Determination of CKM angle γ and introduction to Dalitz analysis

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Flavor Physics Lectures VII / XII



Winter Semester 2022/2023 27. January, 2023

Reading material and references

Lecture material based on several textbooks and online lectures/notes. Credits for material and figures include:

Literature

- Perkins, Donald H. (2000), Introduction to High Energy Physics.
- Griffiths, David J. (2nd edition), Introduction to Elementary Particles.
- Stone, Sheldon (2nd edition), *B decays*.

Online Resources

- Belle/BaBar Collaborations, The Phyiscs of the B-Factories. http://arxiv.org/abs/1406.6311
- Bona, Marcella (University of London), CP Violation Lecture Notes, http://pprc.qmul.ac.uk/ bona/ulpg/cpv/
- Richman, Jeremy D. (UCSB), Heavy Quark Physics and CP Violation. http://physics.ucsd.edu/students/courses/winter2010/physics222/references/driver_houches12.pdf
- Thomson, Mark (Cambridge University), Particle Physics Lecture Handouts, http://www.hep.phy.cam.ac.uk/ thomson/partIIIparticles/welcome.html
- Grossman, Yuval (Cornell University), Just a Taste. Lectures on Flavor Physics, http://www.lepp.cornell.edu/ pt267/files/notes/FlavorNotes.pdf
- Kooijman, P. & Tuning, N., CP Violation, https://www.nikhef.nl/ h71/Lectures/2015/ppII-cpviolation-29012015.pdf

So far, we:

- Learned how to measure CMK angles β and α through various channels:
- We looked at theoretically clean "benchmark" decays (e.g., $B \to J/\psi K_S^0$) and also several decays dominated by $b \to s$ penguin diagrams where we can probe for new physics.
- We saw how penguin pollution complicates the extraction of α . We learned how to exploit isospin relations to help resolve the ambiguity.

Today, we'll:

- Learn now to measure the final CKM angle γ .
- Along the way, we'll introduce the Dalitz analysis technique.

Credits for additional material and figures: Tim Gershon, Tom Latham, and Brian Lindquist

Measurements of angle $\gamma=\phi_3$

$$\gamma = \phi_3$$

$$\gamma \equiv \arg \left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right] \approx \arg \mathbf{V}_{ub}^*$$

- The standard candle along with $V_{ub}/V_{cb}.$
- Less precise than ϕ_2 .
- Limited by the small branching fractions of the processes used in its measurement.
- ⇒ The one place a large experimental gain in UT metrology can be made



http://ckmfitter.in2p3.fr

Determine γ from CP asymmetries in $B \rightarrow DK$ decays

Tree-level Determination

- b → cus and b → ucs tree amplitudes in the charged-B meson decays to open-charm final states.
 - ⇒ Interference between same final state for D and \overline{D} ⇒ possibility of DCPV.





* No penguin contribution (no theoretical uncertainty)

 \Rightarrow All hadronic unknowns obtainable from experiment:

 r_B = magnitude of the ratio of the amplitudes for $B^- \to \overline{D}{}^0 K^-$ and $B^- \to D^0 K^-$.

 δ_B = the relative strong phase between these 2 amplitudes.

! Challenging: Small overall \mathcal{B} from 5×10^{-6} to 10^{-9} .

 \Rightarrow Precise measurement of ϕ_3 requires a very large data sample.

Determine γ from CP asymmetries in $B \rightarrow DK$ decays



 D^0 and \overline{D}^0 decays to a common final state.

Grouped into 3 categories:

- (1) Flavor eigenstate: $K^+\pi^-, K^+\pi^-\pi^0$
- (2) *CP* eigenstate: $K^+K^-, \pi^+\pi^-, K_S^0\pi^0$
- (3) 3-body decay: $K_S^0 \pi^+ \pi^-, K_S^0 K^+ K^-$

Relative strength of the two B decay amplitudes important for interference

$$r_B = \left| \frac{A(b \to u)}{A(b \to c)} \right| \sim 0.1 - 0.3$$

Large $r_B \Rightarrow$ large CP asymmetry

Each category corresponds to a method, whose names reflect the authors who first described them:

The 3 methods according to final state are:

(1) Cabibbo-favored and double-CS final states $(K^+\pi^-, K^+\pi^-\pi^0)$ ["ADS Method"]

Atwood, Dunietz & Soni Phys. Rev. D 63, 036005 (2001)

