

Determination of CKM angle γ and introduction to Dalitz analysis

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Flavor Physics Lectures
VII / XII



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Reading material and references

Lecture material based on several textbooks and online lectures/notes.

Credits for material and figures include:

Literature

- Perkins, Donald H. (2000), *Introduction to High Energy Physics*.
- Griffiths, David J. (2nd edition), *Introduction to Elementary Particles*.
- Stone, Sheldon (2nd edition), *B decays*.

Online Resources

- Belle/BaBar Collaborations, *The Physics of the B-Factories*.
<http://arxiv.org/abs/1406.6311>
- Bona, Marcella (University of London), *CP Violation Lecture Notes*,
<http://pprc.qmul.ac.uk/bona/ulpg/cpv/>
- Richman, Jeremy D. (UCSB), *Heavy Quark Physics and CP Violation*.
http://physics.ucsd.edu/students/courses/winter2010/physics222/references/driver_houches12.pdf
- Thomson, Mark (Cambridge University), *Particle Physics Lecture Handouts*,
<http://www.hep.phy.cam.ac.uk/thomson/partIIIparticles/welcome.html>
- Grossman, Yuval (Cornell University), *Just a Taste. Lectures on Flavor Physics*,
<http://www.lepp.cornell.edu/pt267/files/notes/FlavorNotes.pdf>
- Kooijman, P. & Tuning, N., *CP Violation*,
<https://www.nikhef.nl/h71/Lectures/2015/ppII-cpviolation-29012015.pdf>

Recap & outline

So far, we:

- Learned how to measure CKM angles β and α through various channels:
 - We looked at theoretically clean “benchmark” decays (e.g., $B \rightarrow J/\psi K_S^0$) and also several decays dominated by $b \rightarrow s$ penguin diagrams where we can probe for new physics.
 - We saw how penguin pollution complicates the extraction of α . We learned how to exploit isospin relations to help resolve the ambiguity.

Today, we'll:

- Learn now to measure the final CKM angle γ .
- Along the way, we'll introduce the Dalitz analysis technique.

Credits for additional material and figures:

Tim Gershon, Tom Latham, and Brian Lindquist

Measurements of angle $\gamma = \phi_3$

Determine γ from CP asymmetries in $B \rightarrow DK$ decays

Tree-level Determination

- $b \rightarrow \bar{c}u$ and $b \rightarrow \bar{u}c$ tree amplitudes in the charged- B meson decays to open-charm final states.

\Rightarrow Interference between same final state for D and $\bar{D} \Rightarrow$ possibility of $DCPV$.

- * No penguin contribution (no theoretical uncertainty)

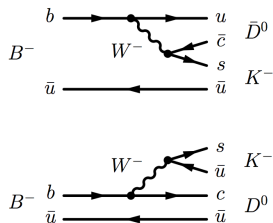
\Rightarrow *All hadronic unknowns obtainable from experiment:*

r_B = magnitude of the ratio of the amplitudes for $B^- \rightarrow \bar{D}^0 K^-$ and $B^- \rightarrow D^0 K^-$.

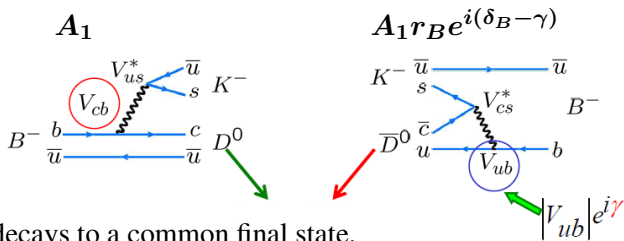
δ_B = the relative strong phase between these 2 amplitudes.

! Challenging: Small overall \mathcal{B} from 5×10^{-6} to 10^{-9} .

\Rightarrow *Precise measurement of ϕ_3 requires a very large data sample.*



Determine γ from CP asymmetries in $B \rightarrow DK$ decays



D^0 and \bar{D}^0 decays to a common final state.

Grouped into 3 categories:

- (1) Flavor eigenstate: $K^+\pi^-$, $K^+\pi^-\pi^0$
- (2) CP eigenstate: K^+K^- , $\pi^+\pi^-$, $K_S^0\pi^0$
- (3) 3-body decay: $K_S^0\pi^+\pi^-$, $K_S^0K^+K^-$

Relative strength of the two B decay amplitudes important for interference

$$r_B = \left| \frac{A(b \rightarrow u)}{A(b \rightarrow c)} \right| \sim 0.1 - 0.3$$

Large $r_B \Rightarrow$ large CP asymmetry

More on these 3 types of final states

Each category corresponds to a method, whose names reflect the authors who first described them:

The 3 methods according to final state are:

- (1) Cabibbo-favored and double-CS final states ($K^+\pi^-$, $K^+\pi^-\pi^0$) [“ADS Method”]
Atwood, Dunietz & Soni *Phys. Rev. D* **63**, 036005 (2001)
- (2) Cabibbo-suppressed (CS) D decays to CP -eigenstates (K^+K^- , $\pi^+\pi^-$, $K_S^0\pi^0$) [“GLW Method”]
Gronau & London *Phys. Lett. B* **253**, 483 (1991), Gronau & Wyler *Phys. Lett. B* **265**, 172 (1991)
- (3) Dalitz plot distribution of the products of D decays to multi-body self-conjugate final states ($K_S^0\pi^+\pi^-$, $K_S^0K^+K^-$) [“GGSZ Method”]
Giri, Grossman, Soffer and Zupan *Phys. Rev. D* **68**, 054018 (2003); Bondar (unpublished)

All methods are statistics-limited **but have common B parameters**

\Rightarrow *Perform a simultaneous fit using the results of all methods.*

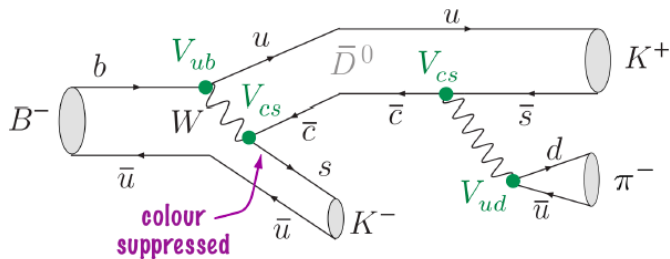
Aside: Cabbibo suppression

Recall the magnitude of the CKM matrix elements (represented by \square)

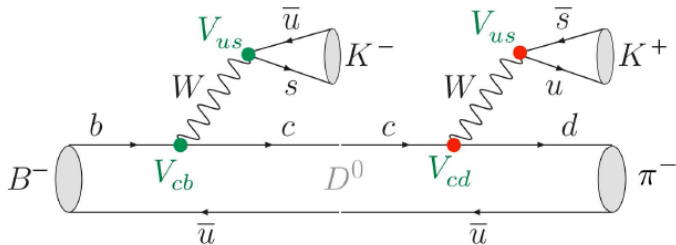
$$\begin{array}{c} \begin{array}{ccc} & d & s & b \\ \begin{array}{c} u \\ c \\ t \end{array} & \left(\begin{array}{ccc} \square & \square & \cdot \\ \square & \square & \square \\ \cdot & \square & \square \end{array} \right) \end{array}$$

- Weak decays whose amplitudes contain only diagonal CKM elements are referred to as Cabbibo favored.
- Those with one factor of V_{us} , V_{cb} , V_{cd} , or V_{ts} are singly Cabbibo suppressed.
- Decays containing 2 of these factors, or one of V_{ub} or V_{td} are doubly Cabbibo suppressed.

