Mixing and CP Violation in the Kaon System

Prof. Dr. Torben Ferber
Dr. Pablo Goldenzweig

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II / XII

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Reading material and references

Lecture material based on several textbooks and online lectures/notes.
Credits for material and figures include:

Literature

- Perkins, Donald H. (2000), *Introduction to High Energy Physics*.

- Griffiths, David J. (2nd edition), *Introduction to Elementary Particles*.

- Stone, Sheldon (2nd edition), *B decays*.

Online Resources

- Belle/BaBar Collaborations, *The Phyiscs of the B-Factories*.
  http://arxiv.org/abs/1406.6311

- Bona, Marcella (University of London), CP *Violation Lecture Notes*,
  http://pprc.qmul.ac.uk/ bona/ulpg/cpv/

- Richman, Jeremy D. (UCSB), *Heavy Quark Phyiscs and CP Violation*.
  https://courses.physics.ucsd.edu/2010/Winter/physics222/references/driver_houches12.pdf

- Thomson, Mark (Cambridge University), *Particle Physics Lecture Handouts*,
  http://www.hep.phy.cam.ac.uk/ thomson/partIIIparticles/welcome.html

- Grossman, Yuval (Cornell University), *Just a Taste. Lectures on Flavor Physics*,
  http://www.lepp.cornell.edu/ pt267/files/notes/FlavorNotes.pdf

- Kooijman, P. & Tuning, N., CP *Violation*,
  https://www.nikhef.nl/ h71/Lectures/2015/ppII-cpviolation-29012015.pdf

Recap & Outline

Last time we showed that:

- CP violation is built into the SM as an irreducible complex phase in the CKM matrix.
- The amount of CP violation predicted by the CKM matrix is several orders of magnitudes too small to account for the observed matter – anti-matter asymmetry in the universe.
- Unitarity of the CKM matrix allows one to construct "unitarity triangles" in the complex plane. The orthogonality of the first and third columns yields a UT with sides of the same order in λ.
- The CKM picture of CP violation can be tested by over-constraining this UT and ensuring that it closes and is not flat.
- This MUST be done by experiment!

- To begin our study of CP violation in specific processes, we'll start with the neutral kaon system, *where CP violation was first discovered in 1964*.
- We'll need to develop several tools along the way:
  - Understand how to determine the CP of a system of particles, to ensure conservation *and look for violation*.
  - We'll see that to do so properly, we'll need to develop the time-dependent formalism of particle – anti-particle mixing (aka oscillations).

Production of neutral Kaons

The neutral kaon system is an ideal laboratory for which to study CP violation, as neutral kaons are copiously produced in <u>strong</u> interaction processes into states of definite strangeness. why? $\Rightarrow$ *because strangeness **is** conserved in strong interactions*

The CPLEAR experiment: CERN 1990-1996 (Low energy anti-proton beam)

- Neutral kaons produced in reactions:

  $\overline{p}p \to K^-\pi^+ K^0(d\overline{s})$

  $\overline{p}p \to K^+\pi^- \overline{K}^0(\overline{d}s)$
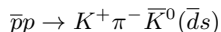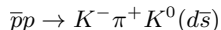
- Low energy, so particles produced almost at rest.

- Observe production and decay processes in the same detector.

- Through a technique called ***tagging*** we can unambiguosly determine the initial kaon.

- Provides a direct probe of *strangeness oscillations*.

CP eigenstates in the neutral kaon system

How do the strong eigenstates transform under $\mathcal{P}$ and $\mathcal{C}$?

- The strong eigenstates $K^0(\bar{s}d)$ and $\overline{K}^0(s\bar{d})$ are pseudoscalars and thus have $J^P = 0^-$:

$$\mathcal{P}|K^0\rangle = -|K^0\rangle \qquad \mathcal{P}|\overline{K}^0\rangle = -|\overline{K}^0\rangle$$

- The charge conjugation operator changes particles $\leftrightarrow$ anti-particles:

$$\mathcal{C}|K^0\rangle = +|\overline{K}^0\rangle \qquad \mathcal{C}|\overline{K}^0\rangle = +|K^0\rangle$$

(where the '+' sign chosen here is purely conventional)

- Combining the $\mathcal{C}$ and $\mathcal{P}$ operations:

$$\mathcal{CP}|K^0\rangle = -|\overline{K}^0\rangle \qquad \mathcal{CP}|\overline{K}^0\rangle = -|K^0\rangle$$

$\Rightarrow$ *Neither K^0 or $\overline{K}^0$ are eigenstates of CP.*

Can form CP eigenstates from linear combinations of strong eigenstates K^0 and $\overline{K}^0$

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right)$$

$$\mathcal{CP}|K_1\rangle = +|K_1\rangle$$

$$\mathcal{CP}|K_2\rangle = -|K_2\rangle$$

Decays of CP eigenstates I

Neutral kaons often decay to pions:
Kaon masses are $\approx$ **498** MeV
Pion masses are $\approx$ **140** MeV

$\Rightarrow$ *Neutral kaons can decay to 2 or 3 pions.*

$K^0 \to \pi^0 \pi^0$

- $J^P : 0^- \to 0^- + 0^-$, and conservation of angular momentum gives $L = 0$

$$\mathcal{P}(\pi^0 \pi^0) = (-1) \times (-1) \times (-1)^L = +1$$

- $|\pi^0\rangle = \frac{1}{\sqrt{2}} \left(|u\bar{u}\rangle - |d\bar{d}\rangle \right)$ is its' own anti-particle, and thus an eigenstate of $\mathcal{C}$

$$\mathcal{C}(\pi^0 \pi^0) = (+1) \times (+1) = +1$$

- The combined effect of $\mathcal{C}$ and $\mathcal{P}$ leaves the *system* unchanged

$$\mathcal{CP}|\pi^0 \pi^0\rangle = +|\pi^0 \pi^0\rangle$$

Neutral kaon decays to 2 pions occur in CP even $(+1)$ eigenstates

(same for $K^0 \to \pi^+ \pi^-$)