(2) Cabibbo-suppressed (CS) D decays to CP-eigenstates $(K^+K^-, \pi^+\pi^-, K_S^0\pi^0)$ ["GLW Method"]

Gronau & London Phys. Lett. B 253, 483 (1991), Gronau & Wyler Phys. Lett. B 265, 172 (1991)

(3) Dalitz plot distribution of the products of *D* decays to multi-body self-conjugate final states $(K_S^0\pi^+\pi^-, K_S^0K^+K^-)$ ["GGSZ Method"]

Giri, Grossman, Soffer and Zupan Phys. Rev. D 68, 054018 (2003); Bondar (unpublished)

All methods are statistics-limited but have common B parameters

 \Rightarrow Perform a simultaneous fit using the results of all methods.

Recall the magnitude of the CKM matrix elements (represented by \Box)



- Weak decays whose amplitudes contain only diagonal CKM elements are referred to as Cabbibo favored.
- Those with one factor of V_{us} , V_{cb} , V_{cd} , or V_{ts} are singly Cabbibo suppressed.
- Decays containing 2 of these factors, or one of V_{ub} or V_{td} are doubly Cabbibo suppressed.

Cabibbo-favored and double-CS final states [ADS]



Cabibbo favoured D decay

> doubly Cabibbo suppressed V decay

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Cabibbo-favored and double-CS final states [ADS]

Favoured $\mathcal{R}_{DK} = \frac{\Gamma([K^{+}\pi^{-}]K^{-}) + \Gamma([K^{-}\pi^{+}]K^{+})}{\Gamma([K^{-}\pi^{+}]K^{-}) + \Gamma([K^{+}\pi^{-}]K^{+})}$ $\xrightarrow{c}{c} \xrightarrow{c}{D^{0}} \xrightarrow{c} K^{-} = r_{B}^{2} + r_{D}^{2} + 2r_{B}r_{D}\cos(\delta_{B} + \delta_{D})\cos\phi_{3}$ Common $\mathcal{A}_{DK} = \frac{\Gamma([K^+\pi^-] K^-) - \Gamma([K^-\pi^+] K^+)}{\Gamma([K^-\pi^+] K^-) + \Gamma([K^+\pi^-] K^+)}$ $=2r_{B}r_{D}\sin(\delta_{B}+\delta_{D})\sin\phi_{3}/\mathcal{R}_{DK}$ where $r_D = \left| \frac{\mathcal{A}(D^0 \to K^+ \pi^-)}{\mathcal{A}(\overline{D}^0 \to K^+ \pi^-)} \right| = 0.0613 \pm 0.0010$

Recall $e^+e^- \rightarrow q \overline{q}$ continuum production



Continuum suppression in ADS decay $B^- \to DK^-$ (I)

$\underline{B^-} \to DK^-$, $D \to K^+ \, \pi^- \; ADS$



Continuum suppression in ADS decay $B^- \rightarrow DK^-$ (II)



Belle result for ADS decay $B^- \rightarrow [K^+\pi^-]_D K^-$

Yields for the ADS mode $B^- \rightarrow [K^+\pi^-]_D K^-$ from 772 million $B\overline{B}$ events PRL 106, 231803 (2011)

Fit ΔE and NB distributions together to extract signal



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 $A_{DK} = -0.39^{+0.26+0.04}_{-0.28-0.03}$

Measurement of γ using Dalitz plot analysis [GGSZ]

The basic idea of this method is to use final states accessible to both D^0 and \overline{D}^0 and to measure the phase of the interference between them in the decay of D mesons produced in $B^{\pm} \rightarrow DK^{\pm}$ transitions

The most convenient decay for this type of measurement is $D \to K_S^0 \pi^+ \pi^-$



$\overline{D \to K^0_S \pi^+} \pi^-$

Unique combination of 3 advantages.

- 1) Large branching fraction.
- 2) Significant overlap of $D \to K_S^0 \pi^+ \pi^-$ and $\overline{D} \to K_S^0 \pi^+ \pi^-$ amplitudes which gives a large interference term sensitive to γ .
- 3) Rich resonant structure which provides large variations of the strong phase in D decays and results in sensitivity to to γ that is only weakly dependent on the values of γ and strong phase δ_B .



Intermission - Dalitz plot formalism

What is a Dalitz plot?