Cabibbo-favored and double-CS final states [ADS]

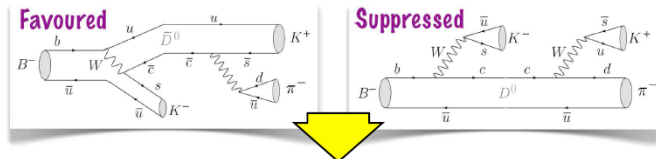


Cabibbo
favoured
 \bar{D} decay

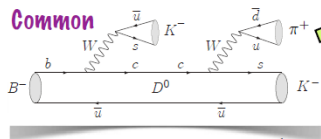


doubly
Cabibbo
suppressed
 \bar{D} decay

Cabibbo-favored and double-CS final states [ADS]



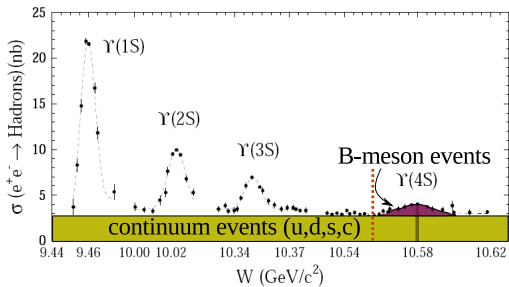
Common



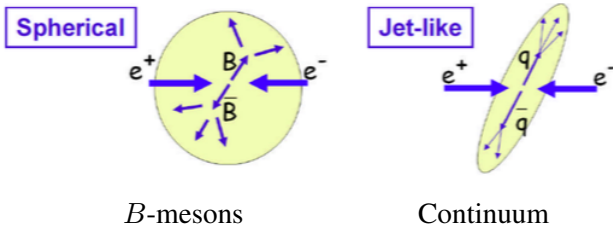
$$\begin{aligned}
 \mathcal{R}_{DK} &= \frac{\Gamma([K^+\pi^-]K^-) + \Gamma([K^-\pi^+]K^+)}{\Gamma([K^-\pi^+]K^-) + \Gamma([K^+\pi^-]K^+)} \\
 &= r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos \phi_3 \\
 \mathcal{A}_{DK} &= \frac{\Gamma([K^+\pi^-]K^-) - \Gamma([K^-\pi^+]K^+)}{\Gamma([K^-\pi^+]K^-) + \Gamma([K^+\pi^-]K^+)} \\
 &= 2r_B r_D \sin(\delta_B + \delta_D) \sin \phi_3 / \mathcal{R}_{DK}
 \end{aligned}$$

where $r_D = \left| \frac{\mathcal{A}(D^0 \rightarrow K^+\pi^-)}{\mathcal{A}(\bar{D}^0 \rightarrow K^+\pi^-)} \right| = 0.0613 \pm 0.0010$

Recall $e^+e^- \rightarrow q\bar{q}$ continuum production



Different event topologies



Continuum suppression in ADS decay $B^- \rightarrow DK^-$ (I)

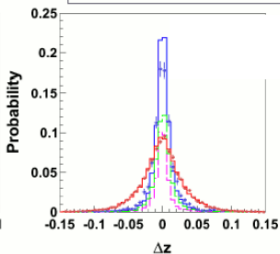
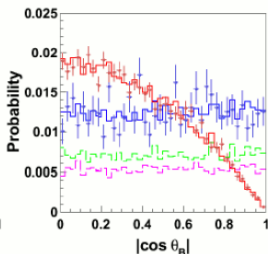
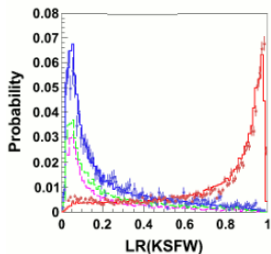
$B^- \rightarrow DK^-, D \rightarrow K^+ \pi^-$ ADS

Main background is $e^+ e^- \rightarrow q\bar{q}$ ($q=u, d, s, c$) continuum
combine 10 variables with neural network:

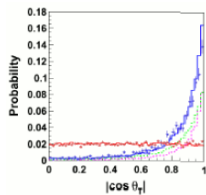
$B^- \rightarrow D\pi^-$
 $\rightarrow K^-\pi^+$
 M_{bc} -sideband

- ▶ Variables which have different distributions for signal and $q\bar{q}$ background are used.

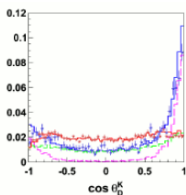
histogram: MC, dots: data
signal
 $qq = \text{charm} + \text{uds}$



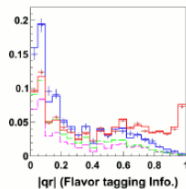
Continuum suppression in ADS decay $B^- \rightarrow DK^-$ (II)



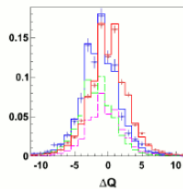
Angle between thrust axes of B decay and remainder.
No full correlation to $LR(KSFV)$.



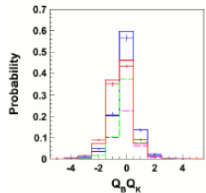
Decay angle of $D \rightarrow K\pi$.



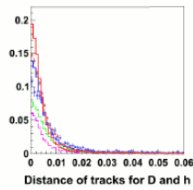
Flavor tagging Info. by MDLH. (NB possible.)



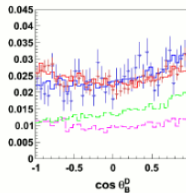
Difference of charges in D hemisphere and opposite hemisphere.



Product of charge of B and sum of charges for K not used in B reconstruction.



Distance of tracks for D and K .



