

# $CP$ violation in the interference between mixing and decay, and an introduction to $B$ -factories

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**Flavor Physics Lectures**  
**IV / XII**



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# Reading material and references

**Lecture material based on several textbooks and online lectures/notes.**

**Credits for material and figures include:**

## Literature

- Perkins, Donald H. (2000), *Introduction to High Energy Physics*.
- Griffiths, David J. (2nd edition), *Introduction to Elementary Particles*.
- Stone, Sheldon (2nd edition), *B decays*.

## Online Resources

- Belle/BaBar Collaborations, *The Physics of the B-Factories*.  
<http://arxiv.org/abs/1406.6311>
- Bona, Marcella (University of London), *CP Violation Lecture Notes*,  
<http://pprc.qmul.ac.uk/bona/ulpg/cpv/>
- Richman, Jeremy D. (UCSB), *Heavy Quark Physics and CP Violation*.  
[https://courses.physics.ucsd.edu/2010/Winter/physics222/references/driver\\_houches12.pdf](https://courses.physics.ucsd.edu/2010/Winter/physics222/references/driver_houches12.pdf)
- Thomson, Mark (Cambridge University), *Particle Physics Lecture Handouts*,  
<http://www.hep.phy.cam.ac.uk/thomson/partIIIparticles/welcome.html>
- Grossman, Yuval (Cornell University), *Just a Taste. Lectures on Flavor Physics*,  
<http://www.lepp.cornell.edu/pt267/files/notes/FlavorNotes.pdf>
- Kooijman, P. & Tuning, N., *CP Violation*,  
<https://www.nikhef.nl/h71/Lectures/2015/ppII-cpviolation-29012015.pdf>

# Recap & outline

So far, we:

- Developed the framework to describe particle anti-particle oscillations, both with and without  $CP$  violation. This was done for the neutral Kaon system, but is applicable to all neutral particle systems.
- Saw that  $CP$ -violation in mixing leads to  $\Gamma(K_{t=0}^0 \rightarrow \bar{K}^0) \neq \Gamma(\bar{K}_{t=0}^0 \rightarrow K^0)$
- Studied direct  $CP$  violation in the  $B$  meson system, and looked at the example of  $B \rightarrow K\pi$  decays which is currently an open question, referred to as the “ $K\pi$   $CP$ -puzzle.”

Today, we'll:

- Study  $CPV$  in  $B^0$ - $\bar{B}^0$  oscillations.
- Finally, we'll introduce a third and the final type of  $CP$  violation,  **$CP$  violation in interference between mixing and decay** and study the “golden mode”  $B^0 \rightarrow J/\psi K_S^0$ .
- We'll also introduce  $B$  factories and experimental techniques along the way.

# Recap: $K^0 - \bar{K}^0$ Oscillations with $CP$ violation

Recall our derivation of mixing in the kaon system including  $CP$  violation

(Lecture II, S29):

- Writing our  $|K_S\rangle$  and  $|K_L\rangle$  states (inc.  $CPV$ ) in terms of the strong eigenstates  $|K^0\rangle$  and  $|\bar{K}^0\rangle$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} [|K_1\rangle + \varepsilon|K_2\rangle] = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\varepsilon|^2}} [(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\bar{K}^0\rangle]$$

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} [|K_2\rangle + \varepsilon|K_1\rangle] = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\varepsilon|^2}} [(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle]$$

- We know from before that the states  $|\Psi(t)\rangle$  propagate as  $|K_S\rangle$  and  $|K_L\rangle$ , i.e., independent particles with definite masses and lifetimes.

