



CP violation in the interference between mixing and decay, and an introduction to B -factories

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Reading material and references

Lecture material based on several textbooks and online lectures/notes.

Credits for material and figures include:

Literature

- Perkins, Donald H. (2000), *Introduction to High Energy Physics*.
- Griffiths, David J. (2nd edition), *Introduction to Elementary Particles*.
- Stone, Sheldon (2nd edition), *B decays*.

Online Resources

- Belle/BaBar Collaborations, *The Physics of the B-Factories*.
<http://arxiv.org/abs/1406.6311>
- Bona, Marcella (University of London), *CP Violation Lecture Notes*,
<http://pprc.qmul.ac.uk/bona/ulpg/cpv/>
- Richman, Jeremy D. (UCSB), *Heavy Quark Physics and CP Violation*.
http://physics.ucsd.edu/students/courses/winter2010/physics222/references/driver_houches12.pdf
- Thomson, Mark (Cambridge University), *Particle Physics Lecture Handouts*,
<http://www.hep.phy.cam.ac.uk/thomson/partIIIparticles/welcome.html>
- Grossman, Yuval (Cornell University), *Just a Taste. Lectures on Flavor Physics*,
<http://www.lepp.cornell.edu/pt267/files/notes/FlavorNotes.pdf>
- Kooijman, P. & Tuning, N., *CP Violation*,
<https://www.nikhef.nl/h71/Lectures/2015/ppII-cpviolation-29012015.pdf>

Recap & outline

So far, we:

- Developed the framework to describe particle anti-particle oscillations, both with and without CP violation. This was done for the neutral Kaon system, but is applicable to all neutral particle systems.
- Saw that CP -violation in mixing leads to $\Gamma(K_{t=0}^0 \rightarrow \bar{K}^0) \neq \Gamma(\bar{K}_{t=0}^0 \rightarrow K^0)$
- Studied direct CP violation in the B meson system, and looked at the example of $B \rightarrow K\pi$ decays which is currently an open question, referred to as the “ $K\pi$ CP -puzzle.”

Today, we'll:

- Study CPV in B^0 - \bar{B}^0 oscillations.
- Finally, we'll introduce a third and the final type of CP violation, **CP violation in interference between mixing and decay** and study the “golden mode” $B^0 \rightarrow J/\psi K_S^0$.
- We'll also introduce B factories and experimental techniques along the way.

Recap: $K^0 - \bar{K}^0$ Oscillations with CP violation

Recall our derivation of mixing in the kaon system including CP violation

(Lecture II, S28):

- Writing our $|K_S\rangle$ and $|K_L\rangle$ states (inc. CPV) in terms of the strong eigenstates $|K^0\rangle$ and $|\bar{K}^0\rangle$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} [|K_1\rangle + \varepsilon|K_2\rangle] = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\varepsilon|^2}} [(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\bar{K}^0\rangle]$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} [|K_2\rangle + \varepsilon|K_1\rangle] = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\varepsilon|^2}} [(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle]$$

- We know from before that the states $|\Psi(t)\rangle$ propagate as $|K_S\rangle$ and $|K_L\rangle$, i.e., independent particles with definite masses and lifetimes.

Invert these expressions to switch to the $|K^0\rangle$ and $|\bar{K}^0\rangle$ basis:

$$|K^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1+\varepsilon} [|K_L\rangle + |K_S\rangle] \quad |\bar{K}^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1-\varepsilon} [|K_L\rangle - |K_S\rangle]$$

- If we add on the time dependence $\theta_S(t)$ and $\theta_L(t)$, we see that states which were initially produced as K^0 , or \bar{K}^0 , evolve with time as

$$|\Psi(t)\rangle_{K^0_{t=0}} = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1+\varepsilon} [\theta_L(t)|K_L\rangle + \theta_S(t)|K_S\rangle] \quad (\text{K8})$$

$$|\Psi(t)\rangle_{\bar{K}^0_{t=0}} = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1-\varepsilon} [\theta_L(t)|K_L\rangle - \theta_S(t)|K_S\rangle] \quad (\text{K9})$$

Recap: $K^0 - \bar{K}^0$ Oscillations with CP violation

If we want to study $K^0 - \bar{K}^0$ oscillations, we need to switch to the $K^0 \bar{K}^0$ basis

- K8 and K9 become.

$$|\Psi(t)\rangle_{K^0_{t=0}} = \frac{1}{2} \left[(\theta_L(t) + \theta_S(t)) |K^0\rangle + (\theta_L(t) - \theta_S(t)) \left(\frac{1-\varepsilon}{1+\varepsilon} \right) |\bar{K}^0\rangle \right]$$

$$|\Psi(t)\rangle_{\bar{K}^0_{t=0}} = \frac{1}{2} \left(\frac{1+\varepsilon}{1-\varepsilon} \right) \left[(\theta_L(t) - \theta_S(t)) |K^0\rangle + (\theta_L(t) + \theta_S(t)) |\bar{K}^0\rangle \right]$$

Now let's change to the B -meson system. Recall we stated that the time-dependent formalism of mixing we derived for the Kaon system applies to all neutral particle systems undergoing oscillation.

- Replace K with B , and for simplicity, let's work in terms of $\frac{q}{p}$ (recall, $\frac{q}{p} = \frac{1-\varepsilon}{1+\varepsilon}$).
- Recall that in the Kaon system, the $|K_{\pm}\rangle$ eigenstates we obtained by solving the time-dependent Schrödinger equation had significantly different lifetimes, which motivated us to name them “short” and “long” lifetime states $|K_S\rangle$ and $|K_L\rangle$.

For the B meson system, the lifetimes are almost identical. Instead the eigenstates have significantly different masses, which motivates us to rename them as “heavy” and “light.”

→ Denote the K_S eigenstate of the Kaon system as the “Heavy” eigenstate of the B system (B_H).

→ Denote the K_L eigenstate of the Kaon system as the “Light” eigenstate of the B system (B_L).

- Write the time-dependence explicitly, switching the subscripts $S \rightarrow H$ for the heavy eigenstate. Recall that for our derivation in the kaon system we had abbreviated this as follows:

$$\theta_S(t) = e^{-\left(im_S + \frac{\Gamma_S}{2}\right)t}, \quad \theta_L(t) = e^{-\left(im_L + \frac{\Gamma_L}{2}\right)t}$$

B meson system

The time dependence of state $|\Psi(t)\rangle$ which was a B^0 at $t = 0$ is given by:

$$|\Psi(t)\rangle_{B^0_{t=0}} = |B^0(t)\rangle = e^{-iMt} e^{-\frac{\Gamma}{2}t} \left[\cos\left(\frac{\Delta Mt}{2}\right) |B^0\rangle + i \sin\left(\frac{\Delta Mt}{2}\right) \frac{q}{p} |\bar{B}^0\rangle \right]$$

and for a state which was a \bar{B}^0 at $t = 0$ is given by:

$$|\Psi(t)\rangle_{\bar{B}^0_{t=0}} = |\bar{B}^0(t)\rangle = e^{-iMt} e^{-\frac{\Gamma}{2}t} \left[\cos\left(\frac{\Delta Mt}{2}\right) |\bar{B}^0\rangle + i \sin\left(\frac{\Delta Mt}{2}\right) \frac{p}{q} |B^0\rangle \right]$$

where we've abbreviated:

$$M = \frac{1}{2}(M_L + M_H), \quad \Delta M = M_H - M_L, \quad \Gamma = \Gamma_L = \Gamma_H$$

since the difference in lifetimes between the B_L and B_H is $\Delta\Gamma/\Gamma \ll 1$.