Decay angle of $B \rightarrow DK$

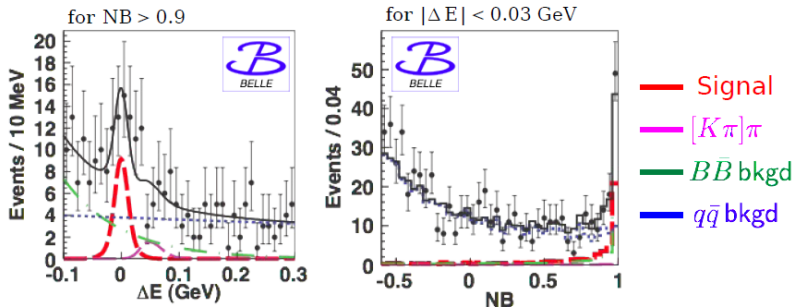
10 variables combined to obtain a single NN output (NB)

for example,
at 99% bckg rej.
signal eff. = 42%
now becomes 60%

Belle result for ADS decay $B^- \rightarrow [K^+\pi^-]_D K^-$

Yields for the ADS mode $B^- \rightarrow [K^+\pi^-]_D K^-$ from 772 million $B\bar{B}$ events
PRL 106, 231803 (2011)

Fit ΔE and NB distributions together to extract signal



$56.0^{+15.1}_{-14.2}$ events

$$R_{DK} = (1.63^{+0.44+0.07}_{-0.41-0.13}) \times 10^{-2}$$

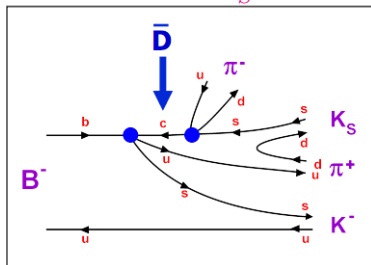
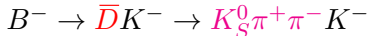
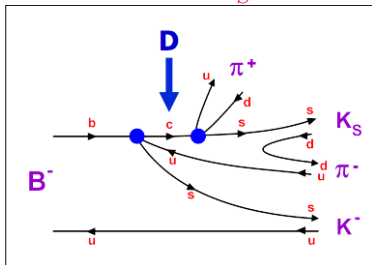
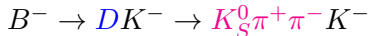
$$A_{DK} = -0.39^{+0.26+0.04}_{-0.28-0.03}$$

First evidence obtained
with a significance of 4.1σ
(including syst.)

Measurement of γ using Dalitz plot analysis [GGSZ]

The basic idea of this method is to use final states accessible to both D^0 and \bar{D}^0 and to measure the phase of the interference between them in the decay of D mesons produced in $B^\pm \rightarrow DK^\pm$ transitions

The most convenient decay for this type of measurement is $D \rightarrow K_S^0 \pi^+ \pi^-$

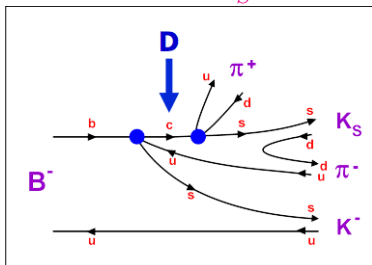


$$D \rightarrow K_S^0 \pi^+ \pi^-$$

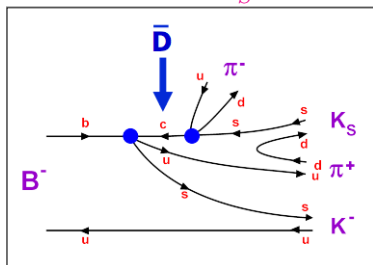
Unique combination of 3 advantages.

- 1) Large branching fraction.
- 2) Significant overlap of $D \rightarrow K_S^0 \pi^+ \pi^-$ and $\bar{D} \rightarrow K_S^0 \pi^+ \pi^-$ amplitudes which gives a large interference term sensitive to γ .
- 3) Rich resonant structure which provides large variations of the strong phase in D decays and results in sensitivity to γ that is only weakly dependent on the values of γ and strong phase δ_B .

$$B^- \rightarrow DK^- \rightarrow K_S^0 \pi^+ \pi^- K^-$$



$$B^- \rightarrow \bar{D}K^- \rightarrow K_S^0 \pi^+ \pi^- K^-$$



Intermission - Dalitz plot formalism

What is a Dalitz plot?

- Visual representation of
 - the phase-space of a three-body decay
 - involving only spin-0 particles
 - (term often abused to refer to phase-space of any multibody decay)
 - Named after it's inventor, Richard Dalitz (1925–2006):
 - “On the analysis of tau-meson data and the nature of the tau-meson.”
 - R.H. Dalitz, Phil. Mag. 44 (1953) 1068
 - (historical reminder: tau meson = charged kaon)
 - For scientific obituary, see
 - I.J.R. Aitchison, F.E. Close, A. Gal, D.J. Millener,
 - Nucl.Phys.A771:8-25,2006

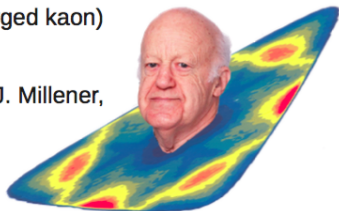
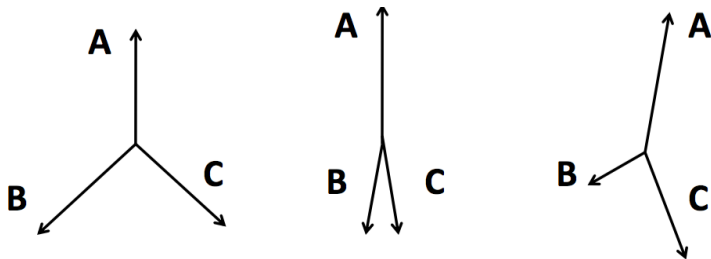


Image credit: Mike Pennington

3-body decays

For 2-body decays, $M \rightarrow ab$, p_a and p_b are completely determined by E , p conservation.

3-body decays have additional degrees of freedom; different values of p_a , p_b and p_c are possible, depending on the decay configuration.



Degrees of freedom (d.o.f)

For 3-body decays, $M \rightarrow abc$, where a , b and c are spin-0, the final state can be described by three 4-vectors: p_a^μ , p_b^μ and p_c^μ .

There are 12 parameters in total, but not all are independent:

- Set $p_{i,z} = 0$ since a , b and c all decay in the same plane; removes 3 d.o.f.
- Remove 3 d.o.f. by $E_i = \sqrt{m_i^2 + p_i^2}$, ($i = a, b, c$)
- Remove 3 d.o.f. by $\vec{p}_M = \vec{p}_a + \vec{p}_b + \vec{p}_c$ and $E_M = E_a + E_b + E_c$.
- Can rotate entire system in $x - y$ plane without effect; removes 1 d.o.f.

\Rightarrow Only 2 d.o.f. left.

