Flavor Physics and the CKM Matrix

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Flavor Physics Lectures
I / XII

Karlsruhe Institute of Technology

Winter Semester 2023/2024
27. October, 2023

Course overview

- Weak interaction of quarks and leptons.
- CKM matrix.
- Measuring and constraining the Unitarity Triangle.
- Charge parity (CP) violation.
- Neutral particle oscillations.
- Quarkonium physics.
- Searches for physics beyond the Standard Model.
- Emphasis on B factories and experimental techniques.

Prerequisites

- Moderne Physik III.
- Not necessary to have taken Experimental Teilchenphysik I.
  Courses are complementary; can be taken together.

Reading material and references

Lecture material based on several textbooks and online lectures/notes.
Credits for material and figures include:

Literature

- Perkins, Donald H. (2000), *Introduction to High Energy Physics*.
- Griffiths, David J. (2nd edition), *Introduction to Elementary Particles*.
- Stone, Sheldon (2nd edition), B *decays*.

Online Resources

- Belle/BaBar Collaborations, *The Phyiscs of the* B*-Factories*.
http://arxiv.org/abs/1406.6311
- Bona, Marcella (University of London), CP *Violation Lecture Notes*,
http://pprc.qmul.ac.uk/ bona/ulpg/cpv/
- Richman, Jeremy D. (UCSB), *Heavy Quark Phyiscs and* CP *Violation*.
https://courses.physics.ucsd.edu/2010/Winter/physics222/references/driver_houches12.pdf
- Thomson, Mark (Cambridge University), *Particle Physics Lecture Handouts*,
http://www.hep.phy.cam.ac.uk/ thomson/partIIIparticles/welcome.html
- Grossman, Yuval (Cornell University), *Just a Taste. Lectures on Flavor Physics*,
http://www.lepp.cornell.edu/ pt267/files/notes/FlavorNotes.pdf
- Kooijman, P. & Tuning, N., CP *Violation*,
https://www.nikhef.nl/ h71/Lectures/2015/ppII-cpviolation-29012015.pdf

Homework assignments and ECTS credits

- Homework uploaded to Ilias every $\sim$2 weeks on Fri.
  First assignment will be posted today.
- Homework due every second Mon. (10 days later) by 10:00AM.
  Can be deposited in the Flavor Physics box (30.23 EG) or emailed to
  Dr. Slavomira (Sally) Stefkova slavomira.stefkova@kit.edu.
  First assigment is due on Nov. 6.
- Reviewed during übungen (Mon. 15:45-17:15) by Sally.
  Due to travel, the first 2 assignments reviewed on Nov. 27.
- Übungen review session held only when there is a homework due the previous week.
- 44 out of 88 HW points needed to obtain 6 ECTS points.
- Option for 8 ECTS points if, in addition to the assignments, you give an oral presentation on a Belle (II) publication at the end of the semester.