Decays of CP eigenstates II

$K^0 \to \pi^0 \, \pi^0 \, \pi^0$

$J^P : 0^- \to 0^- + 0^- + 0^-$, and conservation of angular momentum gives $L_1 \oplus L_2 = 0$, $\Rightarrow L_1 = L_2$

$$\boxed{\mathcal{P}(\pi^0\pi^0\pi^0) = (-1) \times (-1) \times (-1) \times (-1)^{L_1} \times (-1)^{L_2} = -1}$$

$$\boxed{\mathcal{C}(\pi^0\pi^0\pi^0) = (+1) \times (+1) \times (+1) = +1}$$

$$\boxed{\mathcal{CP}|\pi^0\pi^0\pi^0\rangle = -|\pi^0\pi^0\pi^0\rangle}$$

Neutral kaon decays to 3 pions occur in CP odd (-1) eigenstates

(same for $K^0 \to \pi^+ \, \pi^- \, \pi^0$)

Decays of CP eigenstates III

In the absence of CP violation in the weak decays of neutral kaons, we expect decays to pions to occur from the CP eigenstates K_1 and K_2 (i.e., states of definite CP):

CP EVEN $\quad |K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right) \qquad \mathcal{CP}|K_1\rangle = +|K_1\rangle \qquad K_1 \to \pi\pi$

CP ODD $\quad |K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right) \qquad \mathcal{CP}|K_2\rangle = -|K_2\rangle \qquad K_2 \to \pi\pi\pi$

What else can we say about the decays of the CP eigenstates?

- Due to the increased available phase space, decays to 2π should be faster than to 3π.
- Lifetimes of the CP eigenstates should be significantly different due to the energy available:

$$m_K - m_{2\pi} \approx 220\,\text{MeV} \quad \Rightarrow \textit{short}\text{-lifetime kaon } (K_S)$$
$$m_K - m_{3\pi} \approx 80\,\text{MeV} \quad \Rightarrow \textit{long}\text{-lifetime kaon } (K_L)$$

If **CP is conserved**, we can identify

CP EVEN $\quad |K_S\rangle = |K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right) \qquad K_S \to \pi\pi$

CP ODD $\quad |K_L\rangle = |K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right) \qquad K_L \to \pi\pi\pi$

Interlude - Time development in Mixing

- We now see how to construct a set of CP eigenstates K_1 and K_2 (which we've identified as K_S and K_L in the absence of CP violation), written in terms of the strong eigenstates K^0 and $\overline{K}^0$.
    - $\Rightarrow$ If CP is conserved, K_S decays exclusively to $\pi\pi$, while K_L decays exclusively to $\pi\pi\pi$.
- Experimentally though, we can only produce K^0 and $\overline{K}^0$ states of definite strangeness, as, e.g., in the CPLEAR experiment.
- The next question we can ask ourselves is: how does a state which we produce as pure K^0 (or $\overline{K}^0$) develop over time?
- As we'll see in the next slide, to answer this question we must study at the time-evolution of the neutral kaon system in the presence of K^0- $\overline{K}^0$ mixing.

Note: The following derivation applies to all sets of neutral particle pairs which undergo mixing, e.g. $B^0 - \overline{B}^0, B_s - \overline{B}_s, D^0 - \overline{D}^0$. In the following slides we use K, but the formalism holds for the other systems as well.

Particle – anti-Particle Mixing I

Kaons decay via the weak interaction, which allows mixing of $K^0 \leftrightarrow \overline{K}^0$ via *box diagrams*, i.e., allowing transitions between the strong eigenstates K^0, $\overline{K}^0$:
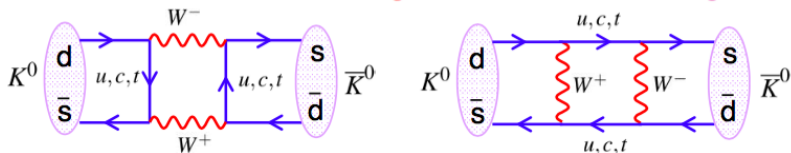
The neutral kaons propagate as eigenstates of the overall **strong + weak** interaction.

$\Rightarrow$ *We need to consider the time evolution of a mixed state of K^0 and $\overline{K}^0$:*

$$\Psi(t) = a(t)K^0 + b(t)\overline{K}^0$$

- The wave-function satisfies the time-dependent wave equation:

$$\mathcal{H}_w|\Psi(t)\rangle = i\frac{\partial}{\partial t}|\Psi(t)\rangle \quad \text{(K1)}$$

where the weak Hamiltonian can be written as a sum of mass and decay matrices:

$$\mathcal{H}_w = \left(\mathbf{M} - i\frac{\mathbf{\Gamma}}{2}\right) = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{1}{2}i\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

Particle – anti-Particle Mixing II

- The time-dependent wave-equation (K1) becomes

$$\left(\begin{array}{cc} M_{11} - \frac{1}{2}i\Gamma_{11} & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{21} - \frac{1}{2}i\Gamma_{21} & M_{22} - \frac{1}{2}i\Gamma_{22} \end{array} \right) \left(\begin{array}{c} |K^0(t)\rangle \\ |\overline{K}^0(t)\rangle \end{array} \right) = i\frac{\partial}{\partial t} \left(\begin{array}{c} |K^0(t)\rangle \\ |\overline{K}^0(t)\rangle \end{array} \right) \quad \text{(K2)}$$

  where the off-diagonal terms are due to mixing

$$M_{12} = \sum_j \frac{\langle K^0|\mathcal{H}_w|j\rangle^* \langle j|\mathcal{H}_w|\overline{K}^0\rangle}{m_{K^0} - E_j}$$
$$(j = \text{intermediate states})$$

  and the decay terms include the effects of interference between decays to a common final state f

$$\Gamma_{12} = 2\pi \sum_f \langle f|\mathcal{H}_w|K^0\rangle^* \langle f|\mathcal{H}_w|\overline{K}^0\rangle \rho_F$$