Note: Here we've introduced $|B^0(t)\rangle$ and $|\bar{B}^0(t)\rangle$ to denote the states $|\Psi(t)\rangle_{B^0_{t=0}}$ and $|\Psi(t)\rangle_{\bar{B}^0_{t=0}}$ for simplicity only.

Sometimes these are also called $|B^0_{\text{phys}}(t)\rangle$ and $|\bar{B}^0_{\text{phys}}(t)\rangle$ to remind readers what the initial state is at $t = 0$ (which is NOT what the state is at time t since it's a superposition of $|B^0\rangle$ & $|\bar{B}^0\rangle$!).

Time-evolution of B decays to a CP eigenstate

Now let these states decay into a common final state $|f_{CP}\rangle$ with eigenvalue $\eta_{CP}(f) = \pm 1$.

The amplitude for $B^0(t) \rightarrow f_{CP}$ is given by:

$$\langle f_{CP}|H|B^0(t)\rangle = e^{-iMt}e^{-\frac{\Gamma}{2}t} \left[\cos\left(\frac{\Delta Mt}{2}\right) \langle f_{CP}|H|B^0\rangle + i \sin\left(\frac{\Delta Mt}{2}\right) \frac{q}{p} \langle f_{CP}|H|\bar{B}^0\rangle \right]$$

while that for a $\bar{B}^0(t) \rightarrow f_{CP}$ is

$$\langle f_{CP}|H|\bar{B}^0(t)\rangle = e^{-iMt}e^{-\frac{\Gamma}{2}t} \left[\cos\left(\frac{\Delta Mt}{2}\right) \langle f_{CP}|H|\bar{B}^0\rangle + i \sin\left(\frac{\Delta Mt}{2}\right) \frac{p}{q} \langle f_{CP}|H|B^0\rangle \right]$$

Here we can explicitly see the 2 amplitudes whose interference can produce a CP asymmetry.

Lets define a quantity λ_f to express the ratio of amplitudes

$$\lambda_f \equiv \frac{q \bar{A}_f}{p A_f} = \frac{q \langle f_{CP}|H|\bar{B}^0\rangle}{p \langle f_{CP}|H|B^0\rangle} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \frac{\langle f_{CP}|H|\bar{B}^0\rangle}{\langle f_{CP}|H|B^0\rangle}$$

(not to be confused with the Wolfenstein parameter λ).

Time-evolution of B decays to a CP eigenstate

To see how this λ_f comes into play, factor out $\langle f_{CP}|H|B^0\rangle$:

$$\langle f_{CP}|H|B^0(t)\rangle = e^{-iMt} e^{-\frac{\Gamma}{2}t} \langle f_{CP}|H|B^0\rangle \left[\cos\left(\frac{\Delta Mt}{2}\right) + i \sin\left(\frac{\Delta Mt}{2}\right) \frac{q}{p} \frac{\langle f_{CP}|H|\bar{B}^0\rangle}{\langle f_{CP}|H|B^0\rangle} \right]$$

$$\langle f_{CP}|H|\bar{B}^0(t)\rangle = e^{-iMt} e^{-\frac{\Gamma}{2}t} \langle f_{CP}|H|B^0\rangle \left[\cos\left(\frac{\Delta Mt}{2}\right) \frac{\langle f_{CP}|H|\bar{B}^0\rangle}{\langle f_{CP}|H|B^0\rangle} + i \sin\left(\frac{\Delta Mt}{2}\right) \frac{p}{q} \right]$$

If the initial state at $t = 0$ is a $|B^0\rangle$, the *probability* to produce $|f_{CP}\rangle$ at time t is thus

$$\begin{aligned} & |\langle f_{CP}|H|B^0(t)\rangle|^2 = \\ & e^{-\Gamma t} |\langle f_{CP}|H|B^0\rangle|^2 \cdot \left[\frac{1}{2}(1 + |\lambda|^2) + \frac{1}{2}(1 - |\lambda|^2) \cos(\Delta Mt) - \text{Im}\lambda \cdot \sin(\Delta Mt) \right] \end{aligned}$$

Likewise for an initial state of $|\bar{B}^0\rangle$ at $t = 0$, the *probability* to produce $|f_{CP}\rangle$ at time t is

$$\begin{aligned} & |\langle f_{CP}|H|\bar{B}^0(t)\rangle|^2 = \\ & e^{-\Gamma t} |\langle f_{CP}|H|B^0\rangle|^2 \left| \frac{p}{q} \right|^2 \cdot \left[\frac{1}{2}(1 + |\lambda|^2) + \frac{1}{2}(1 - |\lambda|^2) \cos(\Delta Mt) + \text{Im}\lambda \cdot \sin(\Delta Mt) \right] \end{aligned}$$

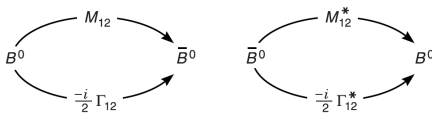
Notice how similar these expressions are. The only differences are the \pm sign of the last term and the presence of $\left| \frac{p}{q} \right|^2$ in the second equation.

What can we say about $\frac{q}{p}$ in the B meson system?

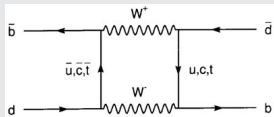
- Recall the definition from **Lecture II**:

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

- The interference occurs between the set of amplitudes with **short-distance, virtual** (aka **off-shell**) intermediate states (M_{12}) and **long-distance, on-shell intermediate states** (Γ_{12}).



M_{12} : The off-shell intermediate states, e.g., $t\bar{t}$, arise from the box-diagrams



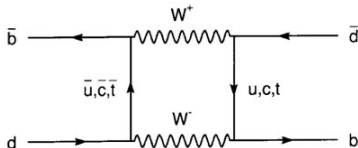
Γ_{12} : While a B can also evolve to a \bar{B}^0 through on-shell intermediate states such as $K^+ K^-$, with mass $M_{K^+ K^-} = M_{B^0}$

The B meson mixing phase $\phi_M(B^0)$

- Γ_{12} is very small for B^0 - \bar{B}^0 mixing, so we can approximate

$$\frac{q}{p} \approx \sqrt{\frac{M_{12}^*}{M_{12}}}$$

- The short-distance, off-shell contribution from M_{12} depends on the size of the CKM-elements at the corners of the box-diagram, and on the mass of the particles in the box.



