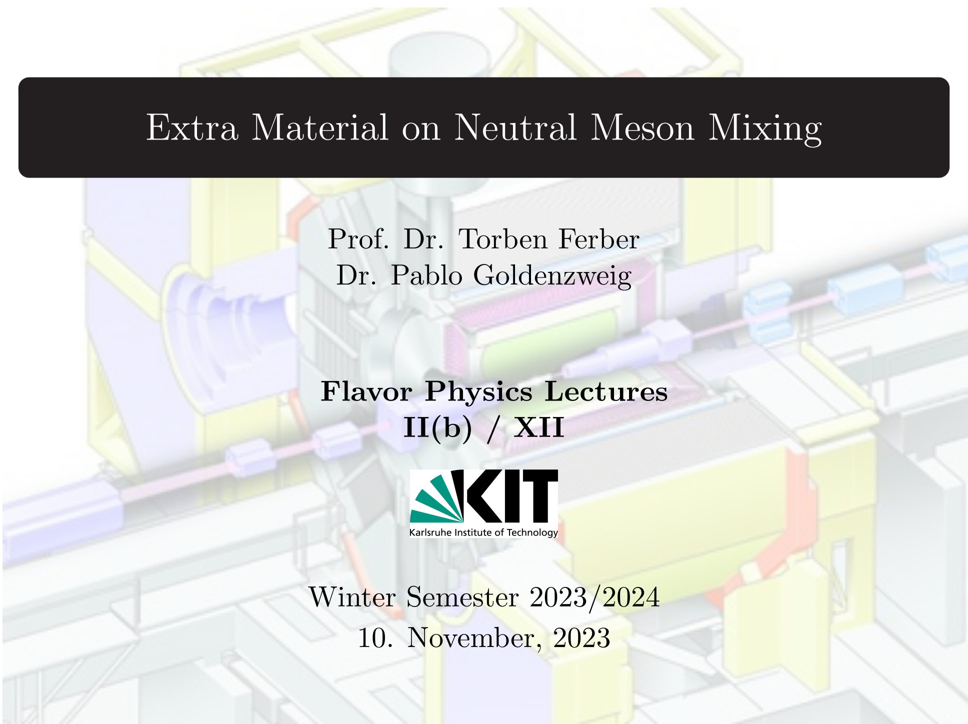

Extra Material on Neutral Meson Mixing

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Flavor Physics Lectures
II(b) / XII

Karlsruhe Institute of Technology

Winter Semester 2023/2024
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Discovery timeline

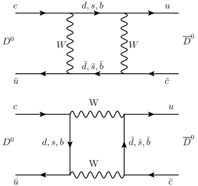
Mixing is a $|\Delta F| = 2$ process

Process in which particle changes to its anti-particle and vice versa

Possible only in flavored neutral particle–anti-particle systems

Meson M	Flavors	Particle discovered	Mixing discovered	Implication
K^0	$\bar{s}d$	1950 (Caltech)	1956 (Columbia)	m_c
B_d^0	$\bar{b}d$	1981 (CESR)	1987 (Desy)	m_t
B_s^0	$\bar{b}s$	1992 (LEP)	2006 (Fermilab)	??
D^0	$c\bar{u}$	1976 (SLAC)	2007 (KEK, SLAC)	??

Short distance contributions



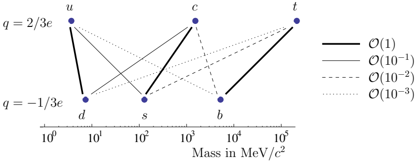
Matrix element

↪ propagators include masses of intermediate quarks

↪ couplings in vertices are given by CKM matrix

$$\boxed{\langle \overline{D}^0 | \mathcal{H}_w | D^0 \rangle \sim \sum_{i,j=d,s,b} V_{ui}^* V_{ci} V_{cj} V_{uj}^* m_i m_j}$$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$K^0 - \overline{K}^0$

$\lvert V_{us} V_{ud} \rvert m_u$	0.0006
$\lvert V_{cs} V_{cd} \rvert m_c$	**0.3**
$\lvert V_{ts} V_{td} \rvert m_t$	0.06

$D^0 - \overline{D}^0$

$\lvert V_{cd} V_{ud} \rvert m_d$	0.001
$\lvert V_{cs} V_{us} \rvert m_s$	**0.02**
$\lvert V_{cb} V_{ub} \rvert m_b$	0.0007