- Both the mass and decay matrices represent observable quantities and are Hermitian:

$$M_{ii} = M_{ii}^*, M_{ij} = M_{ji}^*, \quad \Gamma_{ii} = \Gamma_{ii}^*, \Gamma_{ij} = \Gamma_{ji}^*$$

- Furthermore, conservation of CPT implies that the masses and decay rates of strong eigenstates K^0 and $\overline{K}^0$ are identical:

$$M_{11} = M_{22}^* = M \quad \Gamma_{11} = \Gamma_{22}^* = \Gamma$$

Particle – anti-Particle Mixing III

- Using these identities we can write the time evolution of the system (K2) as

$$\begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12} - \frac{1}{2}i\Gamma_{12} \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M - \frac{1}{2}i\Gamma \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = i\frac{\partial}{\partial t} \begin{pmatrix} a \\ b \end{pmatrix} \quad \text{(K3)}$$

- We can then solve the system of coupled differential equations for $a(t)$ and $b(t)$ by:
  1) Finding the eigenvalues and eigenstates of the weak Hamiltonian $\mathcal{H}_w$, and
  2) Transforming into this basis.

- Solving the eigenvalue equation for $|\mathcal{H}_w - \mu I| = 0$ yields eigenvalues

$$\mu_{\pm} = M - \frac{1}{2}i\Gamma \pm \sqrt{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)(M_{12} - \frac{1}{2}i\Gamma_{12})}$$

- and normalized eigenstates

$$|K_{\pm}\rangle = \frac{1}{\sqrt{1 + |\frac{q}{p}|^2}} (|K^0\rangle \pm \frac{q}{p}|\overline{K}^0\rangle) \qquad \frac{q}{p} = \sqrt{\frac{(M_{12}^* - \frac{1}{2}i\Gamma_{12}^*)}{(M_{12} - \frac{1}{2}i\Gamma_{12})}} \quad \text{(K4)}$$

Look familiar??   *Make a mental note of this $\frac{q}{p}$*
we'll revisit it soon...

- We can now identify these as the $|K_S\rangle$ and $|K_L\rangle$ states:

$$\boxed{|K_S\rangle = \frac{1}{\sqrt{1+|\frac{q}{p}|^2}}(|K^0\rangle - \frac{q}{p}|\overline{K}^0\rangle)} \quad \boxed{|K_L\rangle = \frac{1}{\sqrt{1+|\frac{q}{p}|^2}}(|K^0\rangle + \frac{q}{p}|\overline{K}^0\rangle)} \quad \text{(K5)}$$

where in the limit where M_{12} and Γ_{12} are real, $\frac{q}{p} = 1$, and the $|K_S\rangle$ and $|K_L\rangle$ eigenstates correspond to the CP eigenstates $|K_1\rangle$ and $|K_2\rangle$ we constructed on Slide 8.

- Substituting these states back into $\Psi(t) = a(t)K^0 + b(t)\overline{K}^0$, and evaluating the time-dependence of $a(t)$ and $b(t)$ using (K3), gives

$$\boxed{|\Psi(t)\rangle = A_S e^{-i\left(m_S t - \frac{\Gamma_S}{2}\right)t}|K_S\rangle + A_L e^{-i\left(m_L t - \frac{\Gamma_L}{2}\right)t}|K_L\rangle} \quad \text{(K6)}$$

where A_S and A_L are constants, and we have set

$$\boxed{m_{S,L} = M \mp \mathcal{R}\left\{\sqrt{(M_{12}^* - \tfrac{1}{2}i\Gamma_{12}^*)(M_{12} - \tfrac{1}{2}i\Gamma_{12})}\right\}}$$

$$\boxed{\Gamma_{S,L} = \Gamma \pm 2\mathcal{I}\left\{\sqrt{(M_{12}^* - \tfrac{1}{2}i\Gamma_{12}^*)(M_{12} - \tfrac{1}{2}i\Gamma_{12})}\right\}}$$

Hence, in terms of the $|K_S\rangle$, $|K_L\rangle$ basis, the states propagate as independent particles with definite masses and lifetimes.

Which is the 'real' particle? *Quoting David J. Griffiths*

The neutral Kaon system adds a subtle twist to the old question, 'What is a particle?'

Kaons are typically produced by the strong interactions, in eigenstates of strangeness (K^0 and $\overline{K^0}$), but they decay by the weak interactions, as eigenstates of CP (K_1 and K_2).

Which, then, is the 'real' particle?

If we hold that a 'particle' must have a unique lifetime, then the 'true' particles are K_1 and K_2. But we need not be so dogmatic. In practice, it is sometimes more convenient to use one set, and sometimes, the other.

The situation is in many ways analogous to polarized light. Linear polarization can be regarded as a superposition of left-circular polarization and right-circular polarization. If you imagine a medium that preferentially absorbs right-circularly polarized light, and shine on it a linearly polarized beam, it will become progressively more left-circularly polarized as it passes through the material, just as a K^0 beam turns into a K_2 beam. But whether you choose to analyze the process in terms of states of linear or circular polarization is largely a matter of taste.

Neutral kaon decays to pions *(No CP violation)*

We now have the machinery to determine the decay rate to 2 (or 3) pions for a state which was produced as purely K^0 (or $\overline{K}^0$)

- Again, recall that neutral kaons are produced via $\pi^- + p \rightarrow \Lambda(uds) + K^0(d\overline{s})$, into states of definite strangeness, which means we can create a beam of pure K^0 at $t = 0$.

