Isospin Analysis of *CP* Asymmetries in *B* Decays

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There is some theoretical uncertainty in the predictions for *CP*-violating hadronic asymmetries in neutral-*B* decays to *CP* eigenstates due to the existence of penguin diagrams. Using isospin relations, we show that it is possible to remove this uncertainty for the decays $B_d^0 \to \pi\pi$.

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One of the few remaining problems confronting the standard model of the weak and electromagnetic interactions is whether or not the Cabibbo-Kobayashi-Maskawa (CKM) matrix is the correct explanation of *CP* violation. Probably the best method of verifying this is by measuring *CP*-violating asymmetries in the *B* system.[1] The most promising of these involves a hadronic final state to which both B^0 and $\bar{B}^0$ can decay. Because of B^0-$\bar{B}^0$ mixing, a state which at time $t=0$ was a B^0 (or a $\bar{B}^0$) will evolve in time into a mixture of B^0 and $\bar{B}^0$. *CP* violation then occurs due to the interference between the decay chains $B^0 \to f$ and $B^0 \to \bar{B}^0 \to f$. If the final state is a *CP* eigenstate, then the time-dependent asymmetry

$$a(t) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)} \tag{1}$$

is given solely in terms of CKM matrix parameters; i.e., there are no hadronic uncertainties. (In Eq. (1), $B^0(t)$ $[\bar{B}^0(t)]$ is a state which was a pure B^0 $[\bar{B}^0]$ at $t=0$.)

The above conclusion is, however, predicated on the assumption that only one CKM amplitude contributes to the decay $B \to f$ (and to $\bar{B}^0 \to f$). In fact, there are always additional diagrams with different CKM phase information. For decays such as $B_d^0 \to \Psi K_S, \pi^+\pi^-, K_S\pi^0$, there are contributions from penguin diagrams,[2] while the decays $B_d^0 \to D_{1,2}^0\pi^0, D_{1,2}^0 K_S$ have two tree-level diagrams. For some final states of interest to experimentalists (e.g., $\Psi K_S, \pi^+\pi^-$) the effects of the new diagrams are roughly estimated to be small. Nevertheless, since this involves hadronic matrix elements with rather crude estimates, it introduces a theoretical uncertainty thought to be avoided in this class of *CP* asymmetries.

In this Letter, we will show that it is possible (in principle) to disentangle the effects of the tree and penguin contributions for the final states $\pi^+\pi^-$ and $\pi^0\pi^0$. This is achieved by using isospin to relate the amplitudes of $B_d^0 \to \pi^+\pi^-$, $B_d^0 \to \pi^0\pi^0$, and $B_u^+ \to \pi^+\pi^0$. In order to do this, we note the following features, in complete analogy with the textbook case of $K \to \pi\pi$. First of all, due to Bose statistics the above $\pi\pi$ final states can have only $I=0$ or $I=2$. Second, the tree diagram [Fig. 1(a)], which more generally represents the QCD-corrected left-handed four-fermion terms of the low-energy effective Hamiltonian,[3] can lead to either $I=0$ or $I=2$ final states. On the other hand, the penguin diagrams [Fig. 1(b)], which describe the *W*-loop QCD-induced terms,[4] give $\pi\pi$ states with $I=0$ only. In other words, the $\Delta I = \frac{3}{2}$ operator occurs purely as a tree diagram, but the $\Delta I = \frac{1}{2}$ operator has both tree and penguin contributions. Finally, the $\pi^+\pi^0$ final state can have only $I=2$, so the process $B_u^+ \to \pi^+\pi^0$ arises solely from the tree diagram ($\Delta I = \frac{3}{2}$).

The amplitudes for $B_d^0 \to \pi^+\pi^-$, $B_d^0 \to \pi^0\pi^0$, and $B_u^+ \to \pi^+\pi^0$ (A^{+-}, A^{00}, and A^{+0}, respectively) can now be expanded in terms of the $I=0$ and $I=2$ pieces. Writing $\pi^+\pi^- = (\pi_1^+\pi_2^- + \pi_1^-\pi_2^+)/\sqrt{2}$ (and similarly for $\pi^+\pi^0$), and evaluating the Clebsch-Gordan coefficients, we find

$$\begin{aligned}(1/\sqrt{2})A^{+-} &= A_2 - A_0\,, \\ A^{00} = 2A_2 + A_0, \quad A^{+0} &= 3A_2\,,\end{aligned} \tag{2}$$