$B_d^0 - \overline{B}_d^0$

$\lvert V_{ub} V_{ud} \rvert m_u$	10^{-5}
$\lvert V_{cb} V_{cd} \rvert m_c$	0.01
$\lvert V_{tb} V_{td} \rvert m_t$	**1.4**

$B_s^0 - \overline{B}_s^0$

$\lvert V_{ub} V_{us} \rvert m_u$	10^{-6}
$\lvert V_{cb} V_{cs} \rvert m_c$	0.05
$\lvert V_{tb} V_{ts} \rvert m_t$	**7**

Mixing with generalized parameters

In Lecture 2 we developed the time-dependent formalism of meson mixing in the neutral Kaon system.

In Lecture 4 we will study B^0 mixing and time-dependent CPV in $B^0 \to J/\psi K_S^0$ decays.

There are 2 additional systems (which we will not study in detail) B_s mixing, and D^0 mixing.

To do so, lets introduce some generalized dimensionless mixing parameters [1] which will help us easily quantify the differences in the 4 systems.

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[1] A very nice derivation of this can be found on page 154 of *http://www.lepp.cornell.edu/~pt267/files/notes/FlavorNotes.pdf*

Mixing Phenomenology I - Time evolution

Time evolution of $M^0 - \overline{M}^0$ system

↪ solve time-dependent Schrödinger Eq.

$$\boxed{i\frac{\partial}{\partial t}\begin{pmatrix}|M^0\rangle \\ |\overline{M}^0\rangle\end{pmatrix} = \mathcal{H}_w\begin{pmatrix}|M^0\rangle \\ |\overline{M}^0\rangle\end{pmatrix}} \qquad \mathcal{H}_w = \mathbf{M} - i\frac{\mathbf{\Gamma}}{2}$$

Eigenstates of $\mathcal{H}_w$ are mass eigenstates M_1 and M_2

↪ $\neq$ flavor eigenstates M^0 and $\overline{M}^0$

$$\boxed{|M_{1,2}\rangle = p|M^0\rangle \pm q|\overline{M}^0\rangle} \qquad p^2 + q^2 = 1$$

↪ with eigenvalues $\lambda_{1,2} = m_{1,2} - i\Gamma_{1,2}/2$

Time evolution of $M^0 - \overline{M}^0$ system

$$\boxed{\begin{aligned} |M^0(t)\rangle &= \tfrac{1}{2}\left(g_+(t)|M^0\rangle + \tfrac{q}{p}g_-(t)|\overline{M}^0\rangle\right) \\ |\overline{M}^0(t)\rangle &= \tfrac{1}{2}\left(\tfrac{p}{q}g_-(t)|M^0\rangle + g_+(t)|\overline{M}^0\rangle\right) \end{aligned}} \qquad g_\pm(t) = \left(e^{-i\lambda_1 t} \pm e^{-i\lambda_2 t}\right)$$

Dimensionless mixing parameters

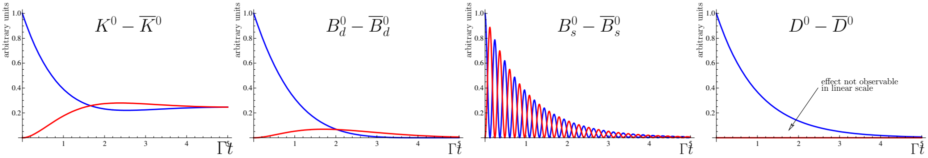
$$\boxed{\boldsymbol{x = \frac{m_1 - m_2}{\bar\Gamma}} \text{ and } \boldsymbol{y = \frac{\Gamma_1 - \Gamma_2}{2\bar\Gamma}}} \qquad \bar\Gamma = \frac{\Gamma_1 + \Gamma_2}{2} \qquad \bar m = \frac{m_1 + m_2}{2}$$

