0(s\bar{d})$ are pseudoscalars and thus have $J^P = 0^-$:

$$\mathcal{P}|K^0\rangle = -|K^0\rangle \qquad \mathcal{P}|\overline{K}^0\rangle = -|\overline{K}^0\rangle$$

- The charge conjugation operator changes particles $\leftrightarrow$ anti-particles:

$$\mathcal{C}|K^0\rangle = +|\overline{K}^0\rangle \qquad \mathcal{C}|\overline{K}^0\rangle = +|K^0\rangle$$

(where the '+' sign chosen here is purely conventional)

- Combining the $\mathcal{C}$ and $\mathcal{P}$ operations:

$$\mathcal{CP}|K^0\rangle = -|\overline{K}^0\rangle \qquad \mathcal{CP}|\overline{K}^0\rangle = -|K^0\rangle$$

$\Rightarrow$ ***Neither $K^0$ or $\overline{K}^0$ are eigenstates of $CP$.***

Can form $CP$ eigenstates from linear combinations of strong eigenstates $K^0$ and $\overline{K}^0$

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right) \qquad \mathcal{CP}|K_1\rangle = +|K_1\rangle$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right) \qquad \mathcal{CP}|K_2\rangle = -|K_2\rangle$$

# Decays of $CP$ eigenstates I

Neutral kaons often decay to pions:

**Kaon** masses are $\approx$ **498** MeV

**Pion** masses are $\approx$ **140** MeV

$\Rightarrow$ *Neutral kaons can decay to 2 or 3 pions.*

$K^0 \to \pi^0 \, \pi^0$

- $J^P : 0^- \to 0^- + 0^-$, and conservation of angular momentum gives $L = 0$

$$\mathcal{P}(\pi^0\pi^0) = (-1) \times (-1) \times (-1)^L = +1$$

- $|\pi^0\rangle = \frac{1}{\sqrt{2}} \left(|u\overline{u}\rangle - |d\overline{d}\rangle\right)$ is its' own anti-particle, and thus an eigenstate of $\mathcal{C}$

$$\mathcal{C}(\pi^0\pi^0) = (+1) \times (+1) = +1$$

- The combined effect of $\mathcal{C}$ and $\mathcal{P}$ leaves the *system* unchanged

$$\mathcal{CP}|\pi^0\pi^0\rangle = +|\pi^0\pi^0\rangle$$

***Neutral kaon decays to 2 pions occur in $CP$ even $(+1)$ eigenstates***

(same for $K^0 \to \pi^+ \, \pi^-$)

# Decays of $CP$ eigenstates II

$K^0 \to \pi^0\ \pi^0\ \pi^0$

$J^P : 0^- \to 0^- + 0^- + 0^-$, and conservation of angular momentum gives $L_1 \oplus L_2 = 0$, $\Rightarrow L_1 = L_2$

$$\mathcal{P}(\pi^0\pi^0\pi^0) = (-1) \times (-1) \times (-1) \times (-1)^{L_1} \times (-1)^{L_2} = -1$$

$$\mathcal{C}(\pi^0\pi^0\pi^0) = (+1) \times (+1) \times (+1) = +1$$

$$\mathcal{CP}|\pi^0\pi^0\pi^0\rangle = -|\pi^0\pi^0\pi^0\rangle$$

***Neutral kaon decays to 3 pions occur in $CP$ odd $(-1)$ eigenstates***

(same for $K^0 \to \pi^+\ \pi^-\ \pi^0$)

# Decays of $CP$ eigenstates III

In the absence of $CP$ violation in the weak decays of neutral kaons, we expect decays to pions to occur from the $CP$ eigenstates $K_1$ and $K_2$ (i.e., states of definite $CP$):

CP EVEN $|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right)$ $\quad \mathcal{CP}|K_1\rangle = +|K_1\rangle$ $\quad K_1 \to \pi\pi$

CP ODD $|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right)$ $\quad \mathcal{CP}|K_2\rangle = -|K_2\rangle$ $\quad K_2 \to \pi\pi\pi$

What else can we say about the decays of the $CP$ eigenstates?

- Due to the increased available phase space, decays to $2\pi$ should be faster than to $3\pi$.
- Lifetimes of the $CP$ eigenstates should be significantly different due to the energy available:

$$m_K - m_{2\pi} \approx 220 \text{ MeV} \quad \Rightarrow \textit{short}\text{-lifetime kaon } (K_S)$$
$$m_K - m_{3\pi} \approx 80 \text{ MeV} \quad \Rightarrow \textit{long}\text{-lifetime kaon } (K_L)$$

If $CP$ **is conserved**, we can identify

CP EVEN $|K_S\rangle = |K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right)$ $\quad K_S \to \pi\pi$

CP ODD $|K_L\rangle = |K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right)$ $\quad K_L \to \pi\pi\pi$

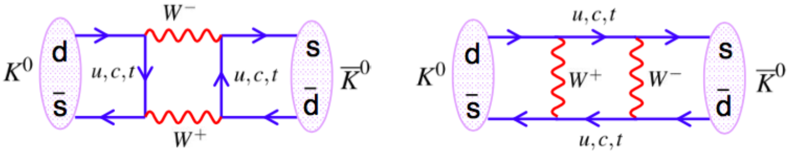
# Interlude - Time development in Mixing

- We now see how to construct a set of $CP$ eigenstates $K_1$ and $K_2$ (which we've identified as $K_S$ and $K_L$ in the absence of $CP$ violation), written in terms of the strong eigenstates $K^0$ and $\overline{K}^0$.
  - $\Rightarrow$ If $CP$ is conserved, $K_S$ decays exclusively to $\pi\pi$, while $K_L$ decays exclusively to $\pi\pi\pi$.
- Experimentally though, we can only produce $K^0$ and $\overline{K}^0$ states of definite strangeness, as, e.g., in the CPLEAR experiment.
- The next question we can ask ourselves is: how does a state which we produce as pure $K^0$ (or $\overline{K}^0$) develop over time?
- As we'll see in the next slide, to answer this question we must study at the time-evolution of the neutral kaon system in the presence of $K^0$- $\overline{K}^0$ mixing.