- In the SM, these amplitudes are completely dominated by the box diagrams with $t\bar{t}$, giving us:

$$M_{12} \propto (V_{tb} V_{td}^*)^2 e^{-2i\theta_{CP}(B)}$$

$$M_{12}^* \propto (V_{tb}^* V_{td})^2 e^{+2i\theta_{CP}(B)}$$

- Thus we have $\frac{q}{p} \approx \sqrt{\frac{M_{12}^*}{M_{12}}} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} e^{2i\theta_{CP}(B)} = e^{2i\phi_M(B^0)} e^{2i\theta_{CP}(B)}$

$$\Rightarrow \left| \frac{q}{p} \right| = 1 \quad \text{Very important result!}$$

Time-dependent CP asymmetry

Using this result, we can write our time-dependent CP asymmetry using the rates we derived on S7, which includes the effects of interference between mixing and decay, as

$$\begin{aligned} \mathcal{A}_{CP}(t) &= \frac{|\langle f_{CP} | H | B^0(t) \rangle|^2 - |\langle f_{CP} | H | \bar{B}^0(t) \rangle|^2}{|\langle f_{CP} | H | B^0(t) \rangle|^2 + |\langle f_{CP} | H | \bar{B}^0(t) \rangle|^2} \\ &= \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \cos(\Delta M t) - \frac{2\text{Im}\lambda}{1 + |\lambda|^2} \sin(\Delta M t) \\ &= A_f \cos(\Delta M t) - S_f \sin(\Delta M t) \end{aligned}$$

$$A_f \equiv \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \quad S_f \equiv \frac{2\text{Im}\lambda}{1 + |\lambda|^2}$$

sometimes called A_{CP} and S_{CP} , or just A and S

How do we calculate λ_f ?

For a given final state f , the magnitude and phase of λ_f fully describe the decay and oscillation of the B^0 - and the \bar{B}^0 -meson.

Our job is to search for decays which we can use to extract and *measure* a meaningful asymmetry.

On slide 10 we looked at $\frac{q}{p}$ and defined the B mixing angle ϕ_M .

Now lets study the decays of $B^0 \rightarrow f_{CP}$ and $\bar{B}^0 \rightarrow f_{CP}$.

Recall from our discussion of CPV in decay, that the total amplitude for the $B^0 \rightarrow f_{CP}$ decay is written as:

$$A \equiv A(B^0 \rightarrow f_{CP}) = \langle f_{CP} | H | B^0 \rangle = \sum_j A_j = \sum_j a_j e^{i\phi_j} = \sum_j |a_j| e^{i\delta_j} e^{i\phi_j}$$

where ϕ_j is the weak phase which changes sign under CP , and δ_j is the CP -conserving strong phase.

How do we calculate λ_f ?

Now lets look at a special case:

When the direct decay to f_{CP} is dominated by a single amplitude.

The $B^0 \rightarrow f_{CP}$ simplifies to one term:

$$\langle f_{CP} | H | B^0 \rangle = |a| e^{i(\delta + \phi)}$$

For the $\bar{B}^0 \rightarrow f_{CP}$ decay, we have

$$\langle f_{CP} | H | \bar{B}^0 \rangle = \eta_{CP}(f) e^{-2i\theta_{CP}(B)} |a| e^{i(\delta - \phi)}$$

What are these extra terms?

- $e^{-2i\theta_{CP}(B)}$ should ring a bell. It's the *intrinsic CP* phase factor associated with B . (Recall **Lecture III S5** when we derived the condition for direct CPV).
- $\eta_{CP}(f)$ is the CP eigenvalue of the final state.

Recall how we determined this in **Lecture II S6-7**, for Kaon decays to $2\pi, 3\pi$:
the 2 pion system has eigenvalue $\eta_{CP}(f) = +1$ [$\mathcal{CP}|\pi^0\pi^0\rangle = +|\pi^0\pi^0\rangle$],
while the 3 pion system has $\eta_{CP}(f) = -1$ [$\mathcal{CP}|\pi^0\pi^0\pi^0\rangle = -|\pi^0\pi^0\pi^0\rangle$]

How do we calculate λ_f ?

Lets put the pieces together

Start with our definition:

$$\lambda_f \equiv \frac{q}{p} \frac{\langle f_{CP} | H | \bar{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle} = \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}} \frac{\langle f_{CP} | H | \bar{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle}$$

Substitute in our result from B^0 - \bar{B}^0 oscillation

$$\lambda_f = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} e^{2i\theta_{CP}(B^0)} \frac{\langle f_{CP} | H | \bar{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle}$$

recalling how we defined the mixing phase $\phi_M(B^0)$

$$\lambda_f = e^{2i\phi_M(B^0)} e^{2i\theta_{CP}(B^0)} \frac{\langle f_{CP} | H | \bar{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle}$$

and including our result from the direct decays $B^0 \rightarrow f_{CP}$ & $\bar{B}^0 \rightarrow f_{CP}$

$$\lambda_f = e^{2i\phi_M(B^0)} e^{2i\theta_{CP}(B^0)} \frac{\eta_{CP}(f) e^{-2i\theta_{CP}(B)} |a| e^{i(\delta-\phi)}}{|a| e^{i(\delta+\phi)}}$$

We arrive at the final expression:

$$\lambda_f = \eta_{CP}(f) e^{2i(\phi_M(B^0) - \phi)}$$

$\mathcal{A}_{CP}(t)$ for a single weak phase in the decay amplitude

What can we obtain from this result?

$$\lambda_f = \eta_{CP}(f) e^{2i(\phi_M(B^0) - \phi)}$$

Immediately we see that $|\lambda_f| = 1$

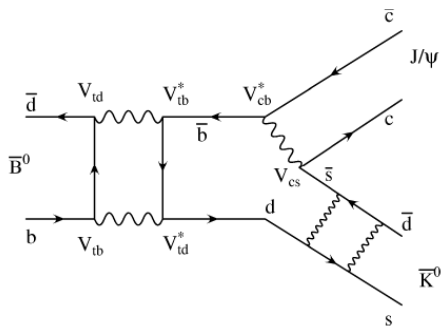
How can we take advantage of this?