where A_0 and A_2 are the amplitudes for a *B* to decay into a $\pi\pi$ pair with $I=0$ and $I=2$, respectively.[5] This immediately yields the complex triangle relation

$$(1/\sqrt{2})A^{+-} + A^{00} = A^{+0}. \tag{3}$$

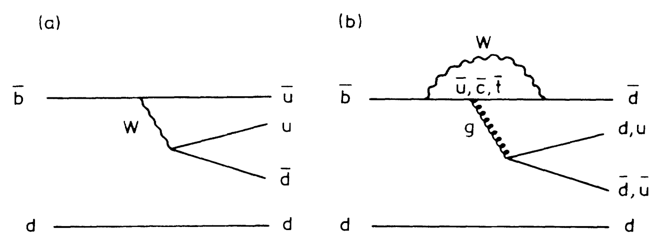

FIG. 1. (a) Tree-level and (b) penguin diagrams for the decay $B_d^0 \to \pi\pi$.

There is a similar triangle relation for the charge-conjugated processes:

$$(1/\sqrt{2})\bar{A}^{+-}+\bar{A}^{00}=\bar{A}^{-0}. \tag{4}$$

Here, $\bar{A}^{+-}$, $\bar{A}^{00}$, and $\bar{A}^{0}$ are the amplitudes for the processes $\bar{B}_d^0\to\pi^+\pi^-$, $\bar{B}_d^0\to\pi^0\pi^0$, and $B_u^-\to\pi^-\pi^0$, respectively. The $\bar{A}$ amplitudes are obtained from the A amplitudes by simply changing the sign of the CKM phases (the strong phases remain the same). As noted above, the A_2 amplitude has only one piece, from the tree-level diagram, so that

$$A_2=|A_2|e^{i\delta_2}e^{i\phi_t},\quad \bar{A}_2=|A_2|e^{i\delta_2}e^{-i\phi_t}, \tag{5}$$

where δ_2 is the $I=2$ final-state-interaction phase, and ϕ_t is the tree-level CKM phase. Thus we have $|A^{+0}|=|\bar{A}^{-0}|$. On the other hand, there are both tree-level and penguin contributions (with u,c,t-quark exchange) to A_0, so that there exists no simple relation between A^{+-} and $\bar{A}^{+-}$, or between A^{00} and $\bar{A}^{00}$.

The magnitudes of the decay amplitudes are obtainable experimentally. For the charged-B decay, $|A^{+0}|$ comes directly from the branching ratio. In the case of neutral-B decays, in order to extract $|A^{+-}|$, $|\bar{A}^{+-}|$, $|A^{00}|$, and $|\bar{A}^{00}|$, one has to take mixing into account. The effect of having more than one amplitude contributing to the decay $B\to f$ has been considered in Ref. 6. With $A_f\equiv A(B_d^0\to f)$ and $\bar{A}_f\equiv A(\bar{B}_d^0\to f)$, the time dependence of the decay is found to be

$$\Gamma(B^0(t)\to f)=\tfrac{1}{2}|A_f|^2e^{-\Gamma t}[(1+|\xi|^2)+(1-|\xi|^2)\cos(\Delta m\,t)-2\,\mathrm{Im}\xi\sin(\Delta m\,t)],$$
$$\Gamma(\bar{B}^0(t)\to f)=\tfrac{1}{2}|A_f|^2e^{-\Gamma t}[(1+|\xi|^2)-(1-|\xi|^2)\cos(\Delta m\,t)+2\,\mathrm{Im}\xi\sin(\Delta m\,t)], \tag{6}$$