  **Note:** The following derivation applies to all sets of neutral particle pairs which undergo mixing, e.g. $B^0 - \overline{B}^0, B_s - \overline{B}_s, D^0 - \overline{D}^0$. In the following slides we use $K$, but the formalism holds for the other systems as well.

# Particle – anti-Particle Mixing I

Kaons decay via the weak interaction, which allows mixing of $K^0 \leftrightarrow \overline{K}^0$ via *box diagrams*, i.e., allowing transitions between the strong eigenstates $K^0$, $\overline{K}^0$:




The neutral kaons propagate as eigenstates of the overall **strong + weak** interaction.

$\Rightarrow$ *We need to consider the time evolution of a mixed state of $K^0$ and $\overline{K}^0$:*

$$\Psi(t) = a(t)K^0 + b(t)\overline{K}^0$$

- The wave-function satisfies the time-dependent wave equation:

$$\mathcal{H}_w|\Psi(t)\rangle = i\frac{\partial}{\partial t}|\Psi(t)\rangle \quad \text{(K1)}$$

where the weak Hamiltonian can be written as a sum of mass and decay matrices:

$$\mathcal{H}_w = (\mathbf{M} - i\frac{\mathbf{\Gamma}}{2}) = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{1}{2}i\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

# Particle – anti-Particle Mixing II

- The time-dependent wave-equation (K1) becomes

$$\begin{pmatrix} M_{11} - \frac{1}{2}i\Gamma_{11} & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{21} - \frac{1}{2}i\Gamma_{21} & M_{22} - \frac{1}{2}i\Gamma_{22} \end{pmatrix} \begin{pmatrix} |K^0(t)\rangle \\ |\overline{K}^0(t)\rangle \end{pmatrix} = i\frac{\partial}{\partial t} \begin{pmatrix} |K^0(t)\rangle \\ |\overline{K}^0(t)\rangle \end{pmatrix} \quad \text{(K2)}$$

  where the off-diagonal terms are due to mixing

$$M_{12} = \sum_j \frac{\langle K^0|\mathcal{H}_w|j\rangle^* \langle j|\mathcal{H}_w|\overline{K}^0\rangle}{m_{K^0} - E_j}$$

  ($j$ = intermediate states)

  and the decay terms include the effects of interference between decays to a common final state $f$

$$\Gamma_{12} = 2\pi \sum_f \langle f|\mathcal{H}_w|K^0\rangle^* \langle f|\mathcal{H}_w|\overline{K}^0\rangle \rho_F$$

- Both the mass and decay matrices represent observable quantities and are Hermitian:

$$M_{ii} = M_{ii}^*, M_{ij} = M_{ji}^*, \quad \Gamma_{ii} = \Gamma_{ii}^*, \Gamma_{ij} = \Gamma_{ji}^*$$

- Furthermore, conservation of $CPT$ implies that the masses and decay rates of strong eigenstates $K^0$ and $\overline{K}^0$ are identical:

$$M_{11} = M_{22}^* = M \quad \Gamma_{11} = \Gamma_{22}^* = \Gamma$$

# Particle – anti-Particle Mixing III

- Using these identities we can write the time evolution of the system (K2) as

$$\begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M - \frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} a \\ b \end{pmatrix} \quad \text{(K3)}$$

- We can then solve the system of coupled differential equations for $a(t)$ and $b(t)$ by:
  1) Finding the eigenvalues and eigenstates of the weak Hamiltonian $\mathcal{H}_w$, and
  2) Transforming into this basis.

- Solving the eigenvalue equation for $|\mathcal{H}_w - \mu I| = 0$ yields eigenvalues

$$\mu_\pm = M - \frac{1}{2}i\Gamma \pm \sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})}$$

- and normalized eigenstates

$$|K_\pm\rangle = \frac{1}{\sqrt{1+|\frac{q}{p}|^2}}(|K^0\rangle \pm \frac{q}{p}|\overline{K}^0\rangle) \qquad \frac{q}{p} = \sqrt{\frac{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)}{(M_{12} - \frac{1}{2}i\Gamma_{12})}} \quad \text{(K4)}$$

*Look familiar??*

*Make a mental note of this $\frac{q}{p}$ we'll revisit it soon...*

# Particle – anti-Particle Mixing IV *(End interlude)*

- We can now identify these as the $|K_S\rangle$ and $|K_L\rangle$ states:

$$|K_S\rangle = \frac{1}{\sqrt{1+|\frac{q}{p}|^2}}(|K^0\rangle - \frac{q}{p}|\overline{K}^0\rangle) \qquad |K_L\rangle = \frac{1}{\sqrt{1+|\frac{q}{p}|^2}}(|K^0\rangle + \frac{q}{p}|\overline{K}^0\rangle) \quad \text{(K5)}$$

where in the limit where $M_{12}$ and $\Gamma_{12}$ are real, $\frac{q}{p} = 1$, and the $|K_S\rangle$ and $|K_L\rangle$ eigenstates correspond to the $CP$ eigenstates $|K_1\rangle$ and $|K_2\rangle$ we constructed on Slide 7.