It simplifies our $\mathcal{A}_{CP}(t)$ considerably

$$\begin{aligned}\mathcal{A}_{CP}(t) &= \frac{|\langle f_{CP} | H | B^0(t) \rangle|^2 - |\langle f_{CP} | H | \bar{B}^0(t) \rangle|^2}{|\langle f_{CP} | H | B^0(t) \rangle|^2 + |\langle f_{CP} | H | \bar{B}^0(t) \rangle|^2} \\ &= \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \cos(\Delta M t) - \frac{2 \operatorname{Im} \lambda}{1 + |\lambda|^2} \sin(\Delta M t) \\ &= -\operatorname{Im} \lambda \sin(\Delta M t)\end{aligned}$$

\Rightarrow Now our task is to find a decay where we can exploit this nice result

The golden mode: $B^0 \rightarrow J/\psi K_S^0$

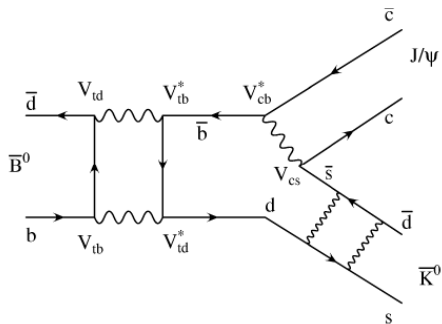


Two important criteria:

- The $J/\psi K_S^0$ is a CP eigenstate accessible to **both** B^0 and \bar{B}^0 .
- This is the only major diagram contributing to this decay, so there is only one weak phase ϕ .

\Rightarrow now lets study this diagram in detail

The golden mode: $B^0 \rightarrow J/\psi K_S^0$



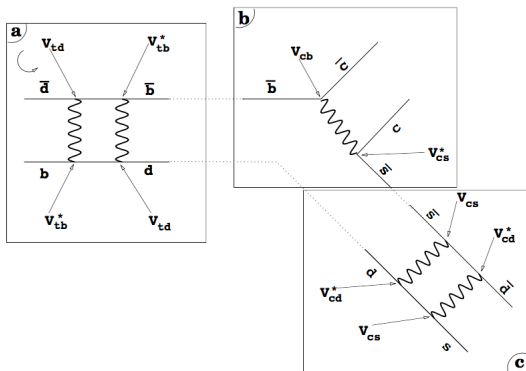
$$\lambda_{J/\psi K_S^0} = \left(\frac{q}{p}\right)_{B^0} \left(\eta_{J/\psi K_S^0} \frac{\bar{A}_{J/\psi K_S^0}}{A_{J/\psi K_S^0}} \right) = - \left(\frac{q}{p}\right)_{B^0} \left(\frac{\bar{A}_{J/\psi \bar{K}^0}}{A_{J/\psi K^0}} \right) \left(\frac{p}{q}\right)_K$$

Do you see why we need the K^0 - \bar{K}^0 mixing?

$B^0 \rightarrow J/\psi K^0$ but $\bar{B}^0 \rightarrow J/\psi \bar{K}^0$, so the K^0 's must mix in order to reach the same final state f for the B^0 and \bar{B}^0 decay (recall $|K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$).

Do you see why the CP eigenvalue of $J/\psi K_S^0$ is -1?

The golden mode: $B^0 \rightarrow J/\psi K_S^0$



In terms of the CKM matrix elements

(a) B^0 - \bar{B}^0 mixing:

$$\begin{pmatrix} q \\ p \end{pmatrix}_{B^0} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}$$

(b) $b \rightarrow c$ decay

$$\begin{pmatrix} \bar{A} \\ A \end{pmatrix} = \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}$$

(c) K^0 - \bar{K}^0 mixing:

$$\begin{pmatrix} p \\ q \end{pmatrix}_K = \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}$$

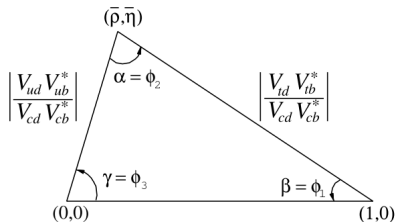
The golden mode: $B^0 \rightarrow J/\psi K_S^0$

Lets put the pieces together

$$\lambda_{J/\psi K_S^0} = (-1) \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \cdot \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \cdot \frac{V_{cs} V_{cd}}{V_{cs}^* V_{cd}} = (-1) \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \cdot \frac{V_{cb} V_{cd}}{V_{cb}^* V_{cd}}$$

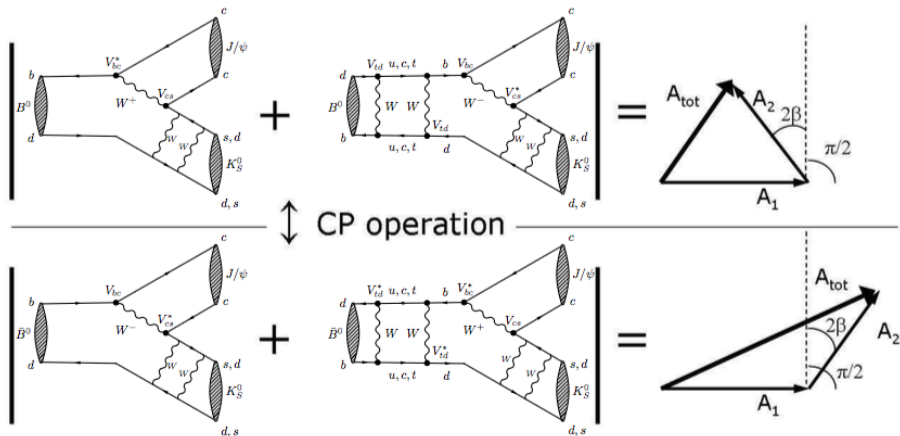
$$\text{Im} \lambda_{J/\psi K_S^0} = -\sin \left\{ \arg \left(\frac{V_{tb}^* V_{td} V_{cb} V_{cd}}{V_{tb} V_{td}^* V_{cb}^* V_{cd}} \right) \right\} = -\sin \left\{ 2 \arg \left(\frac{V_{cb} V_{cd}}{V_{tb} V_{td}^*} \right) \right\} \equiv \sin(2\beta)$$

$$\mathcal{A}_{CP}(t) = -\sin(2\beta) \sin(\Delta M t)$$



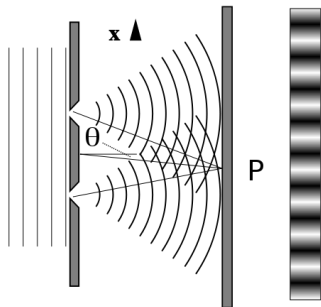
How can we visualize the total amplitude?

Adding two amplitudes results in a A_{tot} with a different magnitude under CP .



Recall your QM basics

The difference in decay rates arises from a different interference term for the matter vs. antimatter process. Analogy to double-slit experiment:



Classical double-slit experiment

Relative phase variation due to different path lengths leads to interference pattern in space.

Observing CP violation with B -mesons is much more difficult than with Kaons

- B_L and B_H have essentially the same lifetimes. No way to get a beam of B_L and look for “forbidden” decay modes.
- It is much harder to produce large quantities of B -mesons than Kaons.