Assignment schedule

- **Assign HW 1** — 11:30 AM – 12:00 PM

FRIDAY, NOVEMBER 10, 2023
- **Assign HW 2** — 11:30 AM – 12:00 PM

FRIDAY, NOVEMBER 24, 2023
- **Assign HW 3** — 11:30 AM – 12:00 PM

FRIDAY, DECEMBER 8, 2023
- **Assign HW 4** — 11:30 AM – 12:00 PM

FRIDAY, DECEMBER 22, 2023
- **Assign HW 5** — 11:30 AM – 12:00 PM

FRIDAY, JANUARY 12, 2024
- **Assign HW 6** — 11:30 AM – 12:00 PM

FRIDAY, JANUARY 26, 2024
- **Assign HW 7** — 11:30 AM – 12:00 PM

MONDAY, NOVEMBER 6, 2023
- **HW 1 due** — 10:00 AM – 10:30 AM

MONDAY, NOVEMBER 20, 2023
- **HW 2 due** — 10:00 AM – 10:30 AM

MONDAY, DECEMBER 4, 2023
- **HW 3 due** — 10:00 AM – 10:30 AM

MONDAY, DECEMBER 18, 2023
- **HW 4 due** — 10:00 AM – 10:30 AM

MONDAY, JANUARY 15, 2024
- **HW 5 due** — 10:00 AM – 10:30 AM

MONDAY, JANUARY 22, 2024
- **HW 6 due** — 10:00 AM – 10:30 AM

MONDAY, FEBRUARY 5, 2024
- **HW 7 due** — 10:00 AM – 10:30 AM

MONDAY, NOVEMBER 27, 2023
- **HW 1 & 2 reviewed** — 3:45 PM – 5:15 PM

MONDAY, DECEMBER 11, 2023
- **HW 3 reviewed** — 3:45 PM – 5:15 PM

MONDAY, JANUARY 8, 2024
- **HW 4 reviewed** — 3:45 PM – 5:15 PM

MONDAY, JANUARY 22, 2024
- **HW 5 reviewed** — 3:45 PM – 5:15 PM

MONDAY, FEBRUARY 5, 2024
- **HW 6 reviewed** — 3:45 PM – 5:15 PM

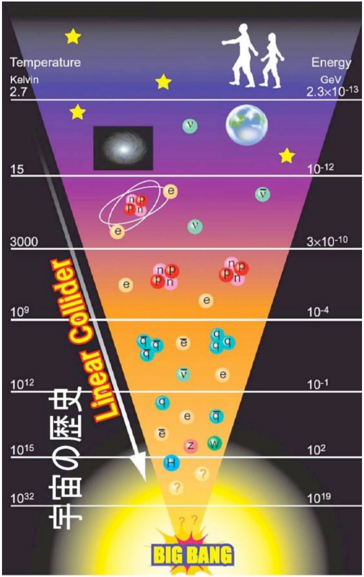
MONDAY, FEBRUARY 12, 2024
- **HW 7 reviewed** — 3:45 PM – 5:15 PM

Setting the stage

Key aims of flavor physics research

- Search for sources of matter-antimatter (CP) asymmetry in flavor to explain cosmological observations.
- Search for new symmetries to explain the mass spectrum of fundamental particles.
- Understand the interplay of mass and CP asymmetries in a coherent theory of flavor and mass generation.

flavor phenomena & possible absence of new physics at LHC point to existence of new symmetries at energies beyond the LHC.



The Standard Model Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\bar{\psi}D\psi$$ Gauge sector

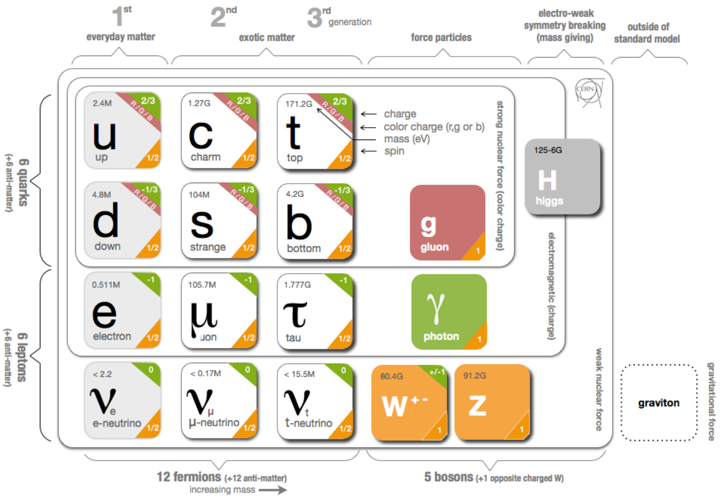
$$+\psi_i\lambda_{ij}\psi_j h + \text{h.c.}$$ Flavor sector

$$+|D_\mu h|^2 - V(h)$$ Electroweak symmetry breaking sector

CP violation only exists in the flavor sector

Moreover the flavor sector contains the majority of the free parameters of the SM
$\Rightarrow$ *Lots to study!*

Standard Model Particles



Flavor

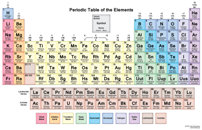
- Quarks come in 6 flavors and are grouped into 3 sets (generations)
  - How do they differ?
  - How do they interact with one another?
  - Are there only 6?

"The term flavor was first used in particle physics in the context of the quark model of hadrons. It was coined in 1971 by Murray Gell-Mann and his students at the time, Harald Fritzsch, at a Baskin-Robbins ice-cream store in Pasadena. Just as ice cream has both color and flavor so do quarks."
RMP 81 (2009) 1887

Centerpiece of the ETP flavor physics poster

The generation problem

Periodic table of the elements (end of 19^{th} century)



Explained by atomic structure (nucleus+electrons, QM and EM forces)

$\Rightarrow$

Hadron table (20^{th} century)

	Q= -1	Q= 0	Q=+1
S= 0		n	p
S= -1	Σ^-	Σ^0 ,Λ	Σ^+
S= -2	Ξ^+	Ξ^0	

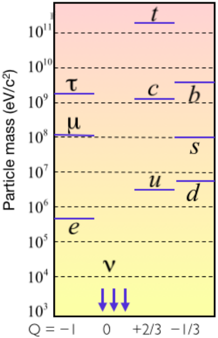
	Q=-1	Q= 0	Q=+1	Q=+2
S= 0	Δ^-	Δ^0	Δ^+	Δ^{++}
S= -1	Σ^{*-}	Σ^{*0}	Σ^{*+}	
S= -2	Ξ^{*-}	Ξ^{*0}		
S= -3	Ω^-			