- · Visual representation of
 - the phase-space of a three-body decay
 - involving only spin-0 particles
 - (term often abused to refer to phase-space of any multibody decay)
 - Named after it's inventor, Richard Dalitz (1925-2006):
 - "On the analysis of tau-meson data and the nature of the tau-meson."
 - R.H. Dalitz, Phil. Mag. 44 (1953) 1068
 - (historical reminder: tau meson = charged kaon)
 - For scientific obituary, see
 - I.J.R. Aitchison, F.E. Close, A. Gal, D.J. Millener,
 - Nucl.Phys.A771:8-25,2006



For 2-body decays, $M \rightarrow ab$, p_a and p_b are completely determined by E, p conservation.

3-body decays have additional degrees of freedom; different values of p_a , p_b and p_c are possible, depending on the decay configuration.



For 3-body decays, $M \to abc$, where a, b and c are spin-0, the final state can be described by three 4-vectors: p_a^{μ} , p_b^{μ} and p_c^{μ} .

There are 12 parameters in total, but not all are independent:

- Set $p_{i,z} = 0$ since a, b and c all decay in the same plane; removes 3 d.o.f.
- Remove 3 d.o.f. by $E_i = \sqrt{m_i^2 + p_i^2}, \ (i = a, b, c)$
- Remove 3 d.o.f. by $\vec{p}_M = \vec{p}_a + \vec{p}_b + \vec{p}_c$ and $E_M = E_a + E_b + E_c$.
- Can rotate entire entire system in x y plane without effect; removes 1 d.o.f.
- \Rightarrow Only 2 d.o.f. left.

What should we use?!

To answer this, lets look at the differential decay probability of $M \rightarrow abc$.

For a particle of mass M decaying into 3 particles denoted as a, b and c, the differential decay probability is:

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M^3} |A|^2 dm_{ab}^2 dm_{bc}^2$$

where m_{ab} and m_{bc} are the invariant masses of the pairs of particles ab and bc, respectively (i.e., $m_{ab}^2 = (p_a^{\mu} + p_b^{\mu})^2$).

Use m_{ab} and m_{bc} (or m_{ab} & m_{ac}) as our 2 d.o.f.

The invariant masses of pairs of final-state particles are related by the linear dependence:

$$m_{ab}^2 + m_{bc}^2 + m_{ac}^2 = M^2 + m_a^2 + m_b^2 + m_c^2$$

Kinematic constraints

The range of invariant masses m_{bc}^2 can be written in terms of either one of the other squared invariant masses (e.g., for m_{ab}^2):

$$\begin{split} (m_{bc}^2)_{\max} &= (E_b^* + E_c^*)^2 - (p_b^* - p_c^*)^2 \\ (m_{bc}^2)_{\min} &= (E_b^* + E_c^*)^2 - (p_b^* + p_c^*)^2 \end{split}$$

where the energies of the particles b and c in the ab rest frame are

$$E_b^* = \frac{m_{ab}^2 - m_a^2 + m_b^2}{2m_{ab}}, E_c^* = \frac{M^2 - m_{ab}^2 + m_c^2}{2m_{ab}}$$

and their corresponding momenta are:

$$p_b^* = \sqrt{E_b^* - m_b^2}, \, p_c^* = \sqrt{E_c^* - m_c^2}$$

Click here for a PDF containing a complete derivation of the kinematic limits (i.e., boundary curve on next slide) of the Dalitz plot [eqn 39]

Kinematic boundaries of the 3-body decay phase space



Decay of M via resonances

Sometimes M will decay directly to a, b, c; this is called non-resonant (NR) or "phase space" decay.

The majority of times M will decay through intermediate particles (or "resonances") \boldsymbol{r}



r is typically very short lived \Rightarrow *can't observe directly*.

But can study r with a Dalitz plot.

E and p conservation imply that if $r \to ab$, then $m_{ab}^2 = m_r^2$ These resonances show up as bands on the Dalitz plot

Dalitz plot for NR decay and for spin-0 resonances



Resonance lifetimes I

- Recall $\Delta E \Delta t \sim \hbar$
- Short-lived resonances have broad peaks.
- Commonly described using a relativistic Breit-Wigner (RBW) parameterization with mass-dependent width.

$$A_{\rm RBW} = \frac{1}{m_r^2 - m_{ab}^2 - im_r \Gamma_{ab}}$$

where the width is inversely proportional to the lifetime $\Gamma = \frac{\hbar}{\tau}$

• Plot of magnitude and phase of $A_{\rm RBW} = |A|e^{i\phi}$



Resonance lifetimes II



Resonance lifetimes III



2.5

• If the resonance has spin S, and M, a, b, and c are S = 0, the decay amplitude is proportional to Legendre polynomials:



- Typically M can decay to the same final state through multiple resonances.
- Results in interference as in Young's double slit experiment.