Invert these expressions to switch to the  $|K_S\rangle$  and  $|K_L\rangle$  basis:

$$|K^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1+\varepsilon} [|K_L\rangle + |K_S\rangle] \quad |\bar{K}^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1-\varepsilon} [|K_L\rangle - |K_S\rangle]$$

- If we add on the time dependence  $\theta_S(t)$  and  $\theta_L(t)$ , we see that states which were initially produced as  $K^0$ , or  $\bar{K}^0$ , evolve with time as

$$|\Psi(t)\rangle_{K^0_{t=0}} = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1+\varepsilon} [\theta_L(t)|K_L\rangle + \theta_S(t)|K_S\rangle] \quad (\text{K8})$$

$$|\Psi(t)\rangle_{\bar{K}^0_{t=0}} = \sqrt{\frac{1+|\varepsilon|^2}{2}} \frac{1}{1-\varepsilon} [\theta_L(t)|K_L\rangle - \theta_S(t)|K_S\rangle] \quad (\text{K9})$$

## Recap: $K^0 - \bar{K}^0$ Oscillations with $CP$ violation

If we want to study  $K^0 - \bar{K}^0$  oscillations, we need to switch to the  $K^0 \bar{K}^0$  basis

- K8 and K9 become.

$$|\Psi(t)\rangle_{K^0_{t=0}} = \frac{1}{2} \left[ (\theta_L(t) + \theta_S(t)) |K^0\rangle + (\theta_L(t) - \theta_S(t)) \left( \frac{1-\varepsilon}{1+\varepsilon} \right) |\bar{K}^0\rangle \right]$$

$$|\Psi(t)\rangle_{\bar{K}^0_{t=0}} = \frac{1}{2} \left( \frac{1+\varepsilon}{1-\varepsilon} \right) [(\theta_L(t) - \theta_S(t)) |K^0\rangle + (\theta_L(t) + \theta_S(t)) |\bar{K}^0\rangle]$$

Now lets change to the  $B$ -meson system. Recall we stated that the time-dependent formalism of mixing we derived for the Kaon system applies to all neutral particle systems undergoing oscillation.

- Replace  $K$  with  $B$ , and for simplicity, lets work in terms of  $\frac{q}{p}$  (recall,  $\frac{q}{p} = \frac{1-\varepsilon}{1+\varepsilon}$ ).
- Recall that in the Kaon system, the  $|K_{\pm}\rangle$  eigenstates we obtained by solving the time-dependent shrodinger equation had significantly different lifetimes, which motivated us to name them “short” and “long” lifetime states  $|K_S\rangle$  and  $|K_L\rangle$ .

For the  $B$  meson system, the lifetimes are almost identical. Instead the eigenstates have significantly different masses, which motivates us to rename them as “heavy” and “light.”

→ Denote the  $K_S$  eigenstate of the Kaon system as the “Heavy” eigenstate of the  $B$  system ( $B_H$ ).

→ Denote the  $K_L$  eigenstate of the Kaon system as the “Light” eigenstate of the  $B$  system ( $B_L$ ).

- Write the time-dependence explicitly, switching the subscripts  $S \rightarrow H$  for the heavy eigenstate. Recall that for our derivation in the kaon system we had abbreviated this as follows:

$$\theta_S(t) = e^{-\left(im_S + \frac{\Gamma_S}{2}\right)t}, \quad \theta_L(t) = e^{-\left(im_L + \frac{\Gamma_L}{2}\right)t}$$

# B meson system

The time dependence of state  $|\Psi(t)\rangle$  which was a  $B^0$  at  $t = 0$  is given by:

$$|\Psi(t)\rangle_{B^0_{t=0}} = |B^0(t)\rangle = e^{-iMt} e^{-\frac{\Gamma}{2}t} \left[ \cos\left(\frac{\Delta Mt}{2}\right) |B^0\rangle + i \sin\left(\frac{\Delta Mt}{2}\right) \frac{q}{p} |\bar{B}^0\rangle \right]$$

and for a state which was a  $\bar{B}^0$  at  $t = 0$  is given by:

$$|\Psi(t)\rangle_{\bar{B}^0_{t=0}} = |\bar{B}^0(t)\rangle = e^{-iMt} e^{-\frac{\Gamma}{2}t} \left[ \cos\left(\frac{\Delta Mt}{2}\right) |\bar{B}^0\rangle + i \sin\left(\frac{\Delta Mt}{2}\right) \frac{p}{q} |B^0\rangle \right]$$

where we've abbreviated:

$$M = \frac{1}{2}(M_L + M_H), \quad \Delta M = M_H - M_L, \quad \Gamma = \Gamma_L = \Gamma_H$$

since the difference in lifetimes between the  $B_L$  and  $B_H$  is  $\Delta\Gamma/\Gamma \ll 1$ .