	Q= -1	Q= 0	Q=+1
S=+1		K^0	K^+
S= 0	π^+	π^0 ,η	π^+
S= -1	K^+	K^0	

Explained by the existence of quarks and nature of strong interactions

$\Rightarrow$

The SM of particle physics



The SM account of the 3 generations is merely a periodic table

Symmetries review & Baryon asymmetry

Symmetries

- Define a quantum mechanical operator $\hat{O}$.
- If $\hat{O}$ describes a good symmetry,
physics looks the same before and after applying the symmetry, i.e., the observed quantity associated with $\hat{O}$ is conserved (it's the same before and after the operator is applied). E.g., the probabilities are the same for matter and anti-matter doing something.
- If this condition is not met, *the symmetry is broken*, i.e., the symmetry is not respected by nature. We can think of $\hat{O}$ as a mathematical tool used to probe our understanding of nature.
- In Moderne Physik III (vorlesung 11), we learned about 3 operators in detail: parity (P), charge (C), and time (T).

Symmetries violation

Parity conjugation reverses the spacial coordinates ($r \to -r$)

- Good symmetry of the strong and electromagnetic interactions.
- Maximally violated in the weak interaction (1957):
  $\Rightarrow$ Observed in the asymmetry in β decays of $^{60}Co \to\ ^{60}Ni + e^{-} + \nu$.

Charge conjugation changes particle into antiparticle (reverses electric charge and other quantum numbers).

- Again, good symmetry of the strong and electromagnetic interactions.
- Maximally violated in the weak interaction (1958):
  $\Rightarrow$ No left-handed anti-neutrino.

Combined Charge and Parity conjugation

- It is not sufficient to consider C and P violation separately in order to distinguish between matter and anti-matter *since the weak interaction is left-right asymmetric*.
- Need to consider CP to remove the convention dependence of what is left or right in nature.
- Product (CP) believed to be a good symmetry...
- until found to be violated in the neutral kaon system in (1964).

Matter dominated universe

In the very early universe might expect equal numbers of baryons (N_B) and anti-baryons ($N_{\overline{B}}$)

- However, no significant amounts of antimatter are observed in universe today.
- Obtain the matter/anti-matter asymmetry from “Big Bang Nucleosynthesis," *which relates the overall number density between B and $\overline{B}$ and the number density of cosmic bkgd radiation photons* (N_γ):

$$\boxed{\frac{N_B - N_{\overline{B}}}{N_\gamma} \approx \frac{N_B}{N_\gamma} \approx 10^{-10}}$$ $\Rightarrow$ *in the universe today, for every baryon there are 10^{10} photons.*

How did this happen?

The conditions *to generate* this initial asymmetry were set by Sakharov in 1967

1. Baryon number violation:
   $N_B - N_{\overline{B}}$ *is not constant*
2. Different interactions of particles and antiparticles (C and CP violation):
   If CP is conserved, for a reaction which generates a net N_B over $N_{\overline{B}}$, there would be a conjugate reaction generating a net $N_{\overline{B}}$.
3. Deviation from thermal equilibrium:
   In thermal eq., any baryon # violating process would be balanced by the inverse reaction.
   (All states with the same energy will be equally populated, so particle and antiparticle populations will be the same).

Dynamic generation of baryon asymmetry

Illustration of Sakharov conditions 1 & 2:

- Start with equal amount of matter (X) and antimatter ($\bar{X}$)
  - X decays to:
    - f_1 (with baryon number N_1) with probability p
    - f_2 (with baryon number N_2) with probability $1-p$
  - $\bar{X}$ decays to:
    - $\bar{f}_1$ (with baryon number $-N_1$) with probability $\bar{p}$
    - $\bar{f}_2$ (with baryon number $-N_2$) with probability $1-\bar{p}$
- Generated baryon asymmetry:
  - $\Delta N_{\text{total}} = \underbrace{N_1 p + N_2(1-p)}_{X\,decays} \underbrace{-N_1\bar{p} - N_2(1-\bar{p})}_{\bar{X}\,decays} = (N_1 - N_2)(p - \bar{p})$

    $\Delta N_{\text{total}} \neq 0$ requires $N_1 \neq N_2$ and $p \neq \bar{p}$

    i.e., there must be a decay mode that has both baryon number violation and a difference in the partial widths f_i and $\bar{f}_i$.