What should we use?!

To answer this, let's look at the differential decay probability of $M \rightarrow abc$.

Kinematic constraints

For a particle of mass M decaying into 3 particles denoted as a , b and c , the differential decay probability is:

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M^3} |A|^2 dm_{ab}^2 dm_{bc}^2$$

where m_{ab} and m_{bc} are the invariant masses of the pairs of particles ab and bc , respectively (i.e., $m_{ab}^2 = (p_a^\mu + p_b^\mu)^2$).

Use m_{ab} and m_{bc} (or m_{ab} & m_{ac}) as our 2 d.o.f.

The invariant masses of pairs of final-state particles are related by the linear dependence:

$$m_{ab}^2 + m_{bc}^2 + m_{ac}^2 = M^2 + m_a^2 + m_b^2 + m_c^2$$

Kinematic constraints

The range of invariant masses m_{bc}^2 can be written in terms of either one of the other squared invariant masses (e.g., for m_{ab}^2):

$$(m_{bc}^2)_{\max} = (E_b^* + E_c^*)^2 - (p_b^* - p_c^*)^2$$
$$(m_{bc}^2)_{\min} = (E_b^* + E_c^*)^2 - (p_b^* + p_c^*)^2$$

where the energies of the particles b and c in the ab rest frame are

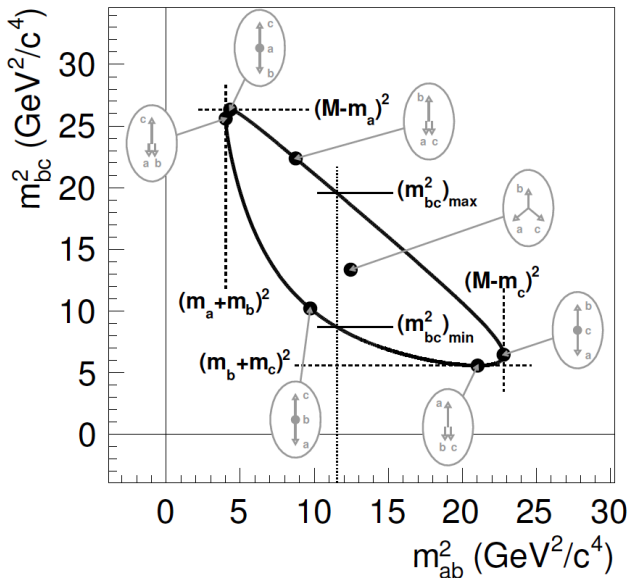
$$E_b^* = \frac{m_{ab}^2 - m_a^2 + m_b^2}{2m_{ab}}, \quad E_c^* = \frac{M^2 - m_{ab}^2 + m_c^2}{2m_{ab}}$$

and their corresponding momenta are:

$$p_b^* = \sqrt{E_b^*^2 - m_b^2}, \quad p_c^* = \sqrt{E_c^*^2 - m_c^2}$$

[Click here for a PDF containing a complete derivation of the kinematic limits \(i.e., boundary curve on next slide\) of the Dalitz plot \[eqn 39\]](#)

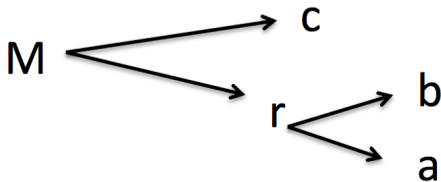
Kinematic boundaries of the 3-body decay phase space



Decay of M via resonances

Sometimes M will decay directly to a, b, c ; this is called non-resonant (NR) or “phase space” decay.

The majority of times M will decay through intermediate particles (or “resonances”) r



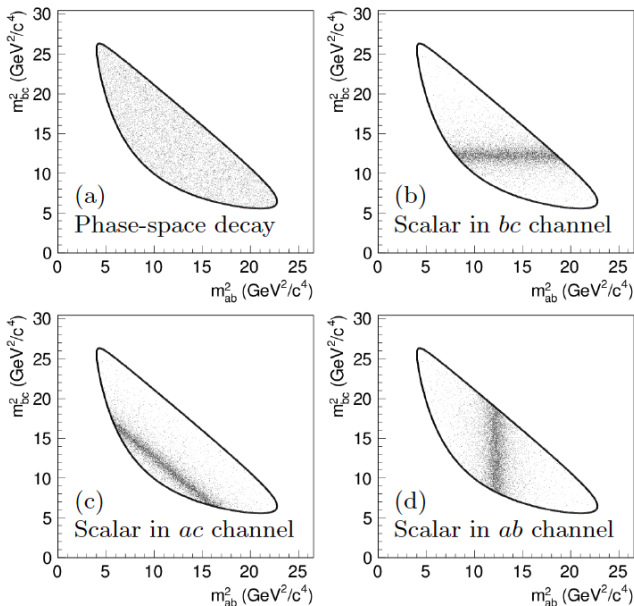
r is typically very short lived \Rightarrow *can't observe directly.*

But can study r with a Dalitz plot.

E and p conservation imply that if $r \rightarrow ab$, then $m_{ab}^2 = m_r^2$

These resonances show up as bands on the Dalitz plot

Dalitz plot for NR decay and for spin-0 resonances



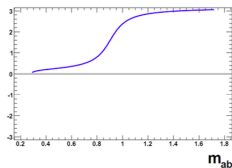
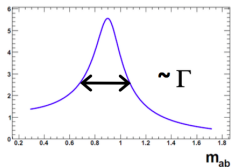
Resonance lifetimes I

- Recall $\Delta E \Delta t \sim \hbar$
- Short-lived resonances have broad peaks.
- Commonly described using a relativistic Breit-Wigner (RBW) parameterization with mass-dependent width.

$$A_{\text{RBW}} = \frac{1}{m_r^2 - m_{ab}^2 - im_r \Gamma_{ab}}$$

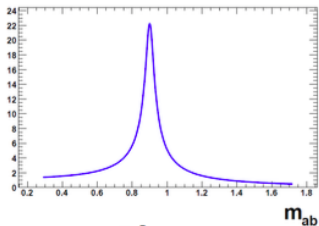
where the width is inversely proportional to the lifetime $\Gamma = \frac{\hbar}{\tau}$