**Note:** Here we've introduced  $|B^0(t)\rangle$  and  $|\bar{B}^0(t)\rangle$  to denote the states  $|\Psi(t)\rangle_{B^0_{t=0}}$  and  $|\Psi(t)\rangle_{\bar{B}^0_{t=0}}$  for simplicity only.

Sometimes these are also called  $|B^0_{\text{phys}}(t)\rangle$  and  $|\bar{B}^0_{\text{phys}}(t)\rangle$  to remind readers what the initial state is at  $t = 0$  (which is NOT what the state is at time  $t$  since it's a superposition of  $|B^0\rangle$  &  $|\bar{B}^0\rangle$ !).

# Time-evolution of $B$ decays to a $CP$ eigenstate

Now let these states decay into a common final state  $|f_{CP}\rangle$  with eigenvalue  $\eta_{CP}(f) = \pm 1$ .

The amplitude for  $B^0(t) \rightarrow f_{CP}$  is given by:

$$\langle f_{CP}|H|B^0(t)\rangle = e^{-iMt}e^{-\frac{\Gamma}{2}t} \left[ \cos\left(\frac{\Delta Mt}{2}\right) \langle f_{CP}|H|B^0\rangle + i \sin\left(\frac{\Delta Mt}{2}\right) \frac{q}{p} \langle f_{CP}|H|\bar{B}^0\rangle \right]$$

while that for a  $\bar{B}^0(t) \rightarrow f_{CP}$  is

$$\langle f_{CP}|H|\bar{B}^0(t)\rangle = e^{-iMt}e^{-\frac{\Gamma}{2}t} \left[ \cos\left(\frac{\Delta Mt}{2}\right) \langle f_{CP}|H|\bar{B}^0\rangle + i \sin\left(\frac{\Delta Mt}{2}\right) \frac{p}{q} \langle f_{CP}|H|B^0\rangle \right]$$

Here we can explicitly see the 2 amplitudes whose interference can produce a  $CP$  asymmetry.

Lets define a quantity  $\lambda_f$  to express the ratio of amplitudes

$$\lambda_f \equiv \frac{q \bar{A}_f}{p A_f} = \frac{q \langle f_{CP}|H|\bar{B}^0\rangle}{p \langle f_{CP}|H|B^0\rangle} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \frac{\langle f_{CP}|H|\bar{B}^0\rangle}{\langle f_{CP}|H|B^0\rangle}}$$

(not to be confused with the Wolfenstein parameter  $\lambda$ ).

# Time-evolution of $B$ decays to a $CP$ eigenstate

To see how this  $\lambda_f$  comes into play, factor out  $\langle f_{CP}|H|B^0\rangle$ :

$$\langle f_{CP}|H|B^0(t)\rangle = e^{-iMt} e^{-\frac{\Gamma}{2}t} \langle f_{CP}|H|B^0\rangle \left[ \cos\left(\frac{\Delta Mt}{2}\right) + i \sin\left(\frac{\Delta Mt}{2}\right) \frac{q}{p} \frac{\langle f_{CP}|H|\bar{B}^0\rangle}{\langle f_{CP}|H|B^0\rangle} \right]$$

$$\langle f_{CP}|H|\bar{B}^0(t)\rangle = e^{-iMt} e^{-\frac{\Gamma}{2}t} \langle f_{CP}|H|B^0\rangle \left[ \cos\left(\frac{\Delta Mt}{2}\right) \frac{\langle f_{CP}|H|\bar{B}^0\rangle}{\langle f_{CP}|H|B^0\rangle} + i \sin\left(\frac{\Delta Mt}{2}\right) \frac{p}{q} \right]$$