    $\rightarrow$ *Baryon number violation alone is not sufficient!*

  Equality of f_i and $\bar{f}_i$ can be guaraneed either by C or CP conservation, as these symmetries relate the particle decay process to the antiparticle decay process.

$\Rightarrow$ ***Must violate both C and CP to violate baryon number!***

Conclusion

- CP violation is a fundamental aspect of our understanding of the universe.
- Question: *Can the SM of particle physics provide the necessary amount of CP violation to explain the universe as we know it?*
- There are 2 places in the SM where CP violation enters:
  - CKM matrix;
  - PMNS matrix.
- Observed (so far) only in the quark sector.

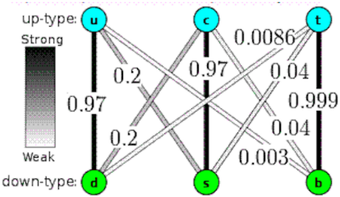
Flavor changing processes & the CKM matrix

Flavor changing transitions

- Quarks can change flavor.

  But which transitions are allowed/favored?

  ⇒ *9 possible direct transitions with varying amplitudes.*

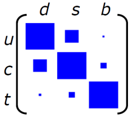


- *How can we think of this this mathematically?*

  ⇒ *as a 3x3 transition matrix.*

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Where each element V_{ij} represents the transition amplitude from quark $i \rightarrow j$



And the size of the □ represents the magnitude $|V_{ij}|$ of the transition amplitude.

The V_{ij} are not predicted by the SM ⇒ must be determined by experiment

What mediates the flavor changing process?

Weak interaction - *Main focus of this course*

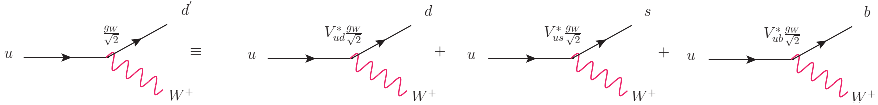
- Caused by the admission or absorbtion of massive W and Z bosons.
  Due to their large mass (80-90GeV*), the* W, Z *are short lived* $\tau = 10^{-24}s$.
- All known fermions interact through the weak interaction.
- All mesons are unstable because of the weak interaction.
- Does not have a binding energy and does not produce bound states, while in comparison: ***G*** does at astro. scales; ***EM*** does at the atomic level; ***Strong*** does inside nucleii.
- Only interaction which violates CP symmetry.
- Why is it called ***weak***?
  Its field strength over distance is several orders of magnitude smaller than that of the strong and EM forces.

Weak Eigenstates & the CKM matrix

The weak interaction couples different generations of quarks

$$\underbrace{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}}_{\text{Weak Eigenstates}} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{CKM matrix}} \underbrace{\begin{pmatrix} d \\ s \\ b \end{pmatrix}}_{\text{Mass Eigenstates}}$$

For example, the weak eigenstate d' is produced in the weak decay of an up quark:



⇒ *i.e., the u-quark couples to a linear combination of s, d, and b quarks, with the probability given by the CKM matrix.*

The CKM matrix is unitary and the elements V_{ij} are complex constants
Unitary matrices preserve normalizations and *thus probability amplitudes*

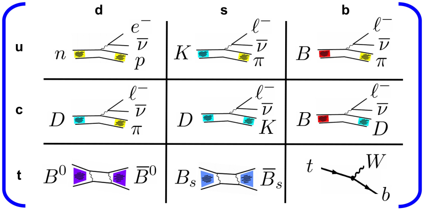
V_{ij} determination

The magnitude of the CKM matrix elements ($|V_{ij}|$) are not predicted by the SM and must be determined by experiment.

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = ?$$

Which processes provide sensitivity to the different V_{ij}?

V =



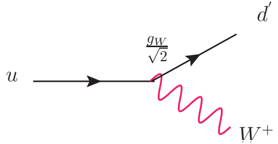
$$\Rightarrow \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.042 \\ 0.009 & 0.041 & 0.999 \end{pmatrix}$$

Combinations of measurements by Belle, BaBar, LHCb, etc., are averaged by the CKM fitter group (http://ckmfitter.in2p3.fr/).