- Substituting these states back into $\Psi(t) = a(t)K^0 + b(t)\overline{K}^0$, and evaluating the time-dependence of $a(t)$ and $b(t)$ using (K3), gives

$$|\Psi(t)\rangle = A_S e^{-i\left(m_S t - \frac{\Gamma_S}{2}\right)t}|K_S\rangle + A_L e^{-i\left(m_L t - \frac{\Gamma_L}{2}\right)t}|K_L\rangle \quad \text{(K6)}$$

where $A_S$ and $A_L$ are constants, and we have set

$$m_{S,L} = M \mp \mathcal{R}\left\{\sqrt{(M^*_{12} - \tfrac{1}{2}i\Gamma^*_{12})(M_{12} - \tfrac{1}{2}i\Gamma_{12})}\right\}$$

$$\Gamma_{S,L} = \Gamma \pm 2\mathcal{I}\left\{\sqrt{(M^*_{12} - \tfrac{1}{2}i\Gamma^*_{12})(M_{12} - \tfrac{1}{2}i\Gamma_{12})}\right\}$$

*Hence, in terms of the $|K_S\rangle$, $|K_L\rangle$ basis, the states propagate as independent particles with definite masses and lifetimes.*

# Which is the 'real' particle? *Quoting David J. Griffiths*

The neutral Kaon system adds a subtle twist to the old question, 'What is a particle?'

Kaons are typically produced by the strong interactions, in eigenstates of strangeness ($K^0$ and $\overline{K}^0$), but they decay by the weak interactions, as eigenstates of CP ($K_1$ and $K_2$).

Which, then, is the 'real' particle?

If we hold that a 'particle' must have a unique lifetime, then the 'true' particles are $K_1$ and $K_2$. But we need not be so dogmatic. In practice, it is sometimes more convenient to use one set, and sometimes, the other.

The situation is in many ways analogous to polarized light. Linear polarization can be regarded as a superposition of left-circular polarization and right-circular polarization. If you imagine a medium that preferentially absorbs right-circularly polarized light, and shine on it a linearly polarized beam, it will become progressively more left-circularly polarized as it passes through the material, just as a $K^0$ beam turns into a $K_2$ beam. But whether you choose to analyze the process in terms of states of linear or circular polarization is largely a matter of taste.

# Neutral kaon decays to pions *(No $CP$ violation)*

*We now have the machinery to determine the decay rate to 2 (or 3) pions for a state which was produced as purely $K^0$ (or $\overline{K}^0$)*

- Again, recall that neutral kaons are produced via $\pi^- + p \rightarrow \Lambda(uds) + K^0(d\bar{s})$, into states of definite strangeness, which means we can create a beam of pure $K^0$ at $t = 0$.

  In the $K_S$ and $K_L$ basis:

  $$|\Psi(t=0)\rangle = \frac{1}{\sqrt{2}}\left[|K_S\rangle + |K_L\rangle\right]$$

  which we've just shown evolves in time as

  $$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left[e^{-\left(im_S+\frac{\Gamma_S}{2}\right)t}|K_S\rangle + e^{-\left(im_L+\frac{\Gamma_L}{2}\right)t}|K_L\rangle\right]$$

- To clean this up a bit, lets write

  $$\theta_S(t) = e^{-\left(im_S+\frac{\Gamma_S}{2}\right)t}, \quad \theta_L(t) = e^{-\left(im_L+\frac{\Gamma_L}{2}\right)t}$$

  which gives us the compact expression for the time dependent state at time $t$

  $$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left[\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle\right]$$

$\Rightarrow$ we can calculate the decay rate to two pions as

$$\Gamma(K^0_{t=0} \rightarrow \pi\pi) \propto |\langle K_S|\Psi(t)\rangle|^2 \propto |\theta_S(t)|^2 = e^{-\Gamma_s t} = e^{-t/\tau_S}$$

i.e., the decays $K_S$, the short lifetime component $\rightarrow$ *as expected*

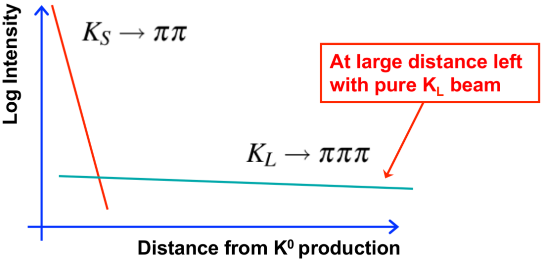
$\Rightarrow$ and to three pions as

$$\Gamma(K^0_{t=0} \rightarrow \pi\pi\pi) \propto |\langle K_L|\Psi(t)\rangle|^2 \propto |\theta_L(t)|^2 = e^{-\Gamma_L t} = e^{-t/\tau_L}$$

i.e., the decays $K_L$, the long lifetime component $\rightarrow$ *as expected*

# Neutral kaon decays to pions *(No $CP$ violation)*

Expect to see predominantly $2\pi$ decays near the start of the beam, and $3\pi$ decays further downstream.