\Rightarrow *Kobayashi-Maskawa mechanism quantitatively predicts large CP violating asymmetries in the decays of the B meson system. Worthwhile to try to measure!*

Time evolution of $B^0\bar{B}^0$ pairs at B -factories

Production mechanism:

$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^0\bar{B}^0$$

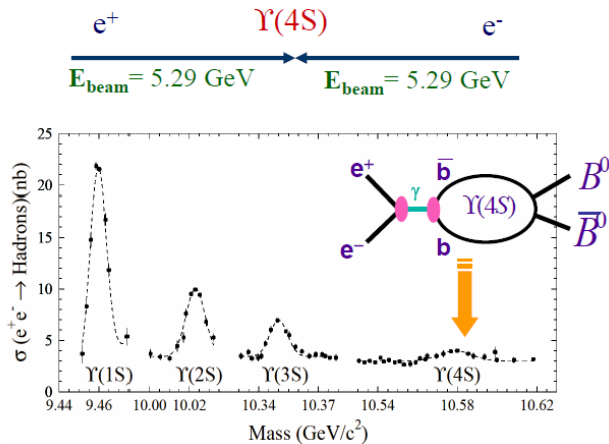
The wave function of the B meson pair in a coherent P – *wave* state:

$$\Psi = \frac{1}{\sqrt{2}} [|B^0\rangle|\bar{B}^0\rangle - |\bar{B}^0\rangle|B^0\rangle]$$

- At all times one B^0 and one \bar{B}^0 meson, until one of them decays.
- The remaining un-decayed B meson will continue to propagate through space-time and mix until it decays.

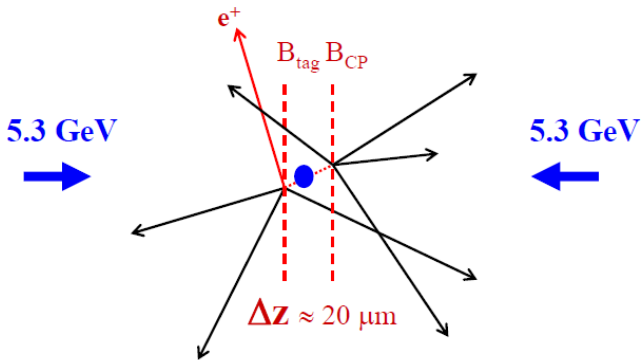
B mesons produced in $\Upsilon(4S)$ decays

What can we say about the B -mesons produced from beams where $E(e^+) = E(e^-)$?



- Enough energy to barely produce 2 B mesons, nothing else!
- B -mesons produced with $\sim 300 \text{ MeV}$ momentum

B mesons produced in $\Upsilon(4S)$ decays



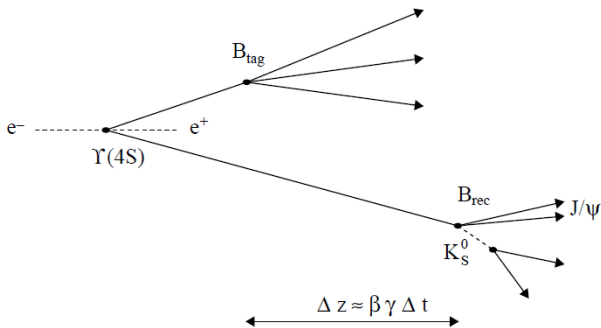
- Experimentally the decay time is measured by measuring the decay length
- Distances of a few $10 \mu\text{m}$ are too small to measure

Asymmetric energy B -factories

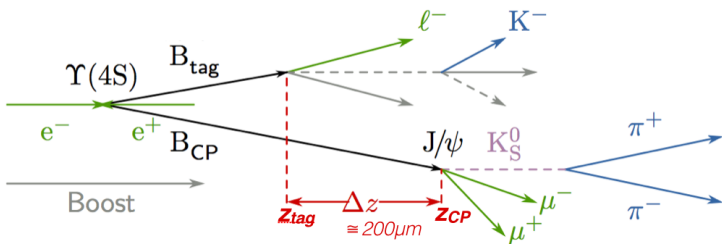
Solution:

- Boost $\Upsilon(4S)$ in laboratory frame by colliding beams of unequal energy but same CM energy

$$E_{CM}^2 = 4E_{LER}E_{HER} = m_{\Upsilon(4S)}^2$$



Asymmetric energy B -factories



- Decay of first B (as B^0) at t_{tag} ensures the other B is \bar{B}^0 .
 \Rightarrow *End of quantum entanglement!*
 Defines a reference time
- At $t > t_{\text{tag}}$, B^0 can mix to \bar{B}^0 before it decays.
- Possible outcomes:

$$B^0 B^0, B^0 \bar{B}^0, \bar{B}^0 \bar{B}^0$$

Asymmetry as a function of Δt

In our derivation earlier, we simply used time t , but now it's clear we need to consider Δt , defined as $t_{CP} - t_{tag}$

Our general time-dependent asymmetry as a function of $\Delta t = t_{CP} - t_{tag}$ is:

$$\mathcal{A}_{CP}(\Delta t) = \frac{f_{B_{tag=B^0}}(\Delta t) - f_{B_{tag=\bar{B}^0}}(\Delta t)}{f_{B_{tag=B^0}}(\Delta t) + f_{B_{tag=\bar{B}^0}}(\Delta t)} = \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \cos(\Delta M \Delta t) - \frac{2 \text{Im}[\lambda]}{1 + |\lambda|^2} \sin(\Delta M \Delta t)$$

\Rightarrow Need to know the flavor of the B_{tag}

Flavor tagging

B^0 or \bar{B}^0 flavor identified from the decay products.

Several different categories of tagging, e.g.,

- ① Lepton tag
- ② Kaon tag
- ③ Pion tag
- ④ Lambda tag
- ...

Primary leptons originate directly from B mesons in semileptonic decays. These modes are due to leading order weak interaction mediated by a charged W^\pm boson. The charge of

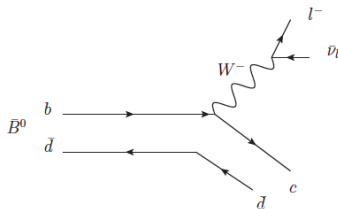


FIG. 3: Production process for direct leptons.

direct leptons is associated with the flavor of its mother particle (Fig. 3). In a $b \rightarrow c l^- \bar{\nu}_l$ semileptonic decay a positively (negatively) charged lepton indicates a B^0 (\bar{B}^0) decay

$$\bar{B}^0 \rightarrow X l^- \bar{\nu}_l, \quad (3)$$

where X indicates another hadronic particle.