⇒ *Many of these decays will be studied in detail throughout this course*

Weak current

Revisit the weak interaction $u \to d' + W$ from s22 and derive the $u \to d$ weak current



Writing the interaction in terms of the weak eigenstates *(where $\overline{d'}$ is the adjoint spinor)*

$$j_{ud'} = \overline{d'}[-i\frac{g_W}{\sqrt{2}}\gamma^u \tfrac{1}{2}(1-\gamma^5)]u$$

and converting to the mass eigenstates

$$\overline{d'} = d'^\dagger\gamma^0 = (V_{ud}d)^\dagger\gamma^0 = V_{ud}^* d^\dagger\gamma^0 = V_{ud}^*\overline{d}$$

gives the $u \to d$ weak current

$$j_{ud} = \overline{d}\underbrace{[-i\frac{g_W}{\sqrt{2}}V_{ud}^*\gamma^u\frac{1}{2}(1-\gamma^5)]}_{vertex factor}u$$

The $d \to u$ weak current can be similarly derived

$$j_{du} = \overline{u}[-i\frac{g_W}{\sqrt{2}}V_{ud}\gamma^u \tfrac{1}{2}(1-\gamma^5)]d$$

by noting that the CKM matrix element enters as either V_{ud}^* or V_{ud} depending on the order of the interaction $u \to d$, or $d \to u$.

Parameters of unitary matrices

A complex $n \times n$ matrix has $2n^2$ parameters

- The unitarity condition imposes n normalization constraints.
- Orthogonality between each pair of columns yields $n(n-1)$ constraints.

$\Rightarrow$ $2n^2 - n - n(n-1) = n^2$

Not all parameters in the CKM matrix have physical meaning

- Given n quark generations, $2n-1$ phases can be absorbed by the freedom to select the phases of the quark fields Each u, c, or t phase allows for multiplying a row of the CKM matrix by a phase, while each d, s, or b phase allows for multiplying a column by a phase.

$\Rightarrow$ $n^2 - (2n-1) = (n-1)^2$

Of the n^2 *real* independent parameters of a general unitary matrix:

- $\frac{1}{2}n(n-1)$ of these parameters can be associated to real rotation angles.
- The number of independent phases is:

$\Rightarrow$ $n^2 - \frac{1}{2}n(n-1) - (2n-1) = \frac{1}{2}(n-1)(n-2)$

n(families)	Total indep. params. $(n-1)^2$	Real rot. angles $\frac{1}{2}n(n-1)$	Complex phase factors $\frac{1}{2}(n-1)(n-2)$
2	1	1	0
3	4	3	1
4	9	6	3

2 vs. 3 generations of quarks

n(families)	Total indep. params. $(n-1)^2$	Real rot. angles $\frac{1}{2}n(n-1)$	Complex phase factors $\frac{1}{2}(n-1)(n-2)$
2	1	1	0
3	4	3	1
4	9	6	3

- In 2 generations the matrix is real
  - $\Rightarrow$ *No complex phase*
  - $\Rightarrow$ *No CP violation*
- In 3 generations:
  - $\Rightarrow$ *3 real numbers (Euler angles)*.
  - $\Rightarrow$ *1 complex phase which gives rise to CP violation*.

    CP violation is *built* into the Standard Model with 3 generations (or more) in this complex phase.

CKM matrix parameterizations

Many different parametrization of the CKM matrix exist, but the important thing is that *the physics results do not depend on choice*

PDG parameterization ⇒ *exact, fully general*

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{13}} & 0 & s_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$s_{ij} \equiv \sin\Theta_{ij}$
$c_{ij} \equiv \cos\Theta_{ij}$
$\delta_{13} \equiv CP$ violating phase

$\Theta_{12} = \Theta_c$ = the Cabibo angle first introduced to explain quark mixing with 2 generations (1963)

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\Theta_c & \sin\Theta_c \\ -\sin\Theta_c & \cos\Theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

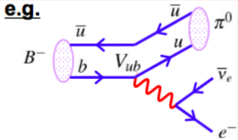
CKM matrix elements

Lets look at some of the experimental results:
(⇒ preview only... to be discussed in detail throughout the course)

$|V_{ub}|$ $\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$

From semi-leptonic B hadron decays

$$\Gamma \propto |V_{ub}|^2$$

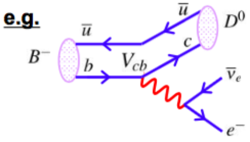


$$|V_{ub}| = 0.0043 \pm 0.0003$$

$|V_{cb}|$ $\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$

From semi-leptonic B hadron decays

$$\Gamma \propto |V_{cb}|^2$$



$$|V_{cb}| = 0.0416 \pm 0.0006$$

- $|V_{ub}| = s_{13} \ll 1$, so $c_{13} \approx 1$
  ⇒ *neglect terms proportional to s_{13} relative to terms of $\mathcal{O}(1)$.*

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

CKM matrix → *simplified*

This allows us to simplify the CKM matrix to:

$$V \approx \begin{pmatrix} c_{12} & s_{12} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} & c_{23}c_{13} \end{pmatrix}$$

where in this approximation, only V_{ub} and V_{td} carry the CP violating phase.