Mixing Phenomenology II - Mixing probability

probability to observe an initial M^0 as M^0 or $\overline{M}^0$ after time t

$$\mathcal{P}_{\mathrm{non-mix}}(t) = |\langle M^0(t)|M^0\rangle|^2 = \frac{1}{4}e^{-(1-y)\Gamma t}\left[1 + e^{-2y\Gamma t} + 2e^{-y\Gamma t}cos(x\Gamma t)\right]$$

$$\mathcal{P}_{\mathrm{mix}}(t) = |\langle M^0(t)|\overline{M}^0\rangle|^2 = \frac{1}{4}e^{-(1-y)\Gamma t}\left[1 + e^{-2y\Gamma t} - 2e^{-y\Gamma t}cos(x\Gamma t)\right]$$



Large sample of D^0 mesons needed to observe $D^0 - \overline{D}^0$ mixing and excellent decay time resolution to resolve oscillations in case of B_s^0 mesons

time integrated mixing rate

$$R_M = \frac{\int_0^\infty \mathcal{P}_{\mathrm{mix}}(t)dt}{\int_0^\infty \mathcal{P}_{\mathrm{non-mix}}(t)dt} = \frac{x^2+y^2}{2+x^2-y^2}$$

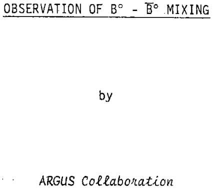
M^0	x	y	R_M
K^0	$-0.946..$	0.997..	0.994..
B_d^0	0.769	0.005..	0.23..
B_s^0	27.03	0.046..	0.997..
D^0	0.0063..	0.0075..	**10^{-4}**

1 out of 10^4 D^0 mesons oscillates before it decays

B^0 mixing – Argus 1987

ARGUS experiment @ DORIS

A Russian-German-United States-Swedish collaboration @ DOppel RIng Speicher



OBSERVATION OF B° - $\bar{B}^\circ$ MIXING

by

ARGUS Collaboration

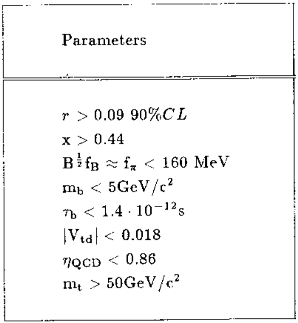
Phys.Lett.B192:245,1987

Observe B^0 mixing:

First evidence of heavy top!

$m_{\text{top}} > 50$ GeV

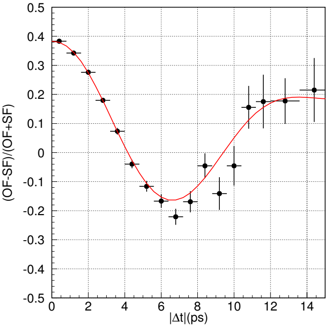
(needed to break GIM mechanism)



Parameters
$r > 0.09$ $90\% CL$
$x > 0.44$
$B^{\frac{1}{2}} f_B \approx f_\pi < 160$ MeV
$m_b < 5\text{GeV}/c^2$
$\tau_b < 1.4 \cdot 10^{-12}$s
$\lvert V_{td} \rvert < 0.018$
$\eta_{QCD} < 0.86$
$m_t > 50\text{GeV}/c^2$

B^0 mixing

Belle, PRD71, 072003 (2005)



Theoretically:

$$\frac{N_{B^0\to B^0}(t) - N_{B^0\to \overline{B}^0}(t)}{N_{B^0\to B^0}(t) + N_{B^0\to \overline{B}^0}(t)} = cos(\Delta m \Delta t)$$

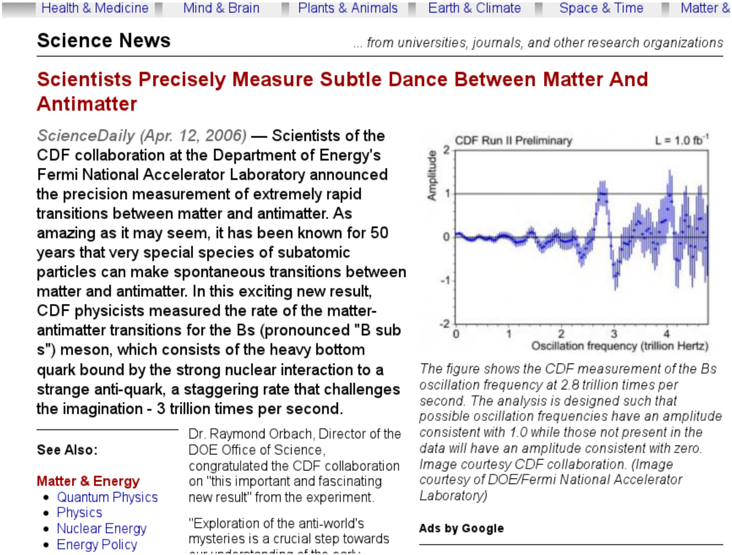
Experimentally:

You can really see this because: B^0 mixing has same time scale as decay

$\tau = 1.53$ ps

$\Delta m = 0.51\ \mathrm{ps}^{-1}$

B_s^0 Mixing: 2006



Health & Medicine | Mind & Brain | Plants & Animals | Earth & Climate | Space & Time | Matter &

Science News *... from universities, journals, and other research organizations*

Scientists Precisely Measure Subtle Dance Between Matter And Antimatter

ScienceDaily (Apr. 12, 2006) — **Scientists of the CDF collaboration at the Department of Energy's Fermi National Accelerator Laboratory announced the precision measurement of extremely rapid transitions between matter and antimatter. As amazing as it may seem, it has been known for 50 years that very special species of subatomic particles can make spontaneous transitions between matter and antimatter. In this exciting new result, CDF physicists measured the rate of the matter-antimatter transitions for the Bs (pronounced "B sub s") meson, which consists of the heavy bottom quark bound by the strong nuclear interaction to a strange anti-quark, a staggering rate that challenges the imagination - 3 trillion times per second.**

See Also:

Matter & Energy

- Quantum Physics
- Physics
- Nuclear Energy
- Energy Policy

Dr. Raymond Orbach, Director of the DOE Office of Science, congratulated the CDF collaboration on "this important and fascinating new result" from the experiment.

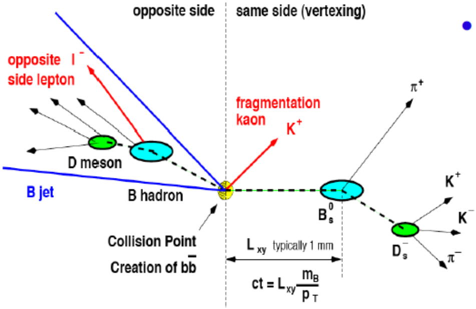
"Exploration of the anti-world's mysteries is a crucial step towards our understanding of the early

The figure shows the CDF measurement of the Bs oscillation frequency at 2.8 trillion times per second. The analysis is designed such that possible oscillation frequencies have an amplitude consistent with 1.0 while those not present in the data will have an amplitude consistent with zero. Image courtesy CDF collaboration. (Image courtesy of DOE/Fermi National Accelerator Laboratory)

Ads by Google

B_s^0 Mixing: 2006

CDF at Tevatron (USA)



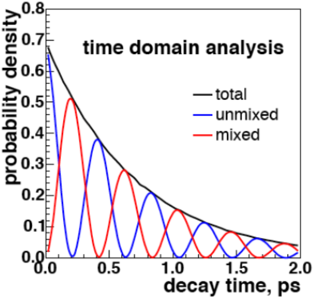
- Need:
  - B flavour at decay
    - Reconstruct decay mode
  - Flavour at production
    - Same- or opposite-side flavour tag
  - Decay proper time
    - vertexing

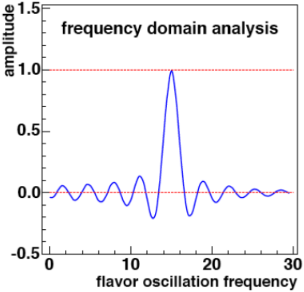
B_s^0 Mixing: 2006

Perform extraction of oscillation frequency by introducing an ad hoc amplitude coefficient *A*, then perform frequency-domain analysis