Secondary leptons descend from semileptonic decaying D mesons via $b \rightarrow c \rightarrow s$ transitions. In this cascade decay the charge of the lepton corresponding to the B meson is reversed: a negatively (positively) charged lepton indicates a B^0 (\bar{B}^0)

$$\bar{B}^0 \rightarrow DX \rightarrow X' l^+ \nu_l. \quad (4)$$

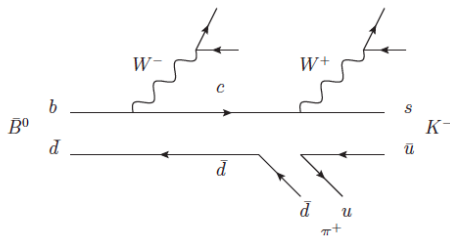
Kaon tag

3. Kaons

Kaons give the most powerful flavor identification and also have a high occurrence in B^0 decays. They mainly originate from $b \rightarrow c \rightarrow s$ cascade decays

$$\begin{aligned} \bar{B}^0 &\rightarrow DX \\ &\rightarrow K^- X'. \end{aligned} \quad (7)$$

A positively (negatively) charged kaon indicates a B^0 (\bar{B}^0), as illustrated in Fig. 7. Since the mother of the kaon is unclear, it can also originate from charm decay or $s\bar{s}$ quark pair popping out of the vacuum, therefore combining the total charge is important. Emerging K_S^0 can indicate a kaon from $s\bar{s}$ quark pair popping. In addition to kinematic variables like p_{cms} and θ_{lab} , the charge and PID can help to identify candidates.



Pion tag

Pions are the most common final state particles. Slow pions originate from a $D^{*\pm}$ decay, where a negatively (positively) charged pion indicates a B^0 (\bar{B}^0)

$$\begin{aligned}\bar{B}^0 &\rightarrow D^{*+} X \\ &\rightarrow D^0 \pi^+.\end{aligned}\tag{5}$$

Because of the low mass difference between the D^{*+} and the D^0 , slow pions are produced nearly at rest in the D^{*+} frame. The pion moves nearly in the same direction as the D^0 in the B_{tag} frame. On the contrary, pions coming from the hadronization of a W boson have

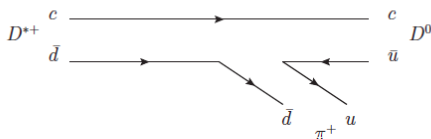


FIG. 5: Production process for slow pions.

a higher momentum, e.g., in the decay

$$\bar{B}^0 \rightarrow D^{*+} \pi^- X.\tag{6}$$

Lambda tag

4. Lambdas

Lambdas are not directly measured as final state particles but have to be reconstructed from protons and pions. They can arise through a $b \rightarrow c \rightarrow s$ cascade decay, such as

$$\begin{aligned}\bar{B}^0 &\rightarrow \bar{\Lambda}_c^+ X \\ &\rightarrow \bar{\Lambda}_0 X'.\end{aligned}\tag{8}$$

Despite their very low occurrence in B meson events, they are valuable for flavor determination. A lambda (anti-lambda) indicates a B^0 (\bar{B}^0). The quality of a lambda candidate depends on a correct reconstruction, therefore the quality of the lambda vertex is of interest. The angle between the lambda momentum, its vertex and the interaction point θ_Λ can also give information about a good candidate, as well as its mass M_Λ .

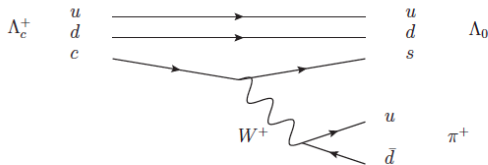
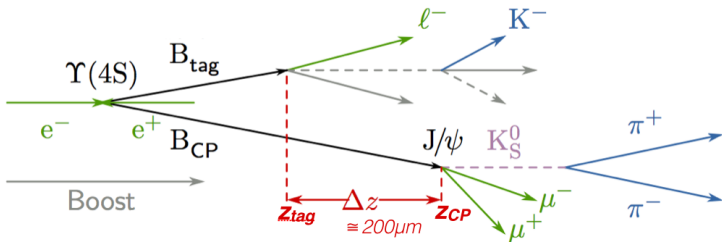


FIG. 8: Production process for lambdas.

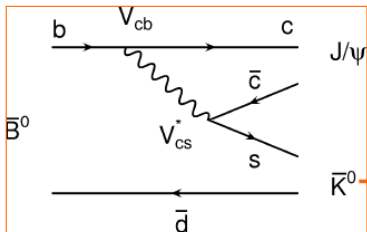
B-meson reconstruction

How do we build our B -mesons?



\Rightarrow Work backwards from the final-state particles, which we can identify in our detector

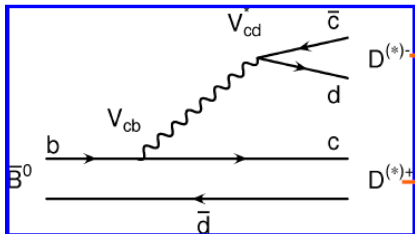
Start with the final-state particles (e, μ, π, K, γ)



$J/\psi \rightarrow \mu^+ \mu^-$ or $e^+ e^-$

$\bar{K}^0 \rightarrow \pi^+ \pi^-$ displaced vertex ($c\tau=2.7\text{cm}$)

⇒ Work Backwards



$D^{(*)-} \rightarrow D^0 \pi^+$
 \searrow
 $K^- \pi^+$

$D^{(*)+} \rightarrow D^- \pi^0$
 $\searrow \searrow \gamma\gamma$
 $K^+ \pi^- \pi^-$

Use energy-momentum to build the intermediate states

- Reconstruct J/ψ (first measurement @ Belle with $5.5fb^{-1}$)

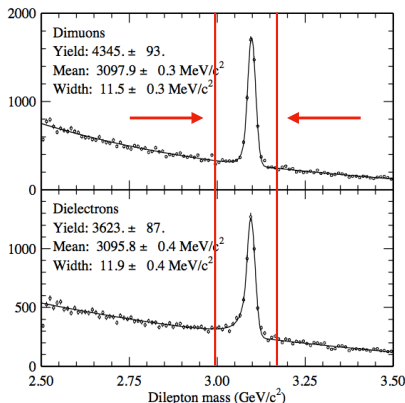


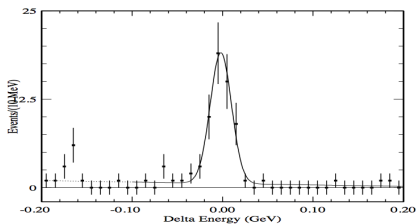
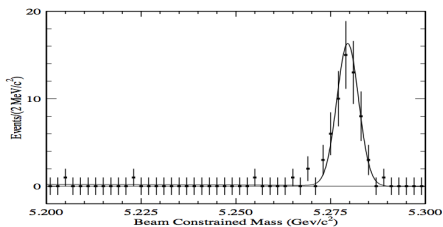
Figure 2: The J/ψ invariant mass distributions for the 5.5 fb^{-1} inclusive J/ψ data.