⇒ *To an excellent accuracy the 4 independent parameters can be chosen as*
$s_{12} = |V_{us}|, \;\; s_{13} = |V_{ub}|, \;\; s_{23} = |V_{cb}|$ *and* δ_{13}

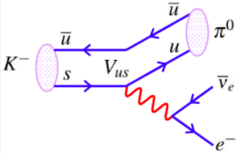
CKM matrix hierarchy

Lets look at another element:

$|V_{us}|$ $\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$

From semi-leptonic Kaon decays

$$\Gamma \propto |V_{us}|^2$$



$|V_{us}| = 0.2257 \pm 0.0021$ $\approx \sin\Theta_c$

Comparing the three elements:

$$\begin{pmatrix} c_{12} & s_{12} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} & c_{23}c_{13} \end{pmatrix}$$

$s_{12} \approx |V_{us}| = 0.2257 \pm 0.0021$

$s_{23} \approx |V_{cb}| = 0.0416 \pm 0.0006$

$s_{13} \approx |V_{ub}| = 0.0043 \pm 0.0003$

Empirically, there is a clear hierarchy of the 3 independent magnitudes of the CKM matrix:

$$1 \gg s_{12} \gg s_{23} \gg s_{13}$$

Wolfenstein parameterizations

- Motivated by this experimentally observed hierarchy, consider a Taylor expansion in powers of $\lambda \equiv |V_{us}|$ up to $\mathcal{O}(\lambda^3)$:

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

  where two new parameters ρ and η must be introduced to preserve unitarity.

  Recognize the upper left 2×2?!

  The elements are the expansion for sine and cosine. This is the 2×2 Cabibo mixing matrix. Also note that complex numbers only appear in the 3-1 mixing element.

- Now go back to the PDG parameterization and *define* the parameters (λ, A, ρ, η) to all orders in λ through:

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}} \quad s_{23} = A\lambda^2 = \lambda \left| \frac{V_{cb}}{V_{us}} \right| \quad s_{13}e^{i\delta} = V_{ub}^* = A\lambda^3(\rho + i\eta)$$

- It follows that $\rho = \frac{s_{13}}{s_{12}s_{23}}\cos\delta$ and $\eta = \frac{s_{13}}{s_{12}s_{23}}\sin\delta$ and we have a change of variables from: PDG $(s_{12}, s_{13}, s_{23}, \delta) \Rightarrow$ Wolfenstein (λ, A, ρ, η)

  Making this change of variables in the PDG parameterization, the CKM matrix is a function of λ, A, ρ, η which satisfies unitarity exactly.

Unitarity triangles

Unitarity implies: $V_{CKM}V_{CKM}^{\dagger} = I$

- Six of the orthogonality relations give rise to triangles in the complex plane with **equal** area (aka unitarity triangles). Which is the most useful?!

  Use the Wolfenstein parameterization to see the $\mathcal{O}(\lambda)$ for each element [1]

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* \approxeq \mathcal{O}(\lambda) + \mathcal{O}(\lambda) + \mathcal{O}(\lambda^5) = 0$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* \approxeq \mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^2) = 0$$

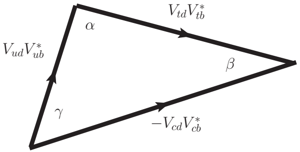
$$\boldsymbol{V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \approxeq \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0}$$

[1] There is another triangle of $\mathcal{O}(\lambda^3)$ which we will return to in our study of B_s decays.

The Unitarity Triangle [2]

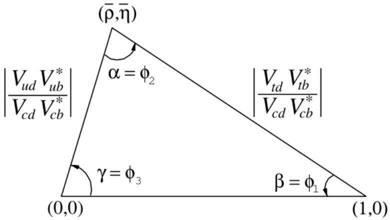
$\boldsymbol{V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0}$

- $V_{id}V_{ib}^* = 0$ represents the orthogonality condition between the first and third column V_{CKM}.
- Orientation depends on the phase convention.