Isobar model

• The usual strategy is to model the total decay amplitude as a sum of individual resonances plus a non-resonant term:

$$A = \sum_{r} a_{r} e^{i\phi_{r}} A_{r} + a_{\rm NR} e^{i\phi_{\rm NR}} A_{\rm NR}$$

where $A_r(m_{ab}^2, m_{ac}^2)$ are the Dalitz plot dependent amplitudes which are of the form

 $A_r = F_P \times \mathcal{F}_r \times A_{\rm RBW} \times W_r$

and a_r and ϕ_r can be measured in a maximum-likelihood fit.

- Here, $A_{\text{RBW}} \times W_r$ is the resonance propagator, where W_r describes the angular distribution of the decay (and recall we used a relativistic BW as the dynamical function of the resonance: A_{RBW})¹.
- F_P and F_r are the transition form factors of the parent particle and resonance, respectively.

\Rightarrow now lets look at an example of 2 interfering amplitudes

¹ Strictly speaking the RBW works well only in the case of narrow states. The use of the mass-dependent width results in the amplitude becoming a non-analytic function. An alternative parametrization proposed by Gounaris and Sakurai (GS) recovers the analyticity of the amplitude and provides a better description for broad vector resonances.

Constructive interference



Destructive interference



Cross-channel interference



Back to measuring γ with $B \rightarrow DK$ decays





MC simulation

Resonance structure in $D \to K_S^0 \pi^+ \pi^-$ Dalitz plane

- Green & blue: $K^{*0}(892)$ [vector]
- Cyan & magenta: $K_2^*(1430)$ [tensor]
- yellow: $\rho(770)$ [vector]
- red: $f_0(980)$ [scalar]
- ... even more which are not simulated here (fit result table)

Main advantage of Dalitz plots is the ability to exploit the interference between different resonances.



Belle measurement

- Define the 2 dalitz plot variables as $m^2_+ \equiv m^2_{K^0_S \pi^+}$ $m^2_- \equiv m^2_{K^0_S \pi^-}$
- The amplitude for the $B^{\pm} \rightarrow DK^{\pm}$ decays are

$$A_{B^{+}}(m_{+}^{2}, m_{-}^{2}) = \overline{A}_{D} + r_{B}e^{i(\delta_{B} + \gamma)}A_{D}$$
$$A_{B^{-}}(m_{+}^{2}, m_{-}^{2}) = A_{D} + r_{B}e^{i(\delta_{B} - \gamma)}\overline{A}_{D}$$

where

 $A_D = A_D(m_+^2, m_-^2)$ is the complex amplitude of $D^0 \to K_S^0 \pi^+ \pi^ \overline{A}_D = \overline{A}_D(m_+^2, m_-^2)$ is the complex amplitude of $\overline{D}^0 \to K_S^0 \pi^+ \pi^-$

Recall:

 r_B = magnitude of the ratio of the amplitudes for $B^- \to \overline{D}{}^0 K^-$ and $B^- \to D^0 K^-.$

 δ_B = the relative strong phase between these 2 amplitudes.