# $K^0 - \overline{K}^0$ Oscillations

*(No $CP$ violation)*

# $K^0 - \overline{K}^0$ Oscillations - Theory I *(No $CP$ violation)*

*We've seen that particles and anti-particles mix, but what can we say about the frequency of oscillations between $K^0$ and $\overline{K}^0$?*

- Again, start with our wave function at time $t$

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left[\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle\right]$$

- We want to study $K^0 - \overline{K}^0$ oscillation, so lets change to this basis by writing $|K_S\rangle$ and $|K_L\rangle$ in terms of the strong eigenstates $|K^0\rangle$ and $|\overline{K}^0\rangle$

$$|\Psi(t)\rangle = \frac{1}{2}\left[\theta_S(t) + \theta_L(t)\right]|K^0\rangle + \frac{1}{2}\left[\theta_L(t) - \theta_S(t)\right]|\overline{K}^0\rangle \quad \text{(K7)}$$

- Now, if we want the decay rate to $K^0$ at time $t$ (i.e., the $K^0$ fraction, aka intensity), we need:

$$\Gamma(K^0_{t=0} \rightarrow K^0) = |\langle K^0|\Psi(t)\rangle|^2 = \frac{1}{4}\left|\theta_S(t) + \theta_L(t)\right|^2$$

- Similarly, for the $\overline{K}^0$ fraction:

$$\Gamma(K^0_{t=0} \rightarrow \overline{K}^0) = |\langle \overline{K}^0|\Psi(t)\rangle|^2 = \frac{1}{4}\left|\theta_S(t) - \theta_L(t)\right|^2$$

# $K^0 - \overline{K}^0$ Oscillations - Theory II *(No $CP$ violation)*

- Using the identity $|a \pm b|^2 = |a^2| + |b^2| \pm 2\mathcal{R}(ab^*)$

$$|\theta_S(t) \pm \theta_L(t)|^2 = \left| e^{-\left(im_S + \frac{\Gamma_S}{2}\right)t} \pm e^{-\left(im_L + \frac{\Gamma_L}{2}\right)t} \right|^2$$

$$= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t} \cos(m_S - m_L)t$$

$$= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t} \cos(\Delta m t)$$

***Frequency of oscillation between neutral kaon states given by the mass splitting***

$$\boldsymbol{\Delta m = m(K_L) - m(K_S)}$$

# $K^0 - \overline{K}^0$ Oscillations - Theory III *(No $CP$ violation)*

We can now write the $K^0$ and $\overline{K}^0$ fraction at time $t$ of a beam initially produced as pure $K^0$ as:

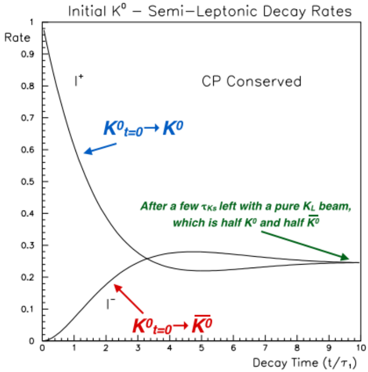
$$\Gamma(K^0_{t=0} \to K^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t}\cos(\Delta m t)\right]$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t}\cos(\Delta m t)\right]$$

Experimentally we find (next slides):

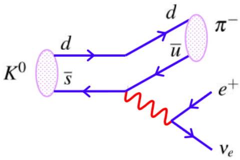
- $\tau(K_S) = 0.9 \times 10^{-10}$s
  $\tau(K_L) = 0.5 \times 10^{-7}$s
  $\Delta m = (3.506 \pm 0.006) \times 10^{-15}$GeV
- The mass difference corresponds to an oscillation period of
  $T = \frac{2\pi\hbar}{\Delta m} \approx 1.2 \times 10^{-9}\ s$

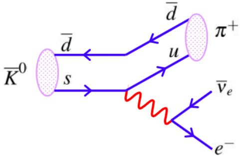
$\Rightarrow$ Relatively long compared to the $K_S$ lifetime, *therefore do not observe very pronounced oscillations*



How we can *measure* the mass splitting?

$\Rightarrow$ ***Lets take a look at semi-leptonic kaon decays.***




| | | |
|---|---|---|
| Direct | $K^0_{t=0} \to K^0 \to \pi^- e^+ \nu_e$ | $\overline{K}^0_{t=0} \to \overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$ |
| via. Mixing | $K^0_{t=0} \to \overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$ | $\overline{K}^0_{t=0} \to K^0 \to \pi^- e^+ \nu_e$ |

For an initial $K^0$ beam, have *both* charge combinations (same for initial $\overline{K}^0$ beam).

If we produce the neutral kaons in $p\overline{p}$ collisions, as was done at the CPLEAR experiment at CERN, how do we *determine* which neutral kaon was initially produced?

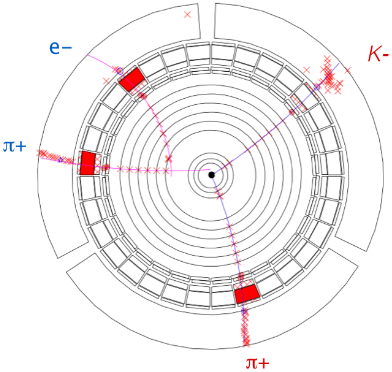
$$\overline{p}p \to K^- \pi^+ K^0 \qquad \text{or} \qquad \overline{p}p \to K^+ \pi^- \overline{K}^0 \;?$$

$\Rightarrow$ *Charge of the $K^\pm \pi^\mp$ in the production process **tags** the initial kaon as either $K^0$ or $\overline{K}^0$*

# $K^0 - \overline{K}^0$ Oscillations - Measurement II *(No $CP$ violation)*

Lets look at an example of a CPLEAR event.