- To excellent accuracy $V_{cd}V_{cb}^*$ is real with $V_{cd}V_{cb}^* = A\lambda^3 + \mathcal{O}(\lambda^7)$.
- *Scale all terms by $A\lambda^3$ and the relation can be represented as a triangle in the complex $(\overline{\rho}, \overline{\eta})$ plane.*

  $\overline{\rho} = \rho(1 - \lambda^2/2), \overline{\eta} = \eta(1 - \lambda^2/2)$



- The angles can be written in terms of CKM matrix elements as:

$$\alpha = \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \beta = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \gamma = \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

[2] 'The' UT for B decays. The B_s UT will be introduced in a later lecture.

The Unitarity Triangle

- The angles β and γ of the UT are related directly to the complex phases of the CKM-elements V_{td} and V_{ub} through

$$V_{td} = |V_{td}|e^{-i\beta}, V_{ub} = |V_{ub}|e^{-i\gamma}.$$

- Thus, we can write the Wolfenstein phase convention of the CKM matrix elements as

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}| & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^4)$$

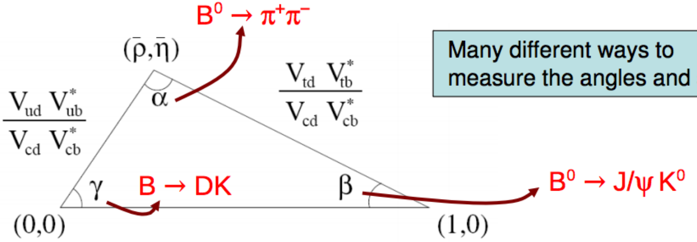
- The angle α can be obtained through the relation $\frac{V_{td}}{V_{ub}^*} = -\frac{|V_{td}|}{|V_{ub}^*|}e^{i\alpha}$, and of course $\alpha + \beta + \gamma = \pi$
- Finally, we can connect α, β, γ with the Wolfenstein parameters ρ and η

$\tan\alpha = \frac{\eta}{\eta^2 - \rho(1-\rho)}$ $\tan\beta = \frac{\eta}{1-\rho}$ $\tan\gamma = \frac{\eta}{\rho}$

Testing the SM

Is the CKM picture of CP violation correct?

- The sides and angles need to be measured to over-constrain the triangle and test that it closes.
- If there is CP violation the triangle is not flat.
  $\Rightarrow$ *Large CP asymmetries predicted $\propto$ UT angles.*

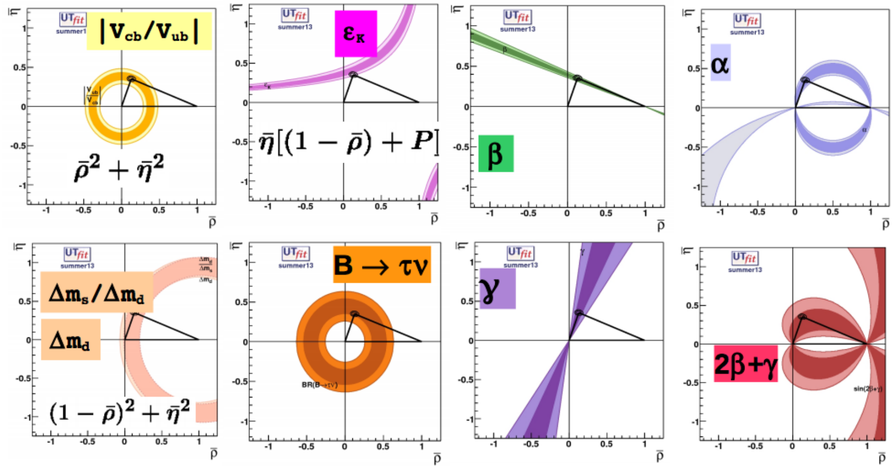


Many different ways to measure the angles and sides.

All lengths involve b decays.

UT Constraints

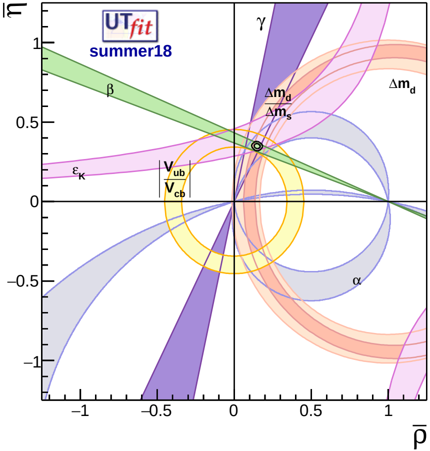
Lets look at some of the constraints from experimental results:



(2013 UT Fit)