  In the K_S and K_L basis:

  $$|\Psi(t=0)\rangle = \tfrac{1}{\sqrt{2}}\left[|K_S\rangle + |K_L\rangle\right]$$

  which we've just shown evolves in time as

  $$|\Psi(t)\rangle = \tfrac{1}{\sqrt{2}}\left[e^{-\left(im_S + \frac{\Gamma_S}{2}\right)t}|K_S\rangle + e^{-\left(im_L + \frac{\Gamma_L}{2}\right)t}|K_L\rangle\right]$$

- To clean this up a bit, lets write

  $$\theta_S(t) = e^{-\left(im_S + \frac{\Gamma_S}{2}\right)t}, \quad \theta_L(t) = e^{-\left(im_L + \frac{\Gamma_L}{2}\right)t}$$

  which gives us the compact expression for the time dependent state at time t

  $$|\Psi(t)\rangle = \tfrac{1}{\sqrt{2}}\left[\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle\right]$$

$\Rightarrow$ we can calculate the decay rate to two pions as

  $$\Gamma(K^0_{t=0} \rightarrow \pi\pi) \propto |\langle K_S|\Psi(t)\rangle|^2 \propto |\theta_S(t)|^2 = e^{-\Gamma_S t} = e^{-t/\tau_S}$$

  i.e., the decays K_S, the short lifetime component $\rightarrow$ *as expected*

$\Rightarrow$ and to three pions as

  $$\Gamma(K^0_{t=0} \rightarrow \pi\pi\pi) \propto |\langle K_L|\Psi(t)\rangle|^2 \propto |\theta_L(t)|^2 = e^{-\Gamma_L t} = e^{-t/\tau_L}$$

  i.e., the decays K_L, the long lifetime component $\rightarrow$ *as expected*

Expect to see predominantly 2π decays near the start of the beam, and 3π decays further downstream.
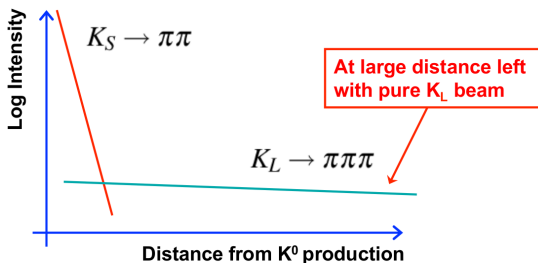
$K^0 - \overline{K}^0$ Oscillations

(No CP violation)

$K^0 - \overline{K}^0$ Oscillations - Theory I   *(No CP violation)*

We've seen that particles and anti-particles mix, but what can we say about the frequency of oscillations between K^0 and $\overline{K}^0$?

- Again, start with our wave function at time t

  $$|\Psi(t)\rangle = \tfrac{1}{\sqrt{2}} \left[\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle\right]$$

- We want to study $K^0 - \overline{K}^0$ oscillation, so lets change to this basis by writing $|K_S\rangle$ and $|K_L\rangle$ in terms of the strong eigenstates $|K^0\rangle$ and $|\overline{K}^0\rangle$

  $$|\Psi(t)\rangle = \tfrac{1}{2} \left[\theta_S(t) + \theta_L(t)\right]|K^0\rangle + \tfrac{1}{2} \left[\theta_L(t) - \theta_S(t)\right]|\overline{K}^0\rangle \quad \text{(K7)}$$

- Now, if we want the decay rate to K^0 at time t (i.e., the K^0 fraction, aka intensity), we need:

  $$\Gamma(K_{t=0}^0 \to K^0) = |\langle K^0|\Psi(t)\rangle|^2 = \tfrac{1}{4} |\theta_S(t) + \theta_L(t)|^2$$

- Similarly, for the $\overline{K}^0$ fraction:

  $$\Gamma(K_{t=0}^0 \to \overline{K}^0) = |\langle \overline{K}^0|\Psi(t)\rangle|^2 = \tfrac{1}{4} |\theta_S(t) - \theta_L(t)|^2$$

$K^0 - \overline{K}^0$ Oscillations - Theory II   *(No CP violation)*

- Using the identity $|a \pm b|^2 = |a^2| + |b^2| \pm 2\mathcal{R}(ab^*)$

$$|\theta_S(t) \pm \theta_L(t)|^2 = \left| e^{-\left(im_S + \frac{\Gamma_S}{2}\right)t} \pm e^{-\left(im_L + \frac{\Gamma_L}{2}\right)t} \right|^2$$

$$= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t} \cos(m_S - m_L)t$$

$$= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t} \cos(\Delta m t)$$

Frequency of oscillation between neutral kaon states given by the mass splitting

$$\Delta m = m(K_L) - m(K_S)$$

$K^0 - \overline{K}^0$ Oscillations - Theory III *(No CP violation)*

We can now write the K^0 and $\overline{K}^0$ fraction at time t of a beam initially produced as pure K^0 as:

$$\Gamma(K^0_{t=0} \to K^0) = \tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2 e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t}\cos(\Delta mt)\right]$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = \tfrac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2 e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t}\cos(\Delta mt)\right]$$

Experimentally we find (next slides):

- $\tau(K_S) = 0.9 \times 10^{-10}$s
  $\tau(K_L) = 0.5 \times 10^{-7}$s
  $\Delta m = (3.506 \pm 0.006) \times 10^{-15}$GeV

- The mass difference corresponds to
  an oscillation period of
  $T = \frac{2\pi\hbar}{\Delta m} \approx 1.2 \times 10^{-9}\ s$

$\Rightarrow$ Relatively long compared to the K_S
  lifetime, *therefore do not observe
  very pronounced oscillations*

Initial K^0 − Semi−Leptonic Decay Rates

CP Conserved

$K^0_{t=0} \to K^0$

After a few τ_{K_S} left with a pure K_L beam, which is half K^0 and half $\overline{K}^0$

$K^0_{t=0} \to \overline{K}^0$

Decay Time (t/τ_1)
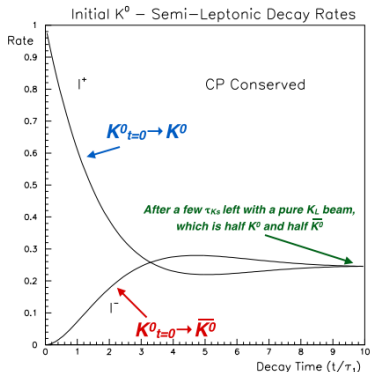
How we can *measure* the mass splitting?