- 4 visible particles in the detector (neutrinos are undetectable).
- We know that there must be an oppositely charged $K\pi$ pair from the initial reaction.
- Here the $K^- \pi^+$ ***tags*** the initial reaction as $\overline{p}p \to K^-\pi^+K^0$ and thus we deduce the initial wave-function to be $|\Psi(t=0)\rangle = |K^0\rangle$
- Now from the remaining $\pi^+ e^-$ we can deduce if the $K^0$ decayed directly or underwent mixing.




Clearly it mixed, and our overall reaction for this event is:

$$\overline{p}p \to K^-\pi^+K^0$$
$$\hookrightarrow \overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$$

- In this way we can measure the decay rates as a function of time for all 4 combinations. For this event, we have the rate:
$\Gamma(K^0_{t=0} \to \overline{K}^0) \propto \Gamma(K^0_{t=0} \to \pi^+ e^- \overline{\nu}_e)$

# $K^0 - \overline{K}^0$ Oscillations - Measurement III *(No $CP$ violation)*

The usual measure is the asymmetry, since efficiencies, and some systematics, cancel out.
*How can we construct a meaningful asymmetry?*

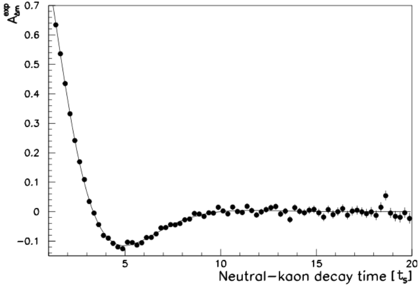
The 4 measureable rates are:

$$\Gamma(K^0_{t=0} \to K^0) \propto \Gamma(K^0_{t=0} \to \pi^- e^+ \nu_e) \equiv R_u \quad \text{Unmixed}$$
$$\Gamma(K^0_{t=0} \to \overline{K}^0) \propto \Gamma(K^0_{t=0} \to \pi^+ e^- \overline{\nu}_e) \equiv R_m \quad \text{Mixed}$$
$$\Gamma(\overline{K}^0_{t=0} \to \overline{K}^0) \propto \Gamma(\overline{K}^0_{t=0} \to \pi^+ e^- \overline{\nu}_e) \equiv \overline{R}_u \quad \text{Unmixed}$$
$$\Gamma(\overline{K}^0_{t=0} \to K^0) \propto \Gamma(K^0_{t=0} \to \pi^- e^+ \nu_e) \equiv \overline{R}_u \quad \text{Mixed}$$

From which we can construct:

$$A(\Delta m) = \frac{(R_u + \overline{R}_u) - (R_m + \overline{R}_m)}{(R_u + \overline{R}_u) + (R_m + \overline{R}_m)}$$
$$= \frac{2e^{-(\Gamma_S + \Gamma_L)t/2}\cos(\Delta m t)}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$

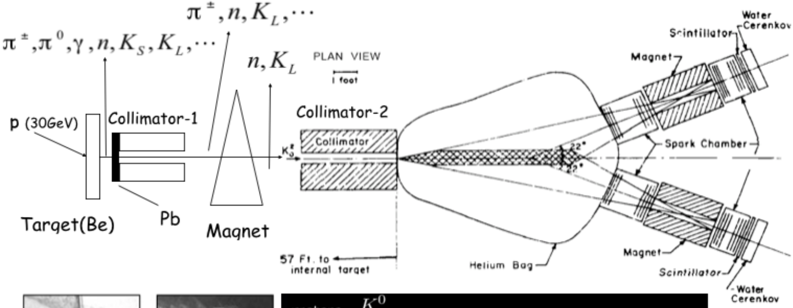
The points are the data and the line shows the theoretical prediction for the value of $\Delta m$ most consistent with the data.



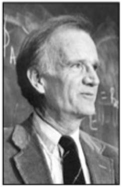

Combining with experiments at Fermilab:

$$\Delta m = m(K_L) - m(K_S) = (3.506 \pm 0.006) \times 10^{-15}\text{GeV}$$

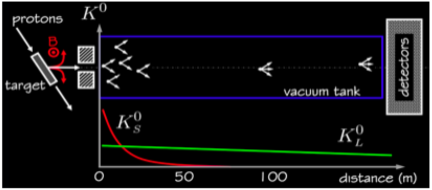
# 1964: Cronin-Fitch Experiment




James Cronin

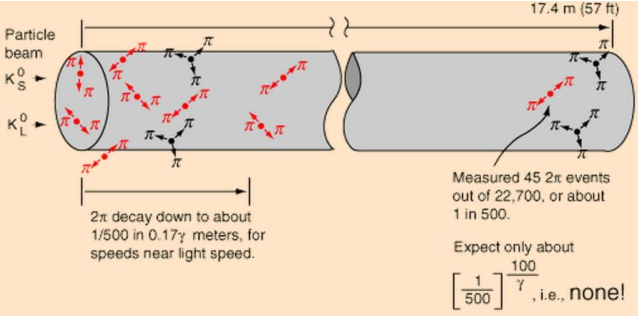


Val Fitch

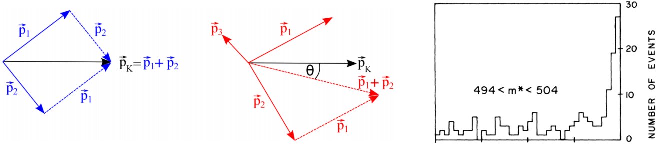



If $CP$ is conserved, we only expect $K_L \to 3\pi$ decays at a long distance from the production point of a beam of neutral kaons (the $K_S$ component will have decayed away).