- Amplitude should reach unity in presence of signal, and be zero elsewhere

$$P(t) \sim (1 \pm \mathcal{A} D \cos \Delta m_s t)$$

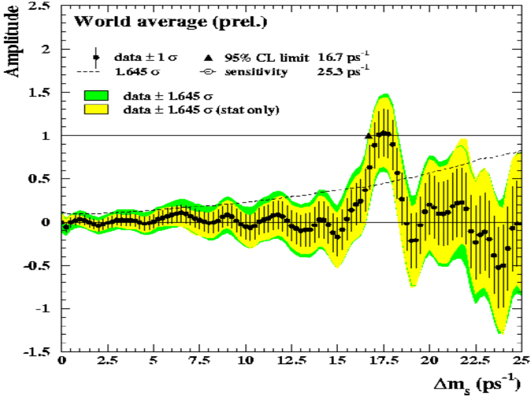




B_s^0 Mixing: 2006

CDF and D0 both reported evidence for mixing in spring 2006

- CDF: $\Delta m_s = 17.33 + 0.42 - 0.21$ (stat) +- 0.07 (syst) ps^{-1}



Reconstruction of D^0 mesons

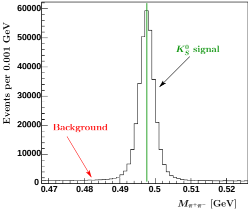
Reconstruction of D^0 mesons

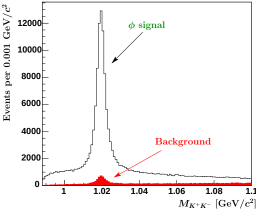
Reconstruct tracks of long lived particles $(e^-, \mu^-, \pi^\pm, K^\pm, K_L^0, p, \gamma)$:

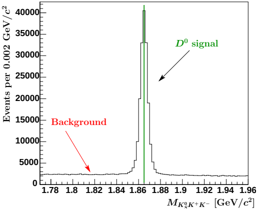
measure their momenta $\vec{p}_i$ or energies E_i

identify them (assign mass m_i)

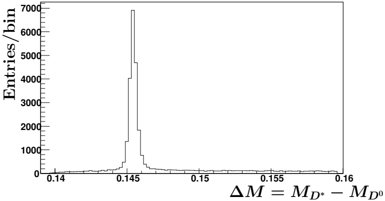
make combinations: $K_S^0 \to \pi^+\pi^-$, $\phi \to K^+K^-$, $D^0 \to \phi K_S^0$



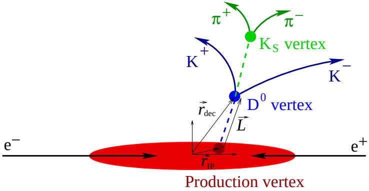




identify the flavor of D^0 meson at production: $D^{*+} \to D^0\pi_s^+$ or $D^{*-} \to \overline{D}^0\pi_s^-$



Decay time distribution

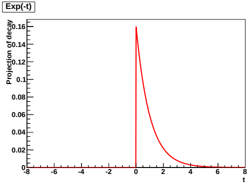


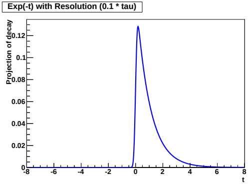
Proper decay time

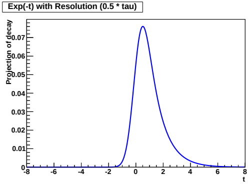
$$t = \frac{m_{D^0}}{p_{D^0}}(\vec{r}_{\mathrm{dec}} - \vec{r}_{\mathrm{IP}}) \cdot \frac{\vec{p}_{D^0}}{p_{D^0}}$$

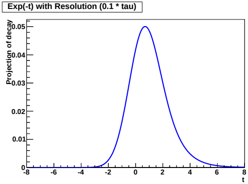
Distribution of proper decay time

$$\mathcal{F}(t) = \int Exp[-t'/\tau] \cdot R(t-t')dt'$$


Exp(-t)


Exp(-t) with Resolution (0.1 * tau)


Exp(-t) with Resolution (0.5 * tau)


Exp(-t) with Resolution (0.1 * tau)

Mixing in semileptonic decays $D^0 \to K^{(*)-}\ell^+\nu_\ell$

Mixing in semileptonic decays $D^0 \to K^{(*)-}\ell^+\nu_\ell$

U. Bitenc *et al.* [Belle Collaboration], PRD77, 112003 (2008). [$\mathcal{L}$ =492 fb^{-1}]

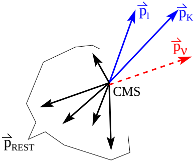
Wrong sign (WS) charge combination accessible only via mixing