- Plot of magnitude and phase of $A_{\text{RBW}} = |A|e^{i\phi}$

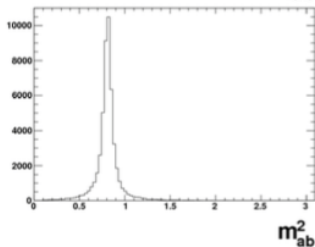
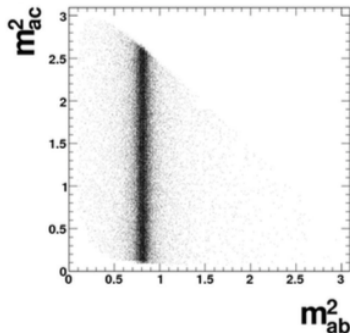
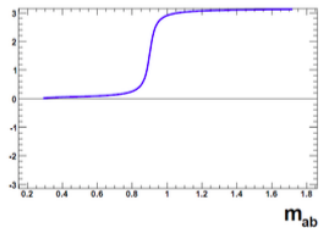


Resonance lifetimes II

Magnitude

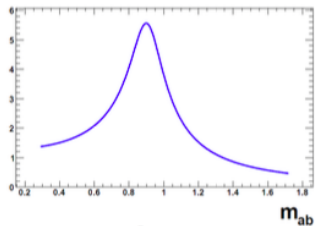


Phase

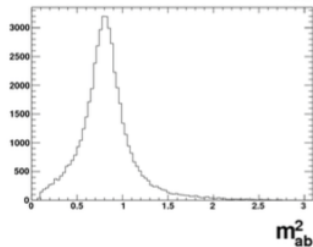
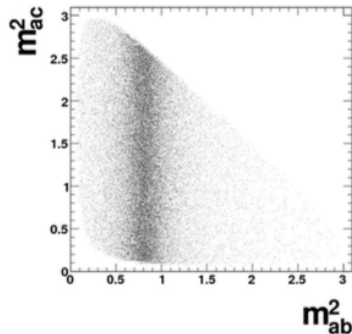
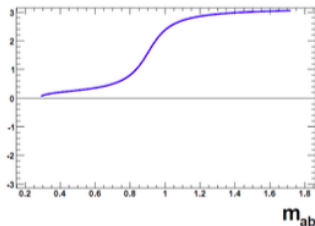


Resonance lifetimes III

Magnitude



Phase



Resonance spins

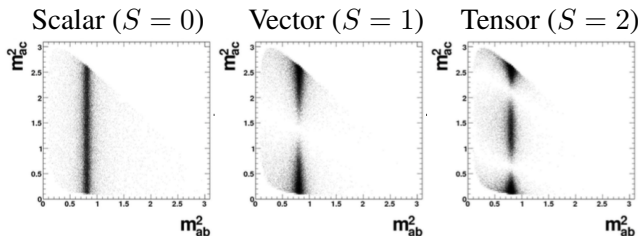
- If the **resonance has spin S** , and M , a , b , and c are $S = 0$, the decay amplitude is proportional to Legendre polynomials:

$$A \propto A_{\text{RBW}}(m_{ab}) P_S(\cos \theta)$$

$$P_0(\cos \theta) = 1$$

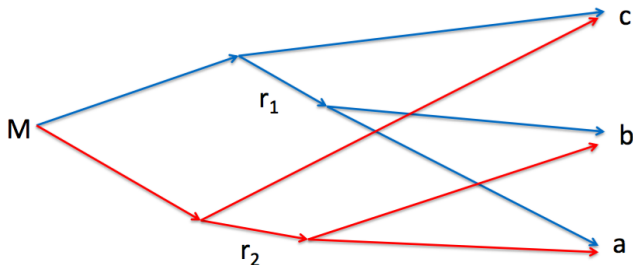
$$P_1(\cos \theta) = \cos \theta$$

$$P_2(\cos \theta) = \frac{1}{2} (3 \cos^2 \theta - 1)$$



Multiple resonances

- Typically M can decay to the same final state through multiple resonances.
- Results in interference as in Young's double slit experiment.



- The usual strategy is to model the total decay amplitude as a **sum of individual resonances** plus a non-resonant term:

$$A = \sum_r a_r e^{i\phi_r} A_r + a_{\text{NR}} e^{i\phi_{\text{NR}}} A_{\text{NR}}$$

where $A_r(m_{ab}^2, m_{ac}^2)$ are the Dalitz plot dependent amplitudes which are of the form

$$A_r = F_P \times \mathcal{F}_r \times A_{\text{RBW}} \times W_r$$

and a_r and ϕ_r can be measured in a maximum-likelihood fit.

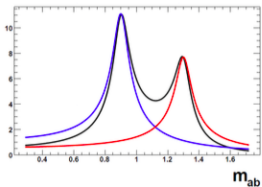
- Here, $A_{\text{RBW}} \times W_r$ is the resonance propagator, where W_r describes the angular distribution of the decay (and recall we used a relativistic BW as the dynamical function of the resonance: A_{RBW})¹.
- F_P and F_r are the transition form factors of the parent particle and resonance, respectively.

⇒ now lets look at an example of 2 interfering amplitudes

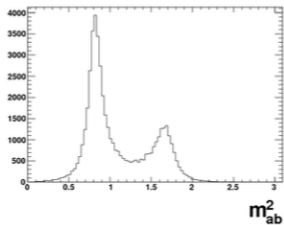
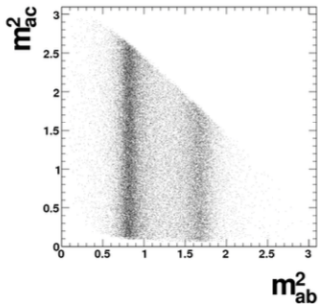
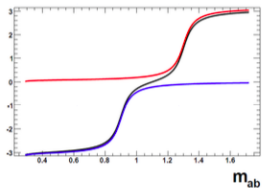
¹Strictly speaking the RBW works well only in the case of narrow states. The use of the mass-dependent width results in the amplitude becoming a non-analytic function. An alternative parametrization proposed by Gounaris and Sakurai (GS) recovers the analyticity of the amplitude and provides a better description for broad vector resonances.

Constructive interference

Magnitude

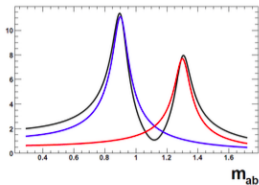


Phase

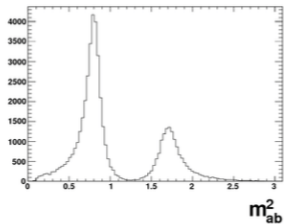
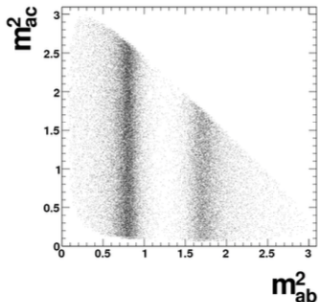
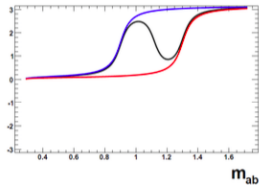