$\Rightarrow$ ***Lets take a look at semi-leptonic kaon decays.***
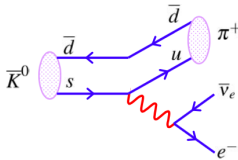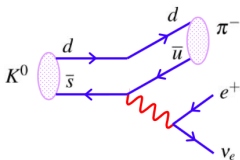
Direct	$K^0_{t=0} \to K^0 \to \pi^- e^+ \nu_e$	$\overline{K}^0_{t=0} \to \overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$
via. Mixing	$K^0_{t=0} \to \overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$	$\overline{K}^0_{t=0} \to K^0 \to \pi^- e^+ \nu_e$

For an initial K^0 beam, have *both* charge combinations (same for initial $\overline{K}^0$ beam).

If we produce the neutral kaons in $p\overline{p}$ collisions, as was done at the CPLEAR experiment at CERN, how do we *determine* which neutral kaon was initially produced?

$$\overline{p}p \to K^- \pi^+ K^0 \quad \text{or} \quad \overline{p}p \to K^+ \pi^- \overline{K}^0 \ ?$$

$\Rightarrow$ *Charge of the $K^\pm \pi^\mp$ in the production process **tags** the initial kaon as either K^0 or $\overline{K}^0$*

$K^0 - \overline{K}^0$ Oscillations - Measurement II *(No CP violation)*

Lets look at an example of a CPLEAR event.

- 4 visible particles in the detector (neutrinos are undetectable).

- We know that there must be an oppositely charged $K\pi$ pair from the initial reaction.

- Here the $K^- \pi^+$ **tags** the initial reaction as $\overline{p}p \to K^- \pi^+ K^0$ and thus we deduce the initial wave-function to be
  $|\Psi(t=0)\rangle = |K^0\rangle$

- Now from the remaining $\pi^+ e^-$ we can deduce if the K^0 decayed directly or underwent mixing.
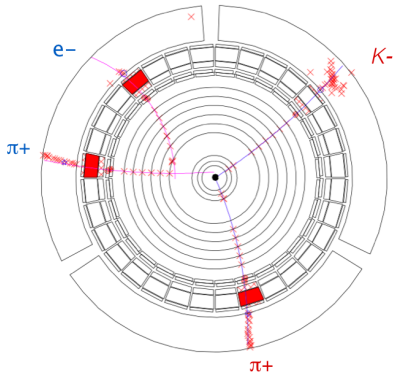
Clearly it mixed, and our overall reaction for this event is:

$$\overline{p}p \to K^- \pi^+ K^0$$
$$\hookrightarrow \overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$$
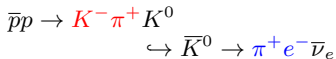
- In this way we can measure the decay rates as a function of time for all 4 combinations. For this event, we have the rate:
  $$\Gamma(K^0_{t=0} \to \overline{K}^0) \propto \Gamma(K^0_{t=0} \to \pi^+ e^- \overline{\nu}_e)$$

$K^0 - \overline{K}^0$ Oscillations - Measurement III *(No CP violation)*

The usual measure is the asymmetry, since efficiencies, and some systematics, cancel out.
How can we construct a meaningful asymmetry?

The 4 measureable rates are:

$$\Gamma(K^0_{t=0} \to K^0) \propto \Gamma(K^0_{t=0} \to \pi^- e^+ \nu_e) \equiv R_u \quad \text{Unmixed}$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) \propto \Gamma(K^0_{t=0} \to \pi^+ e^- \overline{\nu}_e) \equiv R_m \quad \text{Mixed}$$

$$\Gamma(\overline{K}^0_{t=0} \to \overline{K}^0) \propto \Gamma(\overline{K}^0_{t=0} \to \pi^+ e^- \overline{\nu}_e) \equiv \overline{R}_u \quad \text{Unmixed}$$

$$\Gamma(\overline{K}^0_{t=0} \to K^0) \propto \Gamma(K^0_{t=0} \to \pi^- e^+ \nu_e) \equiv \overline{R}_u \quad \text{Mixed}$$

From which we can construct:

$$A(\Delta m) = \frac{(R_u + \overline{R}_u) - (R_m + \overline{R}_m)}{(R_u + \overline{R}_u) + (R_m + \overline{R}_m)}$$

$$= \frac{2e^{-(\Gamma_S + \Gamma_L)t/2}\cos(\Delta m t)}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$

The points are the data and the line shows the theoretical prediction for the value of Δm most consistent with the data.
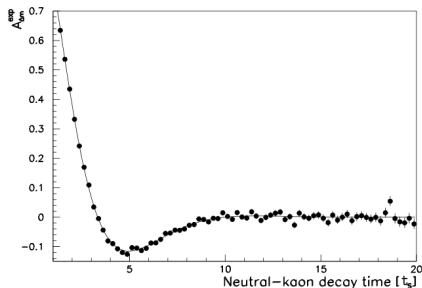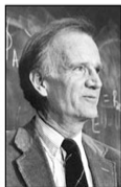

Combining with experiments at Fermilab:

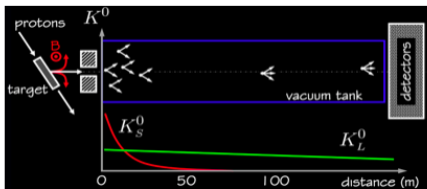$$\Delta m = m(K_L) - m(K_S) = (3.506 \pm 0.006) \times 10^{-15} \text{GeV}$$
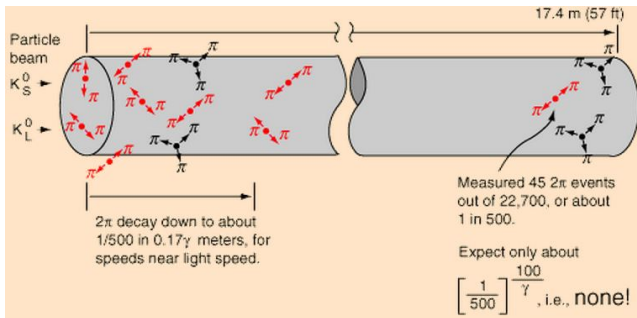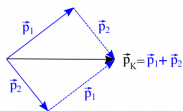
1964: Cronin-Fitch Experiment
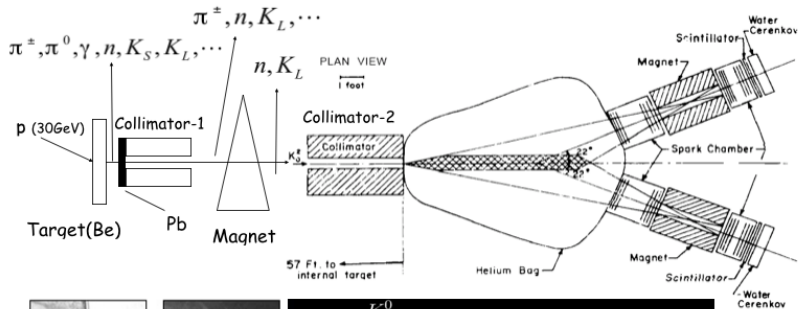
If CP is conserved, we only expect $K_L \to 3\pi$ decays at a long distance from the production point of a beam of neutral kaons (the K_S component will have decayed away).
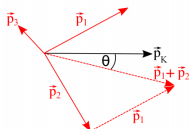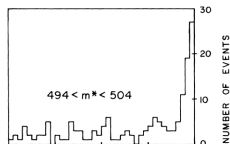
In 1964, C&F observed 45 $K_L \to 2\pi$ decays (out of 22.7K kaon decays) a long distance from the production point.