Make “cuts” (vertical lines) to remove obvious background-dominated regions

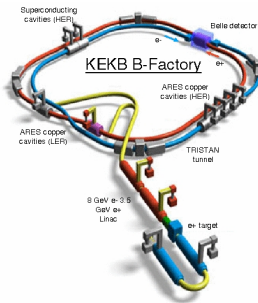
Finally arrive at your B mesons

Main variables:

- Energy difference: $\Delta E = E_B^* - E_{\text{beam}}^*$
- Beam constrained mass: $M_{bc} = \sqrt{(E_{\text{beam}}^*)^2 + (p_B^*)^2}$

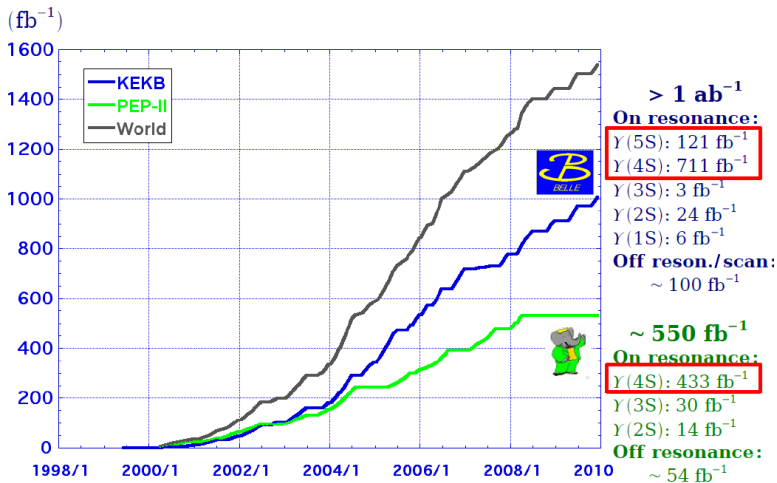


Belle Experiment @ KEKB accelerator



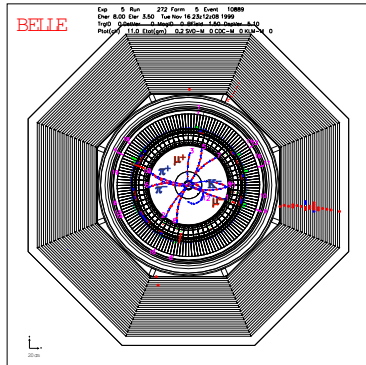
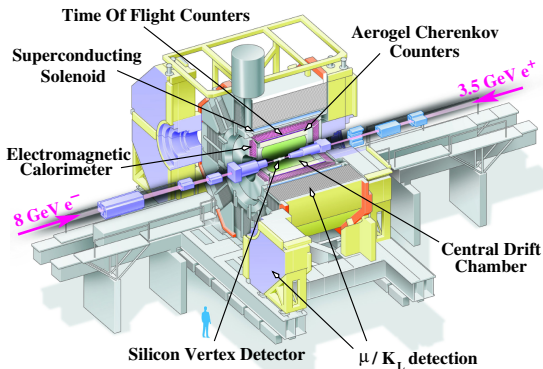
- Asymmetric energy collider: $e^+(3.5\text{GeV}) \rightarrow \leftarrow e^-(8\text{GeV})$
- Energy released in collisions: $\sqrt{s} = 10.58 \text{ GeV} \approx M_{\Upsilon(4S)}$

Total datasets from Belle and BaBar (B -factory at SLAC)



~770 MB \bar{B} for Belle, ~ 470 MB \bar{B} for BaBar
 ~14M B_s also! ($\Upsilon(5S)$ runs)

Belle detector



Precise tracking (SVD)

- D^0 decay point reconstructed with $\sim 70 - 160 \mu\text{m}$ accuracy

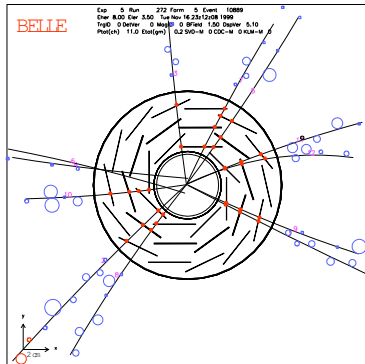
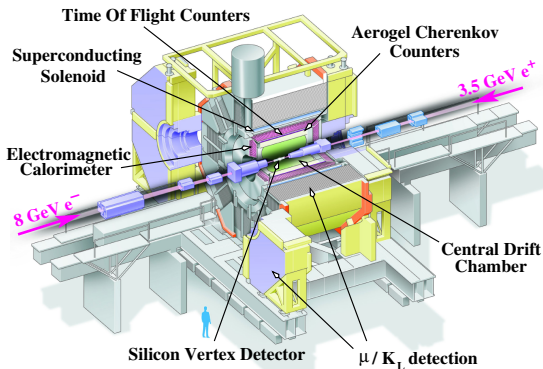
Momentum measurement (CDC)

- $B = 1.5\text{ T}$: Uncertainty of measured p_t between 0.5% (0.5 GeV) and 1.1% (5 GeV)

Particle identification (CDC + ACC + TOF)

- average kaon efficiency is around 88%, while the pion fake rate is below 10%

Belle detector



Precise tracking (SVD)

- D^0 decay point reconstructed with $\sim 70 - 160 \mu\text{m}$ accuracy

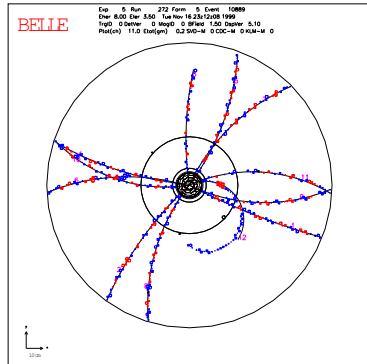
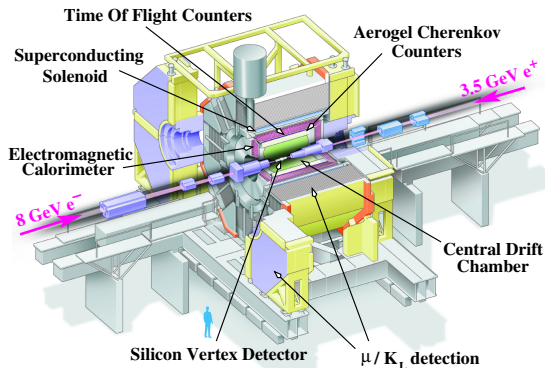
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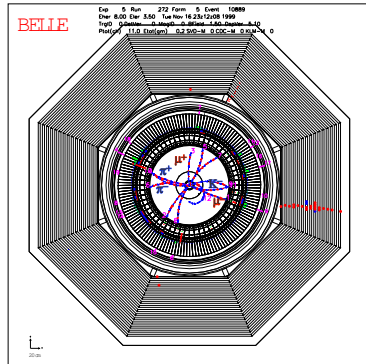
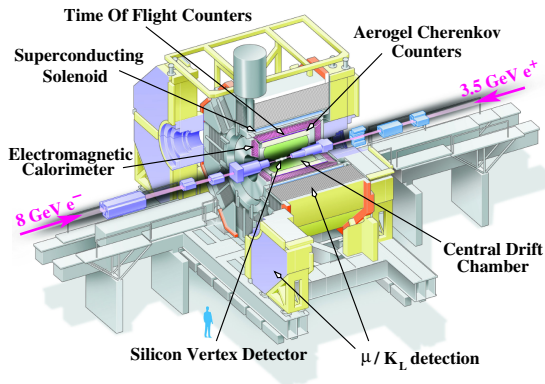
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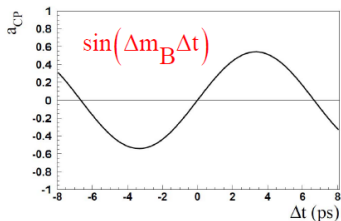
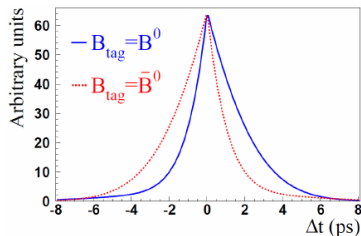
- $B = 1.5\text{ T}$: Uncertainty of measured p_t between 0.5% (0.5 GeV) and 1.1% (5 GeV)