Destructive interference

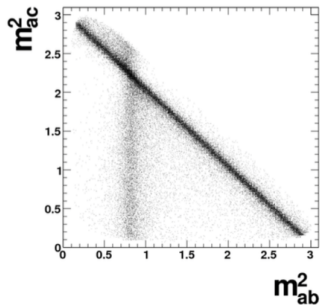
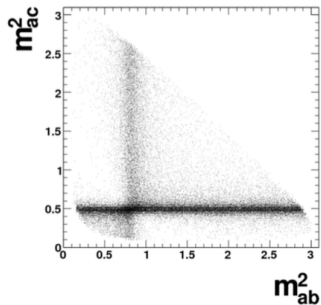
Magnitude



Phase

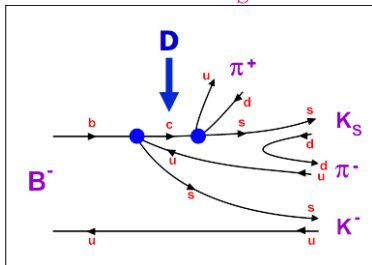


Cross-channel interference

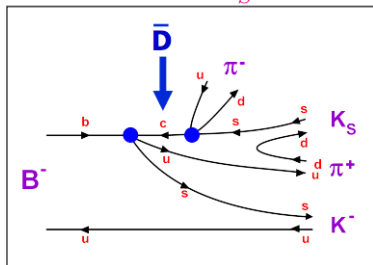


Back to measuring γ with $B \rightarrow DK$ decays

$$B^- \rightarrow DK^- \rightarrow K_S^0 \pi^+ \pi^- K^-$$



$$B^- \rightarrow \bar{D}K^- \rightarrow K_S^0 \pi^+ \pi^- K^-$$

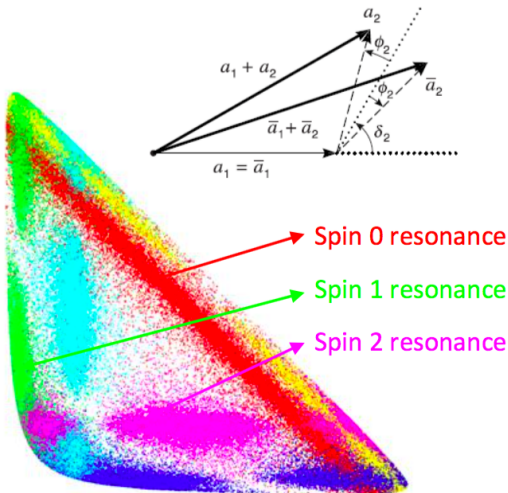


MC simulation

Resonance structure in
 $D \rightarrow K_S^0 \pi^+ \pi^-$ Dalitz plane

- **Green & blue:** $K^{*0}(892)$ [vector]
- **Cyan & magenta:** $K_2^*(1430)$ [tensor]
- **yellow:** $\rho(770)$ [vector]
- **red:** $f_0(980)$ [scalar]
- ... even more which are not simulated here (fit result table)

Main advantage of Dalitz plots is the ability to exploit the interference between different resonances.



- Define the 2 dalitz plot variables as

$$m_+^2 \equiv m_{K_S^0 \pi^+}^2$$

$$m_-^2 \equiv m_{K_S^0 \pi^-}^2$$

- The amplitude for the $B^\pm \rightarrow DK^\pm$ decays are

$$A_{B^+}(m_+^2, m_-^2) = \bar{A}_D + r_B e^{i(\delta_B + \gamma)} A_D$$

$$A_{B^-}(m_+^2, m_-^2) = A_D + r_B e^{i(\delta_B - \gamma)} \bar{A}_D$$

where

$A_D = A_D(m_+^2, m_-^2)$ is the complex amplitude of $D^0 \rightarrow K_S^0 \pi^+ \pi^-$

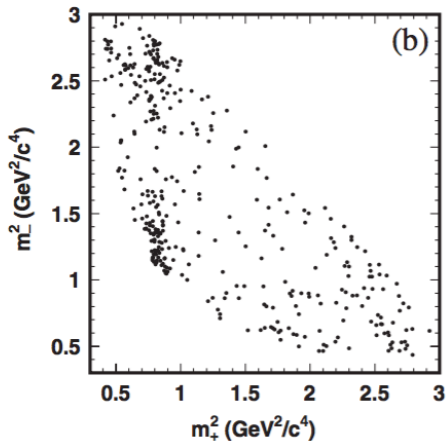
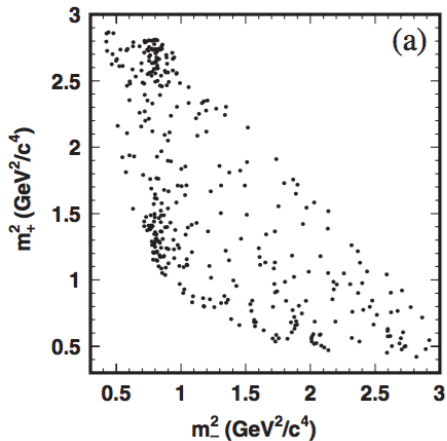
$\bar{A}_D = \bar{A}_D(m_+^2, m_-^2)$ is the complex amplitude of $\bar{D}^0 \rightarrow K_S^0 \pi^+ \pi^-$

Recall:

r_B = magnitude of the ratio of the amplitudes for $B^- \rightarrow \bar{D}^0 K^-$ and $B^- \rightarrow D^0 K^-$.

δ_B = the relative strong phase between these 2 amplitudes.

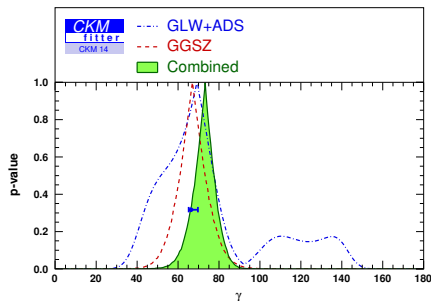
Belle data for $D \rightarrow K_S^0 \pi^+ \pi^-$