2π decay 3π decay $\cos\theta$

CP is violated in hadronic weak interactions

Lesson from Flavor

Unwise to assume 0.1% is "good enough" in flavor.

- **1962:** "A special search at Dubna was carried out by E. Okonov and his group. They have not found a single $K_L \to \pi^+ \pi^-$ event among 600 decays into charged particles (Anikira et al, JETP 1962). At that stage the search was terminated by administration of the Lab. The group was unlucky." L.B. Okun, "Spacetime and vacuum as seen from Moscow" (2002)

- **1964:** Cronin & Fitch observed 45 $K_L \to 2\pi$ decays (out of 22,700 Kaon decays) a long distance from the production point: $\mathcal{B}(K_L \to 2\pi) = 2 \times 10^{-3}$. PRL **13** 138 (1964)

Difficult for some to swallow

Cronin recalls the 1964 ICHEP conference:

> "As the session neared a close, one of my Soviet colleagues suggested that, perhaps, the effect was due to regeneration of short-lived K mesons (K_S) in a fly unfortunately trapped in the bag. We did a quick back of the envelop estimate of the density for the fly necessary to produce the effect. The density required was far in excess of uranium."

A beam with equal fraction of K^0 and $\bar{K}^0$ (e.g. a pure K_L beam) will contain after the passage of matter slightly more K^0 (absorption of $\bar{K}^0$)

$$K_2 = \tfrac{1}{\sqrt{2}}\left(K^0 - \bar{K}^0\right)$$
$$K_L$$

$$\Rightarrow \quad K_2 + \varepsilon K_1$$
$$\qquad K_L \quad K_S$$

Regeneration of K_S from a pure K_L beam after passage of matter.

Matter = ?

CP violation in the Kaon system

Recall that we identified the K_S and K_L states as the CP eigenstates K_1 and K_2 in the limit of no CP violation

CP EVEN (+1) $|K_S\rangle = |K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right)$ $K_S \to \pi\pi$

CP ODD   (-1) $|K_L\rangle = |K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right)$ $K_L \to \pi\pi\pi$

Now that we know CP is violated, we must try to account for it:

One possibility is that the K_S and K_L do not *exactly* correspond to the CP eigenstates K_1 and K_2, but rather a linear combination of them with:

$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_1\rangle + \varepsilon|K_2\rangle\right]$ where $\varepsilon = |\varepsilon|e^{i\phi} = \frac{p-q}{p+q}$ [recall Slide 12 (K4)]

$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]$ and $|\varepsilon| \approx 2 \times 10^{-3}$

           $\hookrightarrow 3\pi$ $\hookrightarrow 2\pi$

           $CP = -1$ $CP = +1$

This is known as CPV in mixing, aka *indirect CPV* ($\Delta S = 2$).

Exercise: Show that these CP violating $|K_S\rangle$, $|K_L\rangle$ in terms of ε are equivalent to the states in eqns. (K4) & (K5) in terms of $\frac{q}{p}$.

$K^0 - \overline{K}^0$ Oscillations

(CP violation in mixing)

$K^0 - \overline{K}^0$ Oscillations - Theory I *(CP violation)*

For the full picture, we must re-visit the development of the $K^0 - \overline{K}^0$ system including CP violation in mixing

- Writing our $|K_S\rangle$ and $|K_L\rangle$ states (inc. CPV) in terms of the strong eigenstates $|K^0\rangle$ and $|\overline{K}^0\rangle$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} \left[|K_1\rangle + \varepsilon|K_2\rangle\right] = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\varepsilon|^2}} \left[(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\overline{K}^0\rangle\right]$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} \left[|K_2\rangle + \varepsilon|K_1\rangle\right] = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\varepsilon|^2}} \left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\overline{K}^0\rangle\right]$$

- We know from before that the states $|\Psi(t)\rangle$ propagate as $|K_S\rangle$ and $|K_L\rangle$, i.e., independent particles with definite masses and lifetimes.
  Invert these expressions to switch to the $|K_S\rangle$ and $|K_L\rangle$ basis:

$$|K^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1+\varepsilon} \left[|K_L\rangle + |K_S\rangle\right] \quad |\overline{K}^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1-\varepsilon} \left[|K_L\rangle - |K_S\rangle\right]$$

- If we add on the time dependence $\theta_S(t)$ and $\theta_L(t)$, we see that states which were initially produced as K^0, or $\overline{K}^0$, evolve with time as

$$|\Psi(t)\rangle_{K^0_{t=0}} = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1+\varepsilon} \left[\theta_L(t)|K_L\rangle + \theta_S(t)|K_S\rangle\right] \quad \text{(K8)}$$

$$|\Psi(t)\rangle_{\overline{K}^0_{t=0}} = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1-\varepsilon} \left[\theta_L(t)|K_L\rangle - \theta_S(t)|K_S\rangle\right] \quad \text{(K9)}$$