Particle identification (CDC + ACC + TOF)

- average kaon efficiency is around 88%, while the pion fake rate is below 10%

Perfect detector

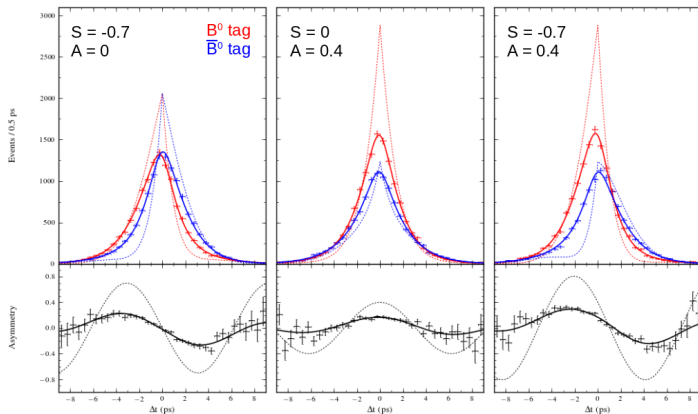
For the case of $B^0 \rightarrow J/\psi K_S^0$, where $A_f = 0$



$$\mathcal{A}_{CP}(\Delta t) = \frac{2\text{Im}\lambda}{1 + |\lambda|^2} \sin(\Delta M \Delta t)$$

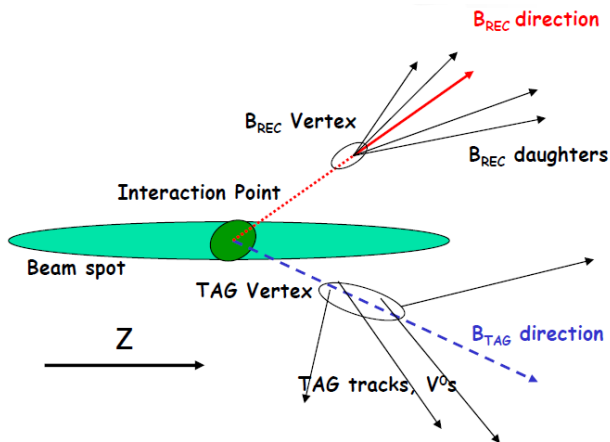
Perfect detector vs. reality

See the effect of different values of A_f and S_f :

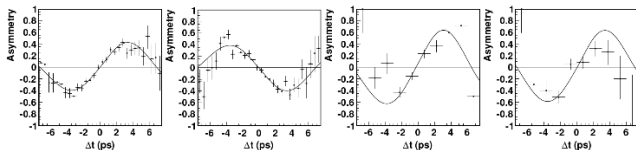
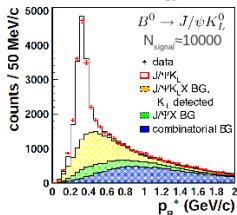
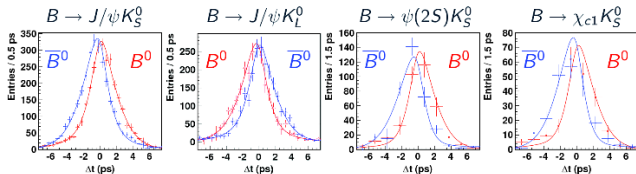
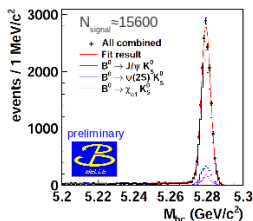


⇒ *Need to take into account mis-tagging and Δt resolution, which smear the Δt distribution*

Vertex and Δt reconstruction



CP violation in $B^0 \rightarrow J/\psi K_S^0, J/\psi K_L^0, \psi(2S)K_S^0, \chi K_S^0$



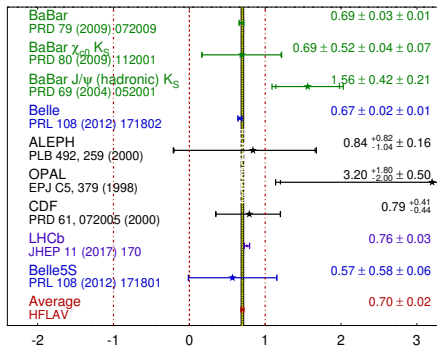
Belle with 772×10^6 BB:

$$\mathcal{A} = 0.007 \pm 0.016 (\text{stat}) \pm 0.013 (\text{syst})$$

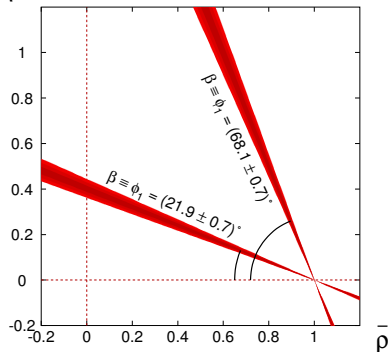
$$\sin(2\phi_1) = 0.668 \pm 0.023 \pm 0.013 \quad \text{preliminary}$$

Combination of results

$\sin(2\beta) \equiv \sin(2\phi_1)$ **HFLAV**
Moriond 2018
PRELIMINARY



$\bar{\eta}$ $\beta \equiv \phi_1$ **HFAG**
Moriond 2015
PRELIMINARY



To summarize:

To measure CP violation at the $\Upsilon(4S)$ using the interference between mixing and decay, one must:

- 1 Determine the flavor of one of the neutral B mesons directly from its decay products (e.g., from semileptonic decays).
- 2 Reconstruct the other B meson in a state that both B^0 and \bar{B}^0 can decay into.
- 3 Measure the time difference, $\Delta t = t_1 - t_2$, between the decays. Requires precise vertexing information to measure Δz .

- Richman, Jeremy D. (UCSB), *Heavy Quark Physics and CP Violation*.
http://physics.ucsd.edu/students/courses/winter2010/physics222/references/driver_houches12.pdf
Pages 199-220.
- Kooijman, P. & Tuning, N., *CP Violation*,
<http://master.particles.nl/LectureNotes/2011-CP.pdf>
Pages 38-46.