# 1964: Cronin-Fitch Experiment – $CP$ Violation




In 1964, C&F observed 45 $K_L \to 2\pi$ decays (out of 22.7K kaon decays) a long distance from the production point.




$2\pi$ decay $\quad$ $3\pi$ decay $\quad$ $\cos\theta$

***CP is violated in hadronic weak interactions***

# Lesson from Flavor

Unwise to assume 0.1% is "good enough" in flavor.

- **1962:** "A special search at Dubna was carried out by E. Okonov and his group. They have not found a single $K_L \to \pi^+\pi^-$ event among 600 decays into charged particles (Anikira et al, JETP 1962). At that stage the search was terminated by administration of the Lab. The group was unlucky." L.B. Okun, "Spacetime and vacuum as seen from Moscow" (2002)

- **1964:** Cronin & Fitch observed 45 $K_L \to 2\pi$ decays (out of 22,700 Kaon decays) a long distance from the production point: $\mathcal{B}(K_L \to 2\pi) = 2 \times 10^{-3}$. PRL **13** 138 (1964)

# $CP$ violation in the Kaon system

Recall that we identified the $K_S$ and $K_L$ states as the $CP$ eigenstates $K_1$ and $K_2$ in the limit of no $CP$ violation

CP EVEN (+1) $|K_S\rangle = |K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right)$ $\quad K_S \to \pi\pi$

CP ODD (-1) $|K_L\rangle = |K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right)$ $\quad K_L \to \pi\pi\pi$

Now that we know $CP$ is violated, we must try to account for it:

**One possibility** is that the $K_S$ and $K_L$ do not *exactly* correspond to the $CP$ eigenstates $K_1$ and $K_2$, but rather a linear combination of them with :

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_1\rangle + \varepsilon|K_2\rangle\right]$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]$$

$|K_2\rangle \hookrightarrow 3\pi$, $CP = -1$ $\quad$ $|K_1\rangle \hookrightarrow 2\pi$, $CP = +1$

where $\varepsilon = |\varepsilon|e^{i\phi} = \frac{p-q}{p+q}$ [recall Slide 13 (K4)]

and $|\varepsilon| \approx 2 \times 10^{-3}$

This is known as $CPV$ in mixing, aka *indirect* $CPV$ $(\Delta S = 2)$.

Exercise: Show that these $CP$ violating $|K_S\rangle$, $|K_L\rangle$ in terms of $\varepsilon$ are equivalent to the states in eqns. (K4) & (K5) in terms of $\frac{q}{p}$.

# $K^0 - \overline{K}^0$ Oscillations

*($CP$ violation in mixing)*

# $K^0 - \overline{K}^0$ Oscillations - Theory I *(CP violation)*

*For the full picture, we must re-visit the development of the $K^0 - \overline{K}^0$ system including $CP$ violation in mixing*

- Writing our $|K_S\rangle$ and $|K_L\rangle$ states (inc. $CPV$) in terms of the strong eigenstates $|K^0\rangle$ and $|\overline{K}^0\rangle$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_1\rangle + \varepsilon|K_2\rangle\right] = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\overline{K}^0\rangle\right]$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right] = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\overline{K}^0\rangle\right]$$

- We know from before that the states $|\Psi(t)\rangle$ propagate as $|K_S\rangle$ and $|K_L\rangle$, i.e., independent particles with definite masses and lifetimes.
  Invert these expressions to switch to the $|K_S\rangle$ and $|K_L\rangle$ basis:

$$|K^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1+\varepsilon}\left[|K_L\rangle + |K_S\rangle\right] \quad |\overline{K}^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1-\varepsilon}\left[|K_L\rangle - |K_S\rangle\right]$$

- If we add on the time dependence $\theta_S(t)$ and $\theta_L(t)$, we see that states which were initially produced as $K^0$, or $\overline{K}^0$, evolve with time as

$$\boldsymbol{|\Psi(t)\rangle_{K^0_{t=0}} = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1+\varepsilon}\left[\theta_L(t)|K_L\rangle + \theta_S(t)|K_S\rangle\right]} \quad \text{(K8)}$$

$$\boldsymbol{|\Psi(t)\rangle_{\overline{K}^0_{t=0}} = \sqrt{\frac{1+|\varepsilon|^2}{2}}\frac{1}{1-\varepsilon}\left[\theta_L(t)|K_L\rangle - \theta_S(t)|K_S\rangle\right]} \quad \text{(K9)}$$

# $K^0 - \overline{K}^0$ Oscillations - Theory II *($CP$ violation)*

*If we want to study $K^0 - \overline{K}^0$ oscillations, we need to switch to the $K^0$ $\overline{K}^0$ basis (as we did before in K7)*

- K8 and K9 become.

$$|\Psi(t)\rangle_{K^0_{t=0}} = \frac{1}{2}\left[(\theta_L(t) + \theta_S(t))\,|K^0\rangle + (\theta_L(t) - \theta_S(t))\left(\frac{1-\varepsilon}{1+\varepsilon}\right)|\overline{K}^0\rangle\right]$$
$$|\Psi(t)\rangle_{\overline{K}^0_{t=0}} = \frac{1}{2}\left(\frac{1+\varepsilon}{1-\varepsilon}\right)\left[(\theta_L(t) - \theta_S(t))\,|K^0\rangle + (\theta_L(t) + \theta_S(t))\,|\overline{K}^0\rangle\right]$$