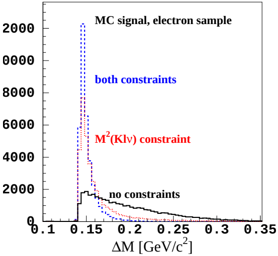
	Flavor at production		Flavor at decay	
without mixing	$D^{*+} \to D^0\pi^+$		$D^0 \to K^-\ell^+\nu_\ell$	**RS**
with mixing	$D^{*+} \to D^0\pi^+$	$D^0 - \overline{D}^0$	$\overline{D}^0 \to K^+\ell^-\nu_\ell$	**WS**

Neutrino reconstruction:

$$P_\nu = P_{\text{cms}} - P_{K\ell} - P_{\text{rest}}$$

$$M^2_{K\ell\nu} = m^2_{D^0} \ \&$$
$$(P^*_\nu)^2 = 0$$

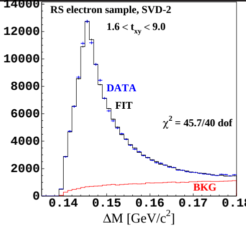




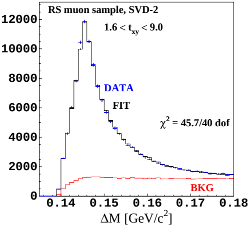
Time integrated mixing rate:

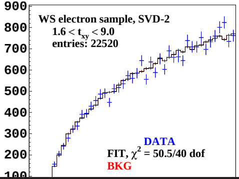
$$R_M \simeq \frac{x^2 + y^2}{2} = \frac{N_{\text{WS}}}{N_{\text{RS}}}$$

Mixing in semileptonic decays $D^0 \to K^{(*)-}\ell^+\nu_\ell$



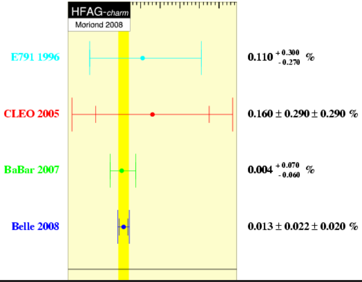
RS events





We observed no WS charge combinations.

$$
\begin{aligned}
\boldsymbol{R_M} &= \mathbf{(1.3 \pm 2.2 \pm 2.0) \times 10^{-4}} \\
\boldsymbol{R_M} &< \mathbf{6.1 \times 10^{-4} @\ 90\%\ C.L.}
\end{aligned}
$$



Mixing and CPV in $D^0 \to K^+K^-,\ \pi^+\pi^-$ *decays*
&
first evidence of $D^0 - \overline{D}^0$ *mixing*

Mixing and CPV in $D^0 \to K^+K^-,\ \pi^+\pi^-$ decays

M. Staric *et al.* [Belle Collaboration], PRL98, 211803 (2007). [$\mathcal{L}$ =540 fb^{-1}]

Measurement of lifetime difference between $D^0 \to K^-\pi^+$ (CP-mixed) and $D^0 \to K^+K^-,\ \pi^+\pi^-$ (CP-odd) decays

Conserved CP symmetry

$$\hookrightarrow \quad \frac{dN(D^0 \to f_{CP})}{dt} \propto e^{-(1+y_{CP})\Gamma t}$$

$$\hookrightarrow \quad \frac{dN(D^0 \to f)}{dt} \propto e^{-\Gamma t}$$

$$\boxed{\boldsymbol{y_{CP} = \frac{\tau(f)}{\tau(f_{CP})} - 1}}$$

$$y_{CP} = y$$

CP Violation

$\hookrightarrow \tau(D^0) \neq \tau(\overline{D}^0)$

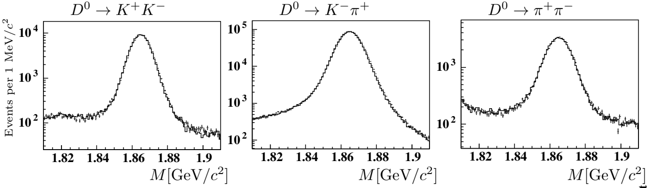
$\hookrightarrow$ CP violating parameter

$$\boxed{\boldsymbol{A_\Gamma = \frac{\tau(\overline{D}^0 \to f_{CP}) - \tau(D^0 \to f_{CP})}{\tau(\overline{D}^0 \to f_{CP}) + \tau(D^0 \to f_{CP})}}}$$

$$y_{CP} = y\cos\phi - \frac{1}{2}A_M x \sin\phi \qquad A_\Gamma = \frac{1}{2}A_M y \cos\phi - x\sin\phi$$