Belle data for $D \to K_S^0 \pi^+ \pi^-$



Fit result to Belle data for $D \rightarrow K_S^0 \pi^+ \pi^-$

Intermediate state	Amplitude	Phase (°)	Fit fraction (%)
$K_S^0 \sigma_1$	1.56 ± 0.06	214 ± 3	11.0 ± 0.7
$K_{S}^{0}f_{0}(980)$	0.385 ± 0.006	207.3 ± 2.3	4.72 ± 0.05
$K^0_S \sigma_2$	0.20 ± 0.02	212 ± 12	0.54 ± 0.10
$K_{s}^{0}f_{0}(1370)$	1.56 ± 0.12	110 ± 4	1.9 ± 0.3
$K_{S}^{0} ho(770)^{0}$	1.0 (fixed)	0 (fixed)	21.2 ± 0.5
$K_{S}^{0}\omega(782)$	0.0343 ± 0.0008	112.0 ± 1.3	0.526 ± 0.014
$K_S^0 f_2(1270)$	1.44 ± 0.04	342.9 ± 1.7	1.82 ± 0.05
$K_{S}^{0} ho^{0}(1450)$	0.49 ± 0.08	64 ± 11	0.11 ± 0.04
$K_0^*(1430)^-\pi^+$	2.21 ± 0.04	358.9 ± 1.1	7.93 ± 0.09
$K_0^*(1430)^+\pi^-$	0.36 ± 0.03	87 ± 4	0.22 ± 0.04
$K^{*}(892)^{-}\pi^{+}$	1.638 ± 0.010	133.2 ± 0.4	62.9 ± 0.8
$K^{*}(892)^{+}\pi^{-}$	0.149 ± 0.004	325.4 ± 1.3	0.526 ± 0.016
$K^*(1410)^-\pi^+$	0.65 ± 0.05	120 ± 4	0.49 ± 0.07
$K^{*}(1410)^{+}\pi^{-}$	0.42 ± 0.04	253 ± 5	0.21 ± 0.03
$K_2^*(1430)^-\pi^+$	0.89 ± 0.03	314.8 ± 1.1	1.40 ± 0.06
$K_2^*(1430)^+\pi^-$	0.23 ± 0.02	275 ± 6	0.093 ± 0.014
$K^*(1680)^-\pi^+$	0.88 ± 0.27	82 ± 17	0.06 ± 0.04
$K^{*}(1680)^{+}\pi^{-}$	2.1 ± 0.2	130 ± 6	0.30 ± 0.07
non-resonant	2.7 ± 0.3	160 ± 5	5.0 ± 1.0

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CKM angle γ and Dalitz analysis

Combination of results from 3 methods



Constraints on γ from world average $B^{\pm} \rightarrow D^{(*)}K^{(*)\pm}$ decays (GLW+ADS) and Dalitz analyses (GGSZ) γ (combined) = $(73.2^{+6.3}_{-7.0})^{\circ}$

Compared to the prediction from the global CKM fit (not including these measurements): $(c_1) = (c_2 o^{\pm 1.0})o^{-1.0}$

 γ (fit) = (66.9^{+1.0}_{-3.7})°

Separated by experiment



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Results for r_B vs. γ



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CKM angle γ and Dalitz analysis

Prospects @ Belle II

All methods reproducible at Belle II

- Improvements in PID and qq suppression using neural networks Nucl. Instrum. Meth. A654: 432 (2011)
- Systematic errors from peaking charmless background, and PDFs from Dπ and sidebands will decrease with statistics.
- Elimination of D model uncertainty using samples of neutral D mesons decaying into CP eigenstates from charm factories CLEO-c and BESIII (via $\psi(3770) \rightarrow DD$).
 - ⇒ Naive scaling of combination with ADS and GLW yeilds an error of 1.5°. Physics at Super B Factory, arXiv:1002.5012 (2010)



(a) Belle at $0.5ab^{-1}$ and (b) Belle II at $50ab^{-1}$

Much more!

- Statistical error will be dominant and can be improved by including *D* decays to, e.g., $K_S^0 K^+ K^-$, $\pi^+ \pi^- \pi^0$, $K_S^0 \pi^+ \pi^- \pi^0$ (2* $\mathcal{B}(K_S^0 \pi^+ \pi^-)!$).
- Use $D\pi$ in addition to DK

Extra material - GLW Method

Cabibbo-suppressed D decays to CP-eigenstates (GLW)

GLW with D^(*)_{CP}**K**

D decays to CP eigenstates

> Amplitude triangle:



Cabibbo-suppressed D decays to CP-eigenstates (GLW)



Cabibbo-suppressed D decays to CP-eigenstates (GLW)



Summary of GLW results

<u>GLW Results</u>

Preliminary (LP 2011)

 $\begin{array}{l} R_{CP+} = 1.03 \pm 0.07 \pm 0.03 \\ R_{CP-} = 1.13 \pm 0.09 \pm 0.05 & \quad \mbox{CP-odd observables} \\ A_{CP+} = +0.29 \pm 0.06 \pm 0.02 \\ A_{CP-} = -0.12 \pm 0.06 \pm 0.01 \end{array}$

(systematics dominated by peaking background, double ratio approximation)