- Finally, as we want the mixing probabilities as a function of time, we calculate, as before:

$$\Gamma(K^0_{t=0} \to K^0) = |\langle K^0|\Psi(t)\rangle_{K^0_{t=0}}|^2$$
$$\Gamma(K^0_{t=0} \to \overline{K}^0) = |\langle \overline{K}^0|\Psi(t)\rangle_{K^0_{t=0}}|^2$$
$$\Gamma(\overline{K}^0_{t=0} \to K^0) = |\langle K^0|\Psi(t)\rangle_{\overline{K}^0_{t=0}}|^2$$
$$\Gamma(\overline{K}^0_{t=0} \to \overline{K}^0) = |\langle \overline{K}^0|\Psi(t)\rangle_{\overline{K}^0_{t=0}}|^2$$

$\Rightarrow$ *Lets see what we can say about the difference in the behavior of matter vs. anti-matter.*

# $K^0 - \overline{K}^0$ Oscillations - Theory III *(CP violation)*

$$\Gamma(K^0_{t=0} \to K^0) = |\langle K^0|\Psi(t)\rangle_{K^0_{t=0}}|^2 = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t}\cos(\Delta m t)\right]$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = |\langle \overline{K}^0|\Psi(t)\rangle_{K^0_{t=0}}|^2 = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t}\cos(\Delta m t)\right]\left|\frac{1-\varepsilon}{1+\varepsilon}\right|^2$$

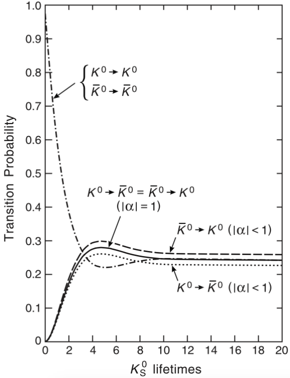
$$\Gamma(\overline{K}^0_{t=0} \to K^0) = |\langle K^0|\Psi(t)\rangle_{\overline{K}^0_{t=0}}|^2 = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t}\cos(\Delta m t)\right]\left|\frac{1+\varepsilon}{1-\varepsilon}\right|^2$$

$$\Gamma(\overline{K}^0_{t=0} \to \overline{K}^0) = |\langle \overline{K}^0|\Psi(t)\rangle_{\overline{K}^0_{t=0}}|^2 = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t}\cos(\Delta m t)\right]$$

In the figure, $\left|\frac{1-\varepsilon}{1+\varepsilon}\right|^2 = |\alpha|$

- $\Gamma(K^0_{t=0} \to K^0) = \Gamma(\overline{K}^0_{t=0} \to \overline{K}^0)$ from $CPT$ symmetry.
- If $|\alpha| = 1$, we recover the equations on S19 for no $CPV$.
- If $|\alpha| \neq 0$, $\Gamma(K^0_{t=0} \to \overline{K}^0) \neq \Gamma(\overline{K}^0_{t=0} \to K^0)$, and if $|\alpha| < 1$, we get less $\overline{K}^0$ than $K^0$, a clear difference as seen in the dashed vs. dotted line.

  (Note: $\varepsilon$ in this model is $10\times$ the measured value, for illustrative purposes)



# $CP$ violation in the $\pi\pi$ system - Theory *($CP$ violation)*

*Lets return to the $\pi\pi$ system where C.&F. first discovered $CPV$ in $K_L \to \pi\pi$.*
*The $\pi\pi$ system is a $CP$ eigenstate, so lets swich to the basis $|K_1\rangle$ and $|K_2\rangle$, states which have definite $CP$.*

- Substituting our new $CP$ violating $|K_S\rangle$ and $|K_L\rangle$ into (K8)

$$|\Psi(t)\rangle_{K^0_{t=0}} = \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}\left[\left(|K_2\rangle + \varepsilon|K_1\rangle\right)\theta_L(t) + \left(|K_1\rangle + \varepsilon|K_2\rangle\right)\theta_S(t)\right]$$

$$= \frac{1}{\sqrt{2}}\frac{1}{1+\varepsilon}\left[\left(\theta_S(t) + \varepsilon\theta_L(t)|K_1\rangle\right) + \left(\theta_L(t) + \varepsilon\theta_S(t)|K_2\rangle\right)\right]$$

$CP$ Eigenstates

- Now, as the $2\pi$ system has $CP = +1$, we know it arises from the decay of the $|K_1\rangle$ eigenstate with $CP = +1$:

$$\Gamma(K^0_{t=0} \to \pi\pi) \propto |\langle K_1||\Psi(t)\rangle_{K^0_{t=0}}|^2 = \frac{1}{2}\left|\frac{1}{1+\varepsilon}\right|^2 |\theta_S(t) + \varepsilon\theta_L(t)|^2$$

- How can we simplify this?

$$\Rightarrow \left|\frac{1}{1+\varepsilon}\right|^2 = \frac{1}{(1+\varepsilon^*)(1+\varepsilon)} \approx \frac{1}{1+2\mathcal{R}\{\varepsilon\}} \approx 1 - 2\mathcal{R}\{\varepsilon\} \qquad \text{for } |\varepsilon| \ll 1$$

$\Rightarrow$ Use the identity $|a \pm b|^2 = |a^2| + |b^2| \pm 2\mathcal{R}(ab^*)$ as before (Slide 18), with $\varepsilon = |\varepsilon|e^{i\phi}$, to evaluate:

$$|\theta_S(t) + \varepsilon\theta_L(t)|^2 = e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon|e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t}\cos(\Delta m t - \phi)$$

Putting it all together:

$$\Gamma(K^0_{t=0} \to \pi\pi) =$$
$$\tfrac{1}{2}\left(1 - 2\mathcal{R}\{\varepsilon\}\right) N_{\pi\pi} \left[ e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon| e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t} \cos(\Delta m t - \phi) \right]$$