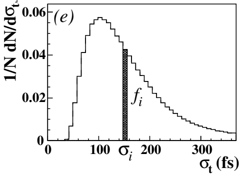
Mixing and CPV in $D^0 \to K^+K^-$, $\pi^+\pi^-$ decays

Signal yield



$D^0 \to$	Sig. yield	Purity
$K^-\pi^+$	1.22M	99%
K^+K^-	111k	98%
$\pi^+\pi^-$	49k	92%

Fit to the proper decay time distribution



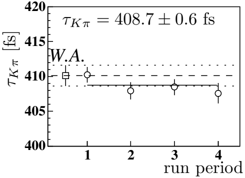
Precise knowledge of resolution function needed

$$R(t) = \frac{t^{\mathrm{rec}} - t^{\mathrm{gen}}}{\sigma_t}$$

$$\boxed{\frac{dN}{dt} \propto \int e^{-t'/\tau} \cdot R(t-t')dt' + B(t)}$$

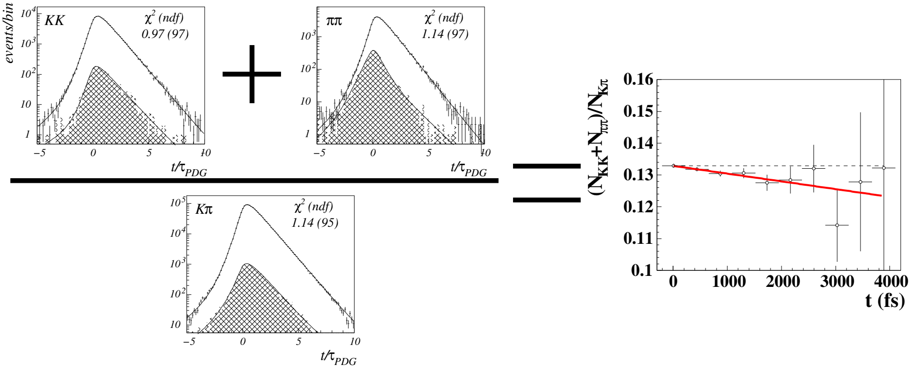
$$\Downarrow$$

$$R(t-t') = \sum_i^N f_i \sum_{k=1}^{3} w_k G(t-t', \sigma_{ik}, t_0)$$



Mixing and CPV in $D^0 \to K^+K^-,\ \pi^+\pi^-$ decays

First evidence for $D^0 - \overline{D}^0$ mixing!



$\boxed{y_{CP} = (1.31 \pm 0.32 \pm 0.25)\%}$

$\boxed{A_\Gamma = (0.01 \pm 0.30 \pm 0.15)\%}$

No evidence for CPV.

Time integrated CPV in $D^0 \to K^+K^-, \pi^+\pi^-$ decays

M. Staric *et al.* [Belle Collaboration], PLB670, 190 (2008). [$\mathcal{L}$ =540 fb^{-1}]

Measure CP-Violating asymmetry

$$\hookrightarrow \quad A_{CP} = \frac{\mathcal{B}(D^0\to f)-\mathcal{B}(\bar{D}^0\to \bar{f})}{\mathcal{B}(D^0\to f)+\mathcal{B}(\bar{D}^0\to \bar{f})}, \quad f = K^+K^- \text{ or } \pi^+\pi^-$$

$$\hookrightarrow \quad A_{CP} = a_f^d + a_f^m + a_f^i \text{ (direct + indirect CPV contributions)}$$

Measured asymmetry:

$$N_{D^0}^{\text{reco}} = N_{D^{*+}}^{\text{prod}} \cdot \mathcal{B}(D^{*+} \to D^0\pi^+) \cdot \mathcal{B}(D^0 \to f) \cdot \varepsilon_f \cdot \varepsilon_{\pi_S^+}$$

Contributions to measured asymmetry:

$$\hookrightarrow \boxed{\boldsymbol{A^{\text{meas}} = A_{FB}^{D^{*+}} + A_{CP}^{f} + A_{\epsilon}^{f} + A_{\epsilon}^{\pi_S}}}$$