Fit result to Belle data for $D \rightarrow K_S^0 \pi^+ \pi^-$

Intermediate state	Amplitude	Phase ($^\circ$)	Fit fraction (%)
$K_S^0 \sigma_1$	1.56 ± 0.06	214 ± 3	11.0 ± 0.7
$K_S^0 f_0(980)$	0.385 ± 0.006	207.3 ± 2.3	4.72 ± 0.05
$K_S^0 \sigma_2$	0.20 ± 0.02	212 ± 12	0.54 ± 0.10
$K_S^0 f_0(1370)$	1.56 ± 0.12	110 ± 4	1.9 ± 0.3
$K_S^0 \rho(770)^0$	1.0 (fixed)	0 (fixed)	21.2 ± 0.5
$K_S^0 \omega(782)$	0.0343 ± 0.0008	112.0 ± 1.3	0.526 ± 0.014
$K_S^0 f_2(1270)$	1.44 ± 0.04	342.9 ± 1.7	1.82 ± 0.05
$K_S^0 \rho^0(1450)$	0.49 ± 0.08	64 ± 11	0.11 ± 0.04
$K_0^*(1430)^- \pi^+$	2.21 ± 0.04	358.9 ± 1.1	7.93 ± 0.09
$K_0^*(1430)^+ \pi^-$	0.36 ± 0.03	87 ± 4	0.22 ± 0.04
$K^*(892)^- \pi^+$	1.638 ± 0.010	133.2 ± 0.4	62.9 ± 0.8
$K^*(892)^+ \pi^-$	0.149 ± 0.004	325.4 ± 1.3	0.526 ± 0.016
$K^*(1410)^- \pi^+$	0.65 ± 0.05	120 ± 4	0.49 ± 0.07
$K^*(1410)^+ \pi^-$	0.42 ± 0.04	253 ± 5	0.21 ± 0.03
$K_2^*(1430)^- \pi^+$	0.89 ± 0.03	314.8 ± 1.1	1.40 ± 0.06
$K_2^*(1430)^+ \pi^-$	0.23 ± 0.02	275 ± 6	0.093 ± 0.014
$K^*(1680)^- \pi^+$	0.88 ± 0.27	82 ± 17	0.06 ± 0.04
$K^*(1680)^+ \pi^-$	2.1 ± 0.2	130 ± 6	0.30 ± 0.07
non-resonant	2.7 ± 0.3	160 ± 5	5.0 ± 1.0

Combination of results from 3 methods



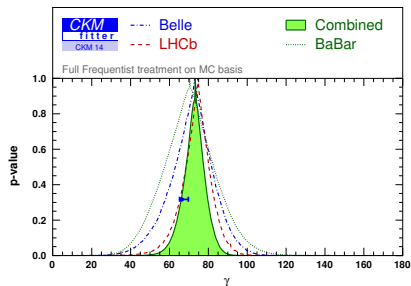
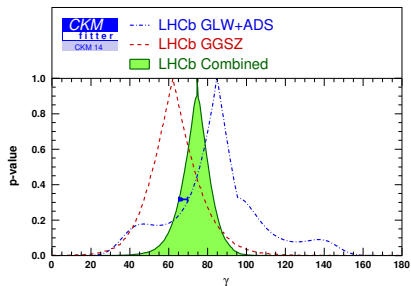
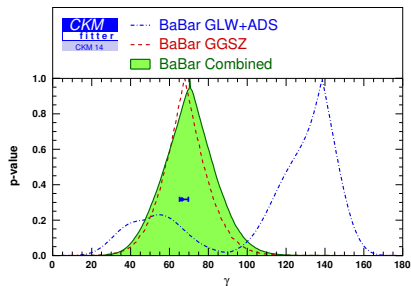
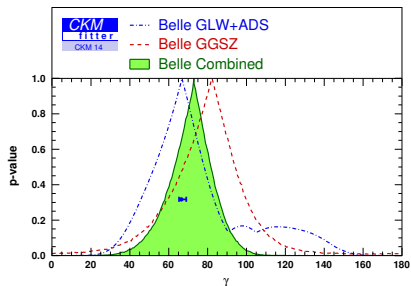
Constraints on γ from world average $B^\pm \rightarrow D^{(*)}K^{(*)\pm}$ decays (GLW+ADS) and Dalitz analyses (GGSZ)

$$\gamma(\text{combined}) = (73.2^{+6.3}_{-7.0})^\circ$$

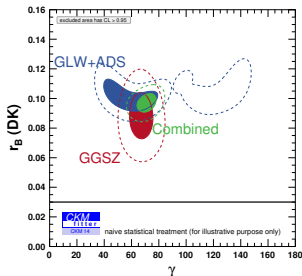
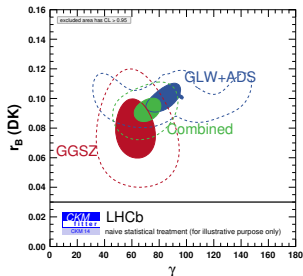
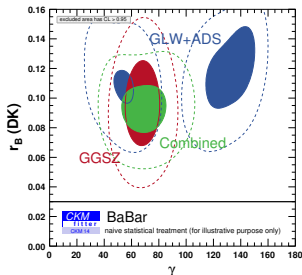
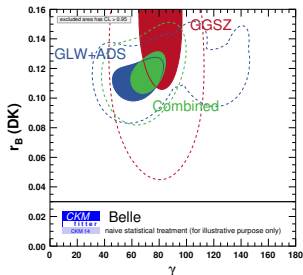
Compared to the prediction from the global CKM fit (not including these measurements):

$$\gamma(\text{fit}) = (66.9^{+1.0}_{-3.7})^\circ$$

Separated by experiment



Results for r_B vs. γ



$$\gamma = (73.2^{+6.3}_{-7.0})^\circ$$

$$r_B = (0.097 \pm 0.006)$$

$$\delta_B = (125.4^{+7.0}_{-7.8})$$

Prospects @ Belle II

All methods reproducible at Belle II

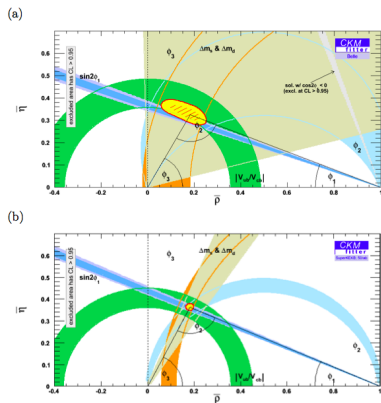
- Improvements in PID and $q\bar{q}$ suppression using neural networks *Nucl. Instrum. Meth. A654: 432 (2011)*
- Systematic errors from peaking charmless background, and PDFs from $D\pi$ and sidebands will decrease with statistics.
- Elimination of D model uncertainty using samples of neutral D mesons decaying into CP eigenstates from charm factories CLEO-c and BESIII (via $\psi(3770) \rightarrow DD$).

\Rightarrow *Naive scaling of combination with ADS and GLW yields an error of 1.5° .*

Physics at Super B Factory, arXiv:1002.5012 (2010)

Much more!