where

$$\xi=e^{-2i\phi_M}\bar{A}_f/A_f. \tag{7}$$

Here, ϕ_M is the phase information from B_d^0-$\bar{B}_d^0$ mixing, $\exp(-2i\phi_M)\equiv V_{tb}^*V_{td}/V_{tb}V_{td}^*$. From Eq. (6), one can see that, by measuring the time dependence of the decays into $\pi^+\pi^-$ and $\pi^0\pi^0$, it is possible to extract $|A^{+-}|$, $|\bar{A}^{+-}|$, $|A^{00}|$, and $|\bar{A}^{00}|$ from the coefficients of the constant and $\cos(\Delta m\,t)$ terms. (For the $\pi^0\pi^0$ final state, this is admittedly rather difficult experimentally. Furthermore, if color suppression holds, then one might expect the branching ratio of $B_d^0\to\pi^0\pi^0$ to be about an order of magnitude smaller than that of $B_d^0\to\pi^+\pi^-$.[7,8] Nevertheless, these measurements should eventually be possible.) The existence of a $\cos(\Delta m\,t)$ term is due to direct *CP* violation, i.e., the interference between tree and penguin diagrams with different CKM phases and different hadronic final-state-interaction phases. In the approximation of neglecting the penguin contributions, $|A_f|=|\bar{A}_f|$ and $|\xi|=1$, so that this term disappears. In addition, in this limit the triangles defined in Eqs. (3) and (4) are congruent and have identical orientations.

The $\sin(\Delta m\,t)$ term corresponds to the existence of *CP* violation due to A_f-$\bar{A}_f$ interference via mixing. For the $\pi^+\pi^-$ final state, its coefficient is given by

$$\mathrm{Im}\xi_{+-}=\mathrm{Im}\left(e^{-2i(\phi_M+\phi_t)}\left[\frac{1-\bar{z}}{1-z}\right]\right), \tag{8}$$

where Eqs. (2) and (5) have been used, and

$$z\equiv A_0/A_2,\quad \bar{z}\equiv\bar{A}_0/\bar{A}_2. \tag{9}$$

Denoting the three angles of the unitarity triangle by α, β, and γ,[9] we have $\phi_t=\gamma$, $\phi_{M_d}=\beta$, and $\beta+\gamma=\pi-\alpha$:

$$\mathrm{Im}\xi_{+-}=\mathrm{Im}\left(e^{2i\alpha}\left[\frac{1-\bar{z}}{1-z}\right]\right). \tag{10}$$

In the limit in which penguin effects are neglected, $z=\bar{z}$, so that $\mathrm{Im}\xi_{+-}=\sin2\alpha$ directly measures the angle α of the unitarity triangle.[1] In the presence of penguins, z and $\bar{z}$ are not equal, so that knowledge of their magnitudes and phases is necessary in order to extract α.

These are obtainable, however, from the triangle relations and from the knowledge of the magnitudes of the decay amplitudes (the sides of the triangles). Consider the triangle shown in Fig. 2(a) [which corresponds to Eq. (3)]. The magnitude $|A_2|$ is obtained directly from $|A^{+0}|$ [Eq. (2)]. Simple geometrical considerations allow one to obtain $|A_0|$ and $\cos\theta$ from the triangle, where θ is the angle between A_0 and A_2. Note that $\sin\theta$ cannot be determined, which means that, although the magnitude of θ is known, the sign is not. The point is that the triangle can be up or down; i.e., it can be reflected through the A^{+0} axis. Therefore z is determined up to a twofold ambiguity in the sign of its phase. Similarly, $\bar{z}$ can be determined from the triangle in Fig. 2(b) [corresponding to Eq. (4)], but there is again a twofold ambiguity in its phase. We find from Eq. (10),

$$\mathrm{Im}\xi_{+-}=\mathrm{Im}\left(e^{2i\alpha}\left[\frac{1-|\bar{z}|e^{\pm i\bar{\theta}}}{1-|z|e^{\pm i\theta}}\right]\right), \tag{11}$$

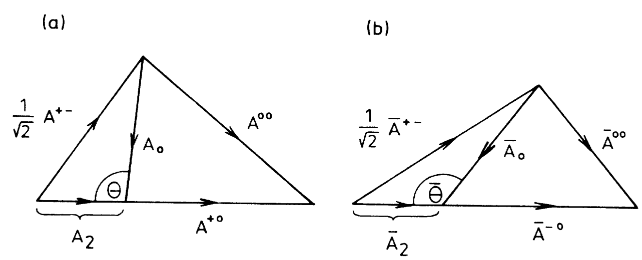

FIG. 2. Complex triangles of (a) Eq. (3) and (b) Eq. (4).

with $|z|$, $|\bar{z}|$, θ, and $\bar{\theta}$ all known. The terms within the square brackets in Eq. (11) have a magnitude, m_{+-}, which is unambiguously determined in terms of these four quantities. On the other hand, its phase can obtain four different values. We denote these phases by $\pm\epsilon_{+-}$ and $\pm\eta_{+-}$, where ϵ_{+-} and η_{+-} are known functions of $|z|$, $|\bar{z}|$, θ, and $\bar{\theta}$. α is a solution of one of the following four equations:

$$\sin(2\alpha\pm\epsilon_{+-})=(\mathrm{Im}\xi_{+-})/m_{+-}\,,$$
$$\sin(2\alpha\pm\eta_{+-})=(\mathrm{Im}\xi_{+-})/m_{+-}\,. \qquad (12)$$

This leaves a fourfold ambiguity in the determination of $\sin 2\alpha$.