$\hookrightarrow K_S \to \pi\pi$ $\quad \hookrightarrow CPV\, K_L \to \pi\pi$ $\quad \hookrightarrow$ interference

Likewise for a beam of initial $\overline{K}^0$:

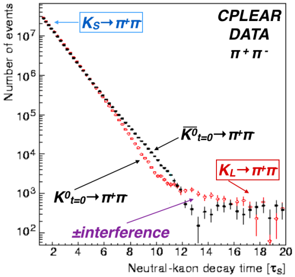
$$\Gamma(\overline{K}^0_{t=0} \to \pi\pi) =$$
$$\tfrac{1}{2}\left(1 + 2\mathcal{R}\{\varepsilon\}\right) N_{\pi\pi} \left[ e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} - 2|\varepsilon| e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t} \cos(\Delta m t - \phi) \right]$$

where the interference term changes sign

Lets look at CPLEAR data for $K^0(\overline{K}^0) \to \pi^+\pi^-$, where we can:

- Distinguish the $K^0_{t=0}$ & $\overline{K}^0_{t=0}$ through tagging
- See the effects of the interference term.
- And clearly see the $CP$ violating $K_L \to \pi^+\pi^-$ at large proper times, where only the long lifetime remains:

$$\Gamma(K^0_{t=0} \to \pi\pi) \to \tfrac{1}{2}\left(1 - 2\mathcal{R}\{\varepsilon\}\right) N_{\pi\pi} |\varepsilon|^2 e^{-\Gamma_L t}$$



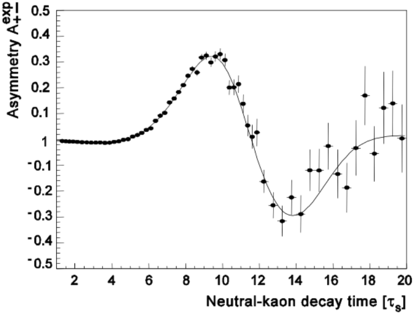
***Can we use the CPLEAR data to measure $\varepsilon = |\varepsilon| e^{i\phi}$?***

Construct the asymmetry:

$$A = \frac{\Gamma(\overline{K}^0_{t=0} \to \pi\pi) - \Gamma(K^0_{t=0} \to \pi\pi)}{\Gamma(\overline{K}^0_{t=0} \to \pi\pi) + \Gamma(K^0_{t=0} \to \pi\pi)} = 2\mathcal{R}\{\varepsilon\} - \frac{2|\varepsilon| e^{(\Gamma_S - \Gamma_L)t/2} \cos(\Delta m t - \phi)}{1 + |\varepsilon|^2 e^{(\Gamma_S - \Gamma_L)t}}$$

Best fit to the data:

$$|\varepsilon| = (2.264 \pm 0.035) \times 10^{-3})$$
$$\phi = (43.19 \pm 0.73)^\circ$$



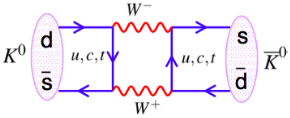
# $K^0 - \overline{K}^0$ Oscillations and the CKM Matrix *(CP violation)*

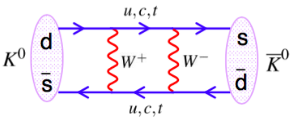
We've now seen the $CPV$ in mixing leads to the inequality:

$$\Gamma(K^0_{t=0} \to \overline{K}^0) \neq \Gamma(\overline{K}^0_{t=0} \to K^0)$$

**How can we explain this in terms of the CKM matrix?**

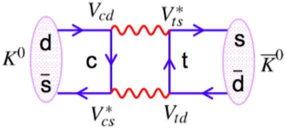
Recall the box diagrams responsible for mixing, *where we need to sum over all possible quark exchanges in the box.*

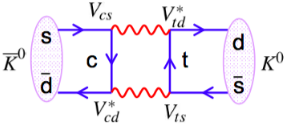



$K^0 \to \overline{K}^0$ $\qquad$ $\overline{K}^0 \to K^0$

For illustration, lets consider only this diagram, and compare the equivalent box diagrams for $K^0 \to \overline{K}^0$ and $\overline{K}^0 \to K^0$.




$M_{fi} \propto A_{ct} V_{cd} V^*_{cs} V_{td} V^*_{ts}$ $\qquad$ $M_{if} \propto A_{ct} V^*_{cd} V_{cs} V^*_{td} V_{ts} = M^*_{if}$

The difference in rates:

$\Gamma(K^0 \to \overline{K}^0) - \Gamma(\overline{K}^0 \to K^0) \propto M_{fi} - M^*_{if} = 2\mathcal{I}\{M_{fi}\}$

is non-zero only if the CKM matrix has an imaginary component

# Extra reading


- Richman, Jeremy D. (UCSB), *Heavy Quark Phyiscs and $CP$ Violation*.
  http://physics.ucsd.edu/students/courses/winter2010/physics222/references/driver_houches12.pdf
  Pages 183-204.
- Thomson, Mark (Cambridge University), *Particle Physics Lecture Handouts*,
  http://www.hep.phy.cam.ac.uk/ thomson/partIIIparticles/welcome.html
  The CKM Matrix and CP Violation, Pages 416-437.
- Kooijman, P. & Tuning, N., *$CP$ Violation*,
  http://master.particles.nl/LectureNotes/2011-CP.pdf
  Pages 55-64.
- The CPLEAR Collaboration, *The CPLEAR-Experiment at CERN.*