- Statistical error will be dominant and can be improved by including D decays to, e.g., $K_S^0 K^+ K^-$, $\pi^+ \pi^- \pi^0$, $K_S^0 \pi^+ \pi^- \pi^0$ ($2 * \mathcal{B}(K_S^0 \pi^+ \pi^-)$!).
- Use $D\pi$ in addition to DK



(a) Belle at 0.5 ab^{-1} and (b) Belle II at 50 ab^{-1}

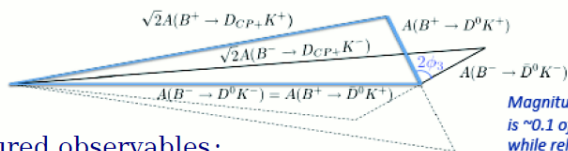
Extra material - GLW Method

Cabibbo-suppressed D decays to CP -eigenstates (GLW)

GLW with $D_{CP}^{(*)}K$

D decays to CP eigenstates

➤ Amplitude triangle:



Magnitude of one side is ~ 0.1 of the others while relative magnitude of the others help ϕ_3 constraint.

measured observables:

$$R_{CP\pm} \equiv \frac{\text{Br}(B^- \rightarrow D_{CP\pm} K^-) + \text{Br}(B^+ \rightarrow D_{CP\pm} K^+)}{\text{Br}(B^- \rightarrow D^0 K^-) + \text{Br}(B^+ \rightarrow \bar{D}^0 K^+)}$$

$$A_{CP\pm} \equiv \frac{\text{Br}(B^- \rightarrow D_{CP\pm} K^-) - \text{Br}(B^+ \rightarrow D_{CP\pm} K^+)}{\text{Br}(B^- \rightarrow D_{CP\pm} K^-) + \text{Br}(B^+ \rightarrow D_{CP\pm} K^+)}$$

Relation between $(A_{CP+}, A_{CP-}, R_{CP+}, R_{CP-})$ and (γ, r_B, δ_B)

$$A_{CP+} = \frac{+2 r_B \sin \delta_B \sin \gamma}{1 + r_B^2 + 2 r_B \cos \delta_B \cos \gamma}$$

$$A_{CP-} = \frac{-2 r_B \sin \delta_B \sin \gamma}{1 + r_B^2 - 2 r_B \cos \delta_B \cos \gamma}$$

$$R_{CP+} = 1 + r_B^2 + 2 r_B \cos \delta_B \cos \gamma$$

$$R_{CP-} = 1 + r_B^2 - 2 r_B \cos \delta_B \cos \gamma$$

⇒ look for $R_{CP\pm} \neq 1$ and $A_{CP\pm} \neq 0$

⇒ $\neq CP$, \neq sign of asymmetry

Cabibbo-suppressed D decays to CP -eigenstates (GLW)

$$\underline{B \rightarrow Dh, D \rightarrow K^+ K^-, \pi^+ \pi^- \rightarrow R_+}$$

Preliminary
LP 2011

(772 MB \bar{B})

$B \rightarrow D\pi$

$B \rightarrow DK$

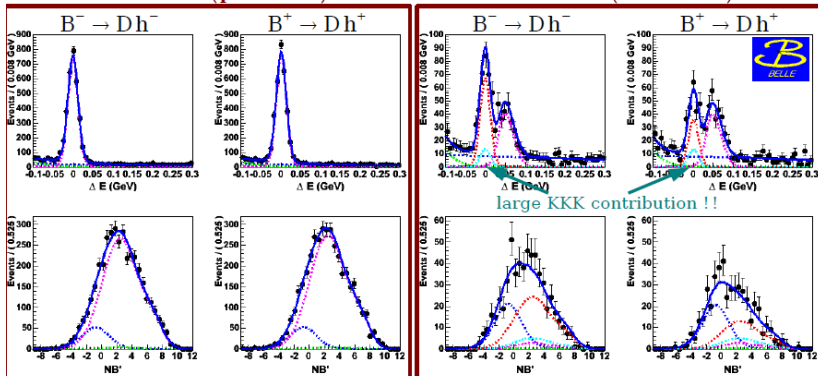
$B\bar{B}$

continuum

Yields	$B \rightarrow D\pi$	$B \rightarrow DK$	
$D \rightarrow K\pi$	50432 ± 243	3692 ± 83	$= R_{D\pi} = (7.32 \pm 0.16)\%$
$D \rightarrow KK, \pi\pi$	7696 ± 106	582 ± 40	$A(DK) = (1.4 \pm 2.0)\%$

KID < 0.6 (pion-like)

KID > 0.6 (kaon-like)



$$\Rightarrow R_{D_{CP^+}} = (7.56 \pm 0.51)\%, \quad A_{D_{CP^+}} = (28.7 \pm 6.0)\%$$

large asymmetry !!

Cabibbo-suppressed D decays to CP -eigenstates (GLW)

$$\underline{B \rightarrow Dh, D \rightarrow K_S \pi^0, K_S \eta \rightarrow R_-}$$

Preliminary
LP 2011

(772 MB \bar{B})

$B \rightarrow D\pi$

$B \rightarrow DK$

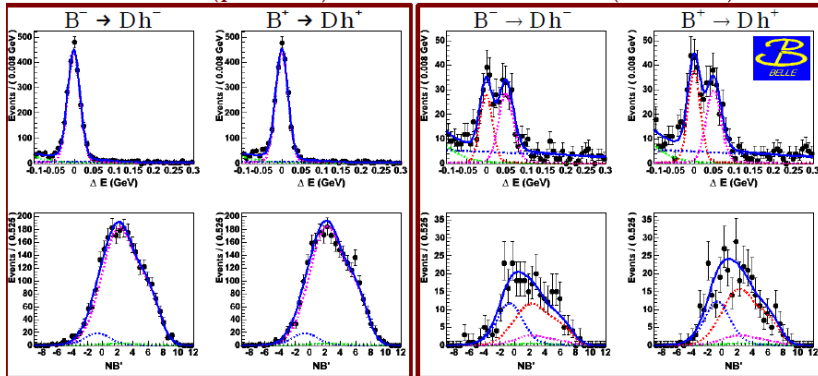
$B\bar{B}$

continuum

Yields
 $D \rightarrow K_S \pi^0, K_S \eta$ 5745 \pm 91 476 \pm 37

KID < 0.6 (pion-like)

KID > 0.6 (kaon-like)



$$\Rightarrow R_{D_{CP^-}} = (8.29 \pm 0.63)\%, \quad A_{D_{CP^-}} = (-12.4 \pm 6.4)\%$$

opposite asymmetry !!

Summary of GLW results

GLW Results

Preliminary (LP 2011)

$$R_{CP+} = 1.03 \pm 0.07 \pm 0.03$$

$$R_{CP-} = 1.13 \pm 0.09 \pm 0.05$$

$$A_{CP+} = +0.29 \pm 0.06 \pm 0.02$$

$$A_{CP-} = -0.12 \pm 0.06 \pm 0.01$$

CP-odd observables
only available at B-factories

(systematics dominated by peaking background, double ratio approximation)