In a similar manner, the coefficient of the $\sin(\Delta m t)$ term in the decay to $\pi^0\pi^0$ is given by

$$\mathrm{Im}\xi_{00}=\mathrm{Im}\left(e^{2i\alpha}\left[\frac{1+\frac{1}{2}|\bar{z}|e^{\pm i\bar{\theta}}}{1+\frac{1}{2}|z|e^{\pm i\theta}}\right]\right). \qquad (13)$$

Denoting the magnitude and phases of the terms within the square brackets by m_{00} and $\pm\epsilon_{00},\pm\eta_{00}$, respectively, this implies that α must also be a solution of one of the following equations:

$$\sin(2\alpha\pm\epsilon_{00})=(\mathrm{Im}\xi_{00})/m_{00}\,,$$
$$\sin(2\alpha\pm\eta_{00})=(\mathrm{Im}\xi_{00})/m_{00}\,. \qquad (14)$$

In general, Eqs. (12) and (14) determine $\sin 2\alpha$ unambiguously. The only exceptions are the very special cases in which some of the ambiguities of Eqs. (12) overlap with those of Eqs. (14). This happens when either ϵ_{+-} or η_{+-} equals one of the four phases, $\pm\epsilon_{00}$, $\pm\eta_{00}(\mathrm{mod}\,\pi)$. In these cases $\sin 2\alpha$ retains a twofold ambiguity.

One might wonder whether this kind of isospin analysis can be applied to other final states, such as ΨK_S, D^+D^-, $K_S\pi^0$, $\rho^0\pi^0$, or $D^0_{1,2}\pi^0$. It turns out that in all cases of interest the answer is negative. For the first two cases there is only one isospin operator—both tree and penguin diagrams are purely $\Delta I=0$ (ΨK_S) or $\Delta I=\frac{1}{2}$ (D^+D^-).[10] For $K_S\pi^0$, the tree and penguin operators have different isospin structures as in the $\pi\pi$ case, so that one can do an isospin analysis. However, here there are four amplitudes, so that one obtains a quadrilateral relation instead of a triangle relation. Unfortunately, knowledge of the length of all sides of a quadrilateral gives no information about the angles, so that the correction due to penguin diagrams cannot be extracted. A similar situation occurs in the case of the $\rho^0\pi^0$ final state, except that here five amplitudes are involved. The case $D^0_{1,2}\pi^0$ is peculiar in that there are no penguin diagrams involved—rather, there are two tree-level diagrams with different CKM factors. Nevertheless, an isospin analysis can still be done. Unfortunately, as in the $\rho^0\pi^0$ case, the description here too is in terms of five amplitudes, from which no useful information can be obtained. It may be, however, that it is possible to distinguish the two tree contributions by looking separately at the final states $D^0\pi^0$, $\bar{D}^0\pi^0$, and $D^0_{1,2}\pi^0$. This will be discussed in a separate paper.[11]

In conclusion, we have shown that one can use isospin to eliminate the theoretical uncertainty due to penguin diagrams in the CP asymmetries in $B^0_d\to\pi^+\pi^-$ and $B^0_d\to\pi^0\pi^0$. In general, one obtains a single value for $\sin 2\alpha$, which is free of hadronic final-state uncertainties. In very special cases one retains a twofold ambiguity in the value derived for $\sin 2\alpha$. This is achieved by noting that the amplitudes for $B^0_d\to\pi^+\pi^-$, $B^0_d\to\pi^0\pi^0$, and $B^+_u\to\pi^+\pi^0$ satisfy a triangle relation, as do their charge-conjugated processes. The magnitudes of the neutral-B decay amplitudes can be obtained by looking at the time dependence of their decays to $\pi^+\pi^-$ and $\pi^0\pi^0$. Using this information, one can calculate the correction to the CP asymmetry due to the existence of penguin diagrams with different CKM phase information. Unfortunately, one cannot use a similar isospin analysis to remove the theoretical uncertainty in the CP asymmetries for other interesting final states.

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