$K^0 - \overline{K}^0$ Oscillations - Theory II *(CP violation)*

If we want to study $K^0 - \overline{K}^0$ oscillations, we need to switch to the $K^0 \, \overline{K}^0$ basis (as we did before in K7)

- K8 and K9 become.

$$|\Psi(t)\rangle_{K^0_{t=0}} = \frac{1}{2} \left[(\theta_L(t) + \theta_S(t)) \, |K^0\rangle + (\theta_L(t) - \theta_S(t)) \left(\frac{1-\varepsilon}{1+\varepsilon} \right) |\overline{K}^0\rangle \right]$$

$$|\Psi(t)\rangle_{\overline{K}^0_{t=0}} = \frac{1}{2} \left(\frac{1+\varepsilon}{1-\varepsilon} \right) \left[(\theta_L(t) - \theta_S(t)) \, |K^0\rangle + (\theta_L(t) + \theta_S(t)) \, |\overline{K}^0\rangle \right]$$

- Finally, as we want the mixing probabilities as a function of time, we calculate, as before:

$$\Gamma(K^0_{t=0} \to K^0) = |\langle K^0 | \Psi(t) \rangle_{K^0_{t=0}}|^2$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = |\langle \overline{K}^0 | \Psi(t) \rangle_{K^0_{t=0}}|^2$$

$$\Gamma(\overline{K}^0_{t=0} \to K^0) = |\langle K^0 | \Psi(t) \rangle_{\overline{K}^0_{t=0}}|^2$$

$$\Gamma(\overline{K}^0_{t=0} \to \overline{K}^0) = |\langle \overline{K}^0 | \Psi(t) \rangle_{\overline{K}^0_{t=0}}|^2$$

$\Rightarrow$ *Lets see what we can say about the difference in the behavior of matter vs. anti-matter.*

$K^0 - \overline{K}^0$ Oscillations - Theory III *(CP violation)*

$$\Gamma(K^0_{t=0} \to K^0) = |\langle K^0 | \Psi(t) \rangle_{K^0_{t=0}}|^2 = \tfrac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t} \cos(\Delta m t) \right]$$

$$\Gamma(K^0_{t=0} \to \overline{K}^0) = |\langle \overline{K}^0 | \Psi(t) \rangle_{K^0_{t=0}}|^2 = \tfrac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t} \cos(\Delta m t) \right] \left| \frac{1-\varepsilon}{1+\varepsilon} \right|^2$$

$$\Gamma(\overline{K}^0_{t=0} \to K^0) = |\langle K^0 | \Psi(t) \rangle_{\overline{K}^0_{t=0}}|^2 = \tfrac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t} \cos(\Delta m t) \right] \left| \frac{1+\varepsilon}{1-\varepsilon} \right|^2$$

$$\Gamma(\overline{K}^0_{t=0} \to \overline{K}^0) = |\langle \overline{K}^0 | \Psi(t) \rangle_{\overline{K}^0_{t=0}}|^2 = \tfrac{1}{4} \left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t} \cos(\Delta m t) \right]$$

In the figure, $\left| \frac{1-\varepsilon}{1+\varepsilon} \right|^2 = |\alpha|$

- $\Gamma(K^0_{t=0} \to K^0) = \Gamma(\overline{K}^0_{t=0} \to \overline{K}^0)$ from CPT symmetry.
- If $|\alpha| = 1$, we recover the equations on S20 for no CPV.
- If $|\alpha| \neq 0$, $\Gamma(K^0_{t=0} \to \overline{K}^0) \neq \Gamma(\overline{K}^0_{t=0} \to K^0)$, and if $|\alpha| < 1$, we get less $\overline{K}^0$ than K^0, a clear difference as seen in the dashed vs. dotted line.

  (Note: ε in this model is $10\times$ the measured value, for illustrative purposes)
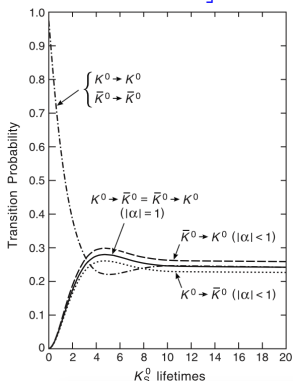
Lets return to the $\pi\pi$ system where C.&F. first discovered CPV in $K_L \to \pi\pi$.
The $\pi\pi$ system is a CP eigenstate, so lets swich to the basis $|K_1\rangle$ and $|K_2\rangle$, states which have definite CP.

- Substituting our new CP violating $|K_S\rangle$ and $|K_L\rangle$ into (K8)

$$|\Psi(t)\rangle_{K_{t=0}^0} = \frac{1}{\sqrt{2}} \frac{1}{1+\varepsilon} \left[(|K_2\rangle + \varepsilon|K_1\rangle)\,\theta_L(t) + (|K_1\rangle + \varepsilon|K_2\rangle)\,\theta_S(t) \right]$$

$$= \frac{1}{\sqrt{2}} \frac{1}{1+\varepsilon} \left[(\theta_S(t) + \varepsilon\theta_L(t)|K_1\rangle) + (\theta_L(t) + \varepsilon\theta_S(t)|K_2\rangle) \right] \quad \boxed{CP \text{ Eigenstates}}$$

- Now, as the 2π system has $CP = +1$, we know it arises from the decay of the $|K_1\rangle$ eigenstate with $CP = +1$:

$$\Gamma(K_{t=0}^0 \to \pi\pi) \propto |\langle K_1||\Psi(t)\rangle_{K_{t=0}^0}|^2 = \frac{1}{2} \left| \frac{1}{1+\varepsilon} \right|^2 |\theta_S(t) + \varepsilon\theta_L(t)|^2$$

- How can we simplify this?

$$\Rightarrow \left| \frac{1}{1+\varepsilon} \right|^2 = \frac{1}{(1+\varepsilon^*)(1+\varepsilon)} \approx \frac{1}{1+2\mathcal{R}\{\varepsilon\}} \approx 1 - 2\mathcal{R}\{\varepsilon\} \qquad \text{for } |\varepsilon| \ll 1$$