$\boldsymbol{A_{FB}^{*+}}$: production asymmetry (anti-symmetric function of $cos\theta^*$)

$\boldsymbol{A_{CP}^{f}}$: CP asymmetry

$\boldsymbol{A_{\epsilon}^{f}}$: $D^0 \to f$ efficiency asymmetry (zero for K^+K^- or $\pi^+\pi^-$)

$\boldsymbol{A_{\epsilon}^{\pi_S}}$: asymmetry in π_S^+/π_S^- reconstruction efficiency (measured and corrected for using tagged and untagged $D^0 \to K^-\pi^+$ decays - $f(p_{\pi_S}, cos\theta_{\pi_S})$) BaBar, PRL100, 061803 (2008)

$$A^{\text{tag}} = A_{FB} + A_{CP}^{K\pi} + A_{\epsilon}^{K\pi} + A_{\epsilon}^{\pi_S}$$

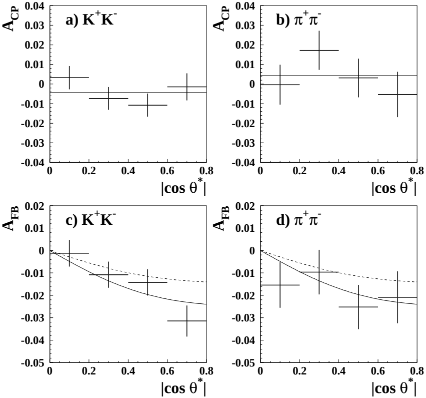
$$A^{\text{untag}} = A_{FB} + A_{CP}^{K\pi} + A_{\epsilon}^{K\pi}$$

Time integrated CPV in $D^0 \to K^+K^-, \pi^+\pi^-$ decays

$$A^{\text{reco}}_{\text{corr}}(cos\theta^*) = \frac{N(D^0) - N(\bar{D}^0)}{N(D^0) + N(\bar{D}^0)} = A^{D^{*+}}_{FB} + A^{f}_{CP}$$

$$A_{CP} = \frac{A^{\text{reco}}_{\text{corr}}(cos\theta^*) + A^{\text{reco}}_{\text{corr}}(-cos\theta^*)}{2}$$

$$A_{FB} = \frac{A^{\text{reco}}_{\text{corr}}(cos\theta^*) - A^{\text{reco}}_{\text{corr}}(-cos\theta^*)}{2}$$



Belle

$$A^{KK}_{CP} = (-0.43 \pm 0.30(\text{stat}) \pm 0.11(\text{syst}))\%$$
$$A^{\pi\pi}_{CP} = (\ \ 0.43 \pm 0.52(\text{stat}) \pm 0.12(\text{syst}))\%$$

BaBar

$$A^{KK}_{CP} = (\ \ 0.00 \pm 0.34(\text{stat}) \pm 0.13(\text{syst}))\%$$
$$A^{\pi\pi}_{CP} = (-0.24 \pm 0.52(\text{stat}) \pm 0.22(\text{syst}))\%$$

BaBar, PRL100, 061803 (2008)

Consistent with no CPV!

SM expectations: $A_{CP} \sim \mathcal{O}(10^{-5} - 10^{-4})$

PRD51,3478(1995); RNCIB,26N7,11(2003)

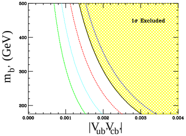
Impact – Constraints on NP models from mixing

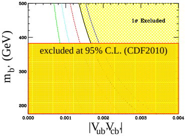
E. Golowich *et al.*, PRD76,095009

Constraints on new physics models from $D^0 - \overline{D}^0$ complementary to those obtained in B and K sector
$\hookrightarrow$ FCNC transitions with *down-like* quarks in charm sector (unique feature)

21 NP models considered → 17 with useful constraints

Example: quark b' from 4th generation





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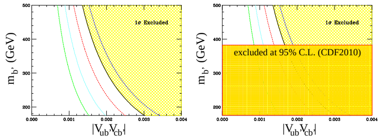

$|V_{ub'}V_{cb'}| < 0.002$
order of magnitude stronger constraint
as from CKM unitarity

Providing complementary and
improved constraints!