Combined UT Fit

Fast-forward to 2018



$$\bar{\rho} = 0.148 \pm 0.013$$
$$\bar{\eta} = 0.348 \pm 0.010$$

Expected CP violation in the CKM matrix

- We stated that the UT must have non-zero area for CPV to exist. We also stated that all UTs have equal area.
- What we need now is a way to quantify it in a manifestly basis-independent way, i.e., we need some kind of invariant that identifies CPV.
- The interference terms that produce CP violation are proportional to the *phase-convention-independent* Jarlskog invariant:

$$\begin{aligned} J_{CP} &= \left|\mathrm{Im}(V_{ij}V_{il}^{*}V_{kj}V_{kl}^{*}\right| \quad i \neq k, j \neq l \text{ (no sum)} \\ &= 2\times \text{ UT triangle area} \\ &= \mathcal{O}(10^{-5}) \end{aligned}$$

- In the Wolfenstein parameterization:

$$J_{CP} \approxeq A^2\lambda^6\eta$$

- Finally, in the PDG parameterization we have:

$$J_{CP} = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta$$

  From this form it is clear why this quantity occurs in all CPV effects:

  It's zero if any of the mixing angles are zero. Would reduce the CKM matrix to a 2×2 matrix and allow the removal of the phase. Also, the it's clear that if the complex phase is zero, no CPV is possible.

Expected CP violation in the CKM matrix

- Now go back to “Big Bang Nucleosynthesis," and calculate[3]:

$$\frac{N_B - N_{\bar{B}}}{N_\gamma} \approx \frac{N_B}{N_\gamma} \approx \frac{J_{CP} \times P_u \times P_d}{M^{12}}$$

$$P_u = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)$$

$$P_d = (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$$

- The mass scale M^{12} taken to be the electroweak scale $\mathcal{O}(100\text{ GeV})$

  $\Rightarrow$ Gives a predicted asymmetry of $\mathcal{O}(10^{-17})$

  $\Rightarrow$ *Well below the value in the observable universe of* $\mathcal{O}(10^{-10})$

Where can we find the remainder?

- Quark sector: discrepancies with KM predictions.
- Lepton sector: CPV in neutrino oscillations.
- Gauge sector, extra dimensions, other new physics.

$\Rightarrow$ *Precision measurements of flavor observables are generically sensitive to BSM physics*

[3] https://cds.cern.ch/record/249423/files/PhysRevD.50.774.pdf

Summary

- CP violation is built into the SM as an irreducible complex phase in the CKM matrix.
- There are many ways for this CP-violating phase to manifest itself experimentally.
- Unitarity of the CKM matrix allows one to construct "unitarity triangles" in the complex plane.
- The amplitudes of CP violating processes are proportional to the area of the UT.
- However, the amount of CP violation predicted by the CKM matrix is several orders of magnitudes too small to account for the observed matter anti-matter asymmetry in the universe.
- The CKM picture of CP violation can be tested by over-constraining this UT and ensuring that it closes and is not flat.
- This MUST be done by experiment!
- If new measurements are not compatible with the CKM framework, they will open the door to physics beyond the SM.

Extra reading

- Richman, Jeremy D. (UCSB), *Heavy Quark Phyiscs and CP Violation*. Pages 14-27 (up to eqn 3.37)
- Thomson, Mark (Cambridge University), *Particle Physics Lecture Handouts*, The CKM Matrix and CP Violation, Pages 406-415.
- Grossman, Yuval (Cornell University), *Just a Taste. Lectures on Flavor Physics*, Pages 29-37 (primary) & 57-75 (secondary)
- Kooijman, P. & Tuning, N., *CP Violation*, Pages 17-26.

Particle Physics Colloquium (Thurs. @ 15:45)

February 2024

- 15 Feb Babette Doebrich (MPI): Axions/beam dumps NEW
- 08 Feb Bjoern Penning (University of Michigan): Dark matter detection using superfluid helium NEW
- 01 Feb Ferruccio Feruglio (INFN Padua): Modular flavour symmetries NEW

January 2024

- 25 Jan Roberto Franceschini (INFN Rome): Future colliders NEW
- 18 Jan Kai Schmidt-Hoberg (DESY): From the furnace, from the cold, or something else - how was dark matter produced in the early universe? NEW
- 11 Jan Jan Kieseler (ETP): AI designed detectors NEW

December 2023

- 21 Dec Jan Fiete Grosse-Oetringhaus (CERN): Heavy Ions
- 14 Dec Matthew McCullough (CERN): Higgs Vacuum metastability, inflatons
- 07 Dec Tevong You (King's College London): Light by light scattering at e+e- colliders

November 2023

- 30 Nov Slavomira Stefkova (ETP): B -> K + invisible at Belle II