$\Rightarrow$ Use the identity $|a \pm b|^2 = |a^2| + |b^2| \pm 2\mathcal{R}(ab^*)$ as before (Slide 18), with $\varepsilon = |\varepsilon|e^{i\phi}$, to evaluate:

$$|\theta_S(t) + \varepsilon\theta_L(t)|^2 = e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon|e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t}\cos(\Delta m t - \phi)$$

Putting it all together:

$$\Gamma(K^0_{t=0} \to \pi\pi) =$$
$$\frac{1}{2}\left(1 - 2\mathcal{R}\{\varepsilon\}\right) N_{\pi\pi}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} + 2|\varepsilon|e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t}\cos(\Delta m t - \phi)\right]$$

$$\hookrightarrow K_S \to \pi\pi \quad \hookrightarrow CPV K_L \to \pi\pi \quad \hookrightarrow \text{interference}$$

Likewise for a beam of initial $\overline{K}^0$:

$$\Gamma(\overline{K}^0_{t=0} \to \pi\pi) =$$
$$\frac{1}{2}\left(1 + 2\mathcal{R}\{\varepsilon\}\right) N_{\pi\pi}\left[e^{-\Gamma_S t} + |\varepsilon|^2 e^{-\Gamma_L t} - 2|\varepsilon|e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t}\cos(\Delta m t - \phi)\right]$$

where the interference term changes sign

Lets look at CPLEAR data for $K^0(\overline{K}^0) \to \pi^+\pi^-$, where we can:

- Distinguish the $K^0_{t=0}$ & $\overline{K}^0_{t=0}$ through tagging
- See the effects of the interference term.
- And clearly see the CP violating $K_L \to \pi^+\pi^-$ at large proper times, where only the long lifetime remains:

$$\Gamma(K^0_{t=0} \to \pi\pi) \to \frac{1}{2}\left(1 - 2\mathcal{R}\{\varepsilon\}\right) N_{\pi\pi}|\varepsilon|^2 e^{-\Gamma_L t}$$
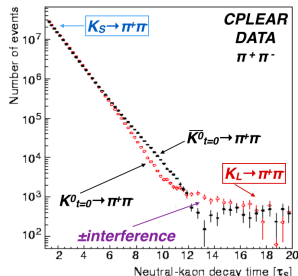
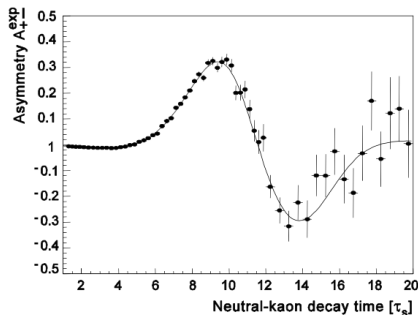***Can we use the CPLEAR data to measure $\varepsilon = |\varepsilon|e^{i\phi}$?***

Construct the asymmetry:

$$A = \frac{\Gamma(\overline{K}^0_{t=0} \to \pi\pi) - \Gamma(K^0_{t=0} \to \pi\pi)}{\Gamma(\overline{K}^0_{t=0} \to \pi\pi) + \Gamma(K^0_{t=0} \to \pi\pi)} = 2\mathcal{R}\{\varepsilon\} - \frac{2|\varepsilon|e^{(\Gamma_S - \Gamma_L)t/2}\cos(\Delta m t - \phi)}{1 + |\varepsilon|^2 e^{(\Gamma_S - \Gamma_L)t}}$$

Best fit to the data:

$$|\varepsilon| = (2.264 \pm 0.035) \times 10^{-3})$$
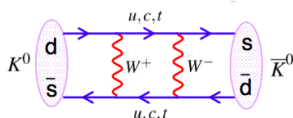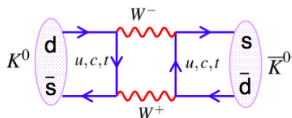$$\phi = (43.19 \pm 0.73)^\circ$$

$K^0 - \overline{K}^0$ Oscillations and the CKM Matrix *(CP violation)*

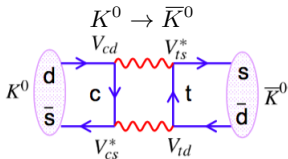We've now seen the CPV in mixing leads to the inequality:
$$\Gamma(K^0_{t=0} \to \overline{K}^0) \neq \Gamma(\overline{K}^0_{t=0} \to K^0)$$
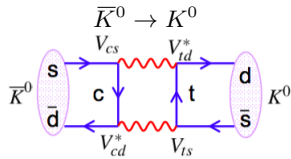**How can we explain this in terms of the CKM matrix?**

Recall the box diagrams responsible for mixing, *where we need to sum over all possible quark exchanges in the box.*

For illustration, lets consider only this diagram, and compare the equivalent box diagrams for $K^0 \to \overline{K}^0$ and $\overline{K}^0 \to K^0$.

$$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^* \qquad M_{if} \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{if}^*$$

The difference in rates:
$$\Gamma(K^0 \to \overline{K}^0) - \Gamma(\overline{K}^0 \to K^0) \propto M_{fi} - M_{if}^* = 2\mathcal{I}\{M_{fi}\}$$
is non-zero only if the CKM matrix has an imaginary component

Extra reading

- Richman, Jeremy D. (UCSB), *Heavy Quark Phyiscs and CP Violation*.
  http://physics.ucsd.edu/students/courses/winter2010/physics222/references/driver_houches12.pdf
  Pages 183-204.

- Thomson, Mark (Cambridge University), *Particle Physics Lecture Handouts*,
  http://www.hep.phy.cam.ac.uk/ thomson/partIIIparticles/welcome.html
  The CKM Matrix and CP Violation, Pages 416-437.

- Kooijman, P. & Tuning, N., *CP Violation*,
  http://master.particles.nl/LectureNotes/2011-CP.pdf
  Pages 55-64.

- The CPLEAR Collaboration, *The CPLEAR-Experiment at CERN*.