If the initial state at  $t = 0$  is a  $|B^0\rangle$ , the *probability* to produce  $|f_{CP}\rangle$  at time  $t$  is thus

$$\begin{aligned} & |\langle f_{CP}|H|B^0(t)\rangle|^2 = \\ & e^{-\Gamma t} |\langle f_{CP}|H|B^0\rangle|^2 \cdot \left[ \frac{1}{2}(1 + |\lambda|^2) + \frac{1}{2}(1 - |\lambda|^2) \cos(\Delta Mt) - \text{Im}\lambda \cdot \sin(\Delta Mt) \right] \end{aligned}$$

Likewise for an initial state of  $|\bar{B}^0\rangle$  at  $t = 0$ , the *probability* to produce  $|f_{CP}\rangle$  at time  $t$  is

$$\begin{aligned} & |\langle f_{CP}|H|\bar{B}^0(t)\rangle|^2 = \\ & e^{-\Gamma t} |\langle f_{CP}|H|B^0\rangle|^2 \left| \frac{p}{q} \right|^2 \cdot \left[ \frac{1}{2}(1 + |\lambda|^2) + \frac{1}{2}(1 - |\lambda|^2) \cos(\Delta Mt) + \text{Im}\lambda \cdot \sin(\Delta Mt) \right] \end{aligned}$$

*Notice how similar these expressions are. The only differences are the  $\pm$  sign of the last term and the presence of  $\left| \frac{p}{q} \right|^2$  in the second equation.*

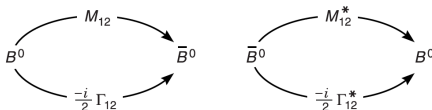


# What can we say about $\frac{q}{p}$ in the $B$ meson system?

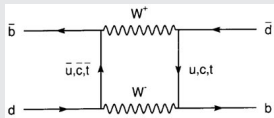
- Recall the definition from **Lecture II**:

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

- The interference occurs between the set of amplitudes with **short-distance, virtual** (aka **off-shell**) intermediate states ( $M_{12}$ ) and **long-distance, on-shell intermediate states** ( $\Gamma_{12}$ ).



$M_{12}$ : The off-shell intermediate states, e.g.,  $t\bar{t}$ , arise from the box-diagrams



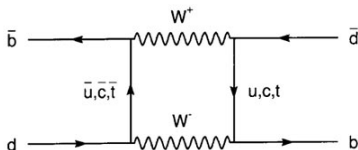
$\Gamma_{12}$ : While a  $B$  can also evolve to a  $\bar{B}^0$  through on-shell intermediate states such as  $K^+ K^-$ , with mass  $M_{K^+ K^-} = M_{B^0}$

# The $B$ meson mixing phase $\phi_M(B^0)$

- $\Gamma_{12}$  is very small for  $B^0$ - $\bar{B}^0$  mixing, so we can approximate

$$\frac{q}{p} \approx \sqrt{\frac{M_{12}^*}{M_{12}}}$$

- The short-distance, off-shell contribution from  $M_{12}$  depends on the size of the CKM-elements at the corners of the box-diagram, and on the mass of the particles in the box.



- In the SM, these amplitudes are completely dominated by the box diagrams with  $t\bar{t}$ , giving us:

$$M_{12} \propto (V_{tb}V_{td}^*)^2 e^{-2i\theta_{CP}(B)}$$

$$M_{12}^* \propto (V_{tb}^*V_{td})^2 e^{+2i\theta_{CP}(B)}$$

- Thus we have  $\frac{q}{p} \approx \sqrt{\frac{M_{12}^*}{M_{12}}} = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} e^{2i\theta_{CP}(B)} = e^{2i\phi_M(B^0)} e^{2i\theta_{CP}(B)}$

$$\Rightarrow \left| \frac{q}{p} \right| = 1 \quad \text{Very important result!}$$

## Time-dependent $CP$ asymmetry

Using this result, we can write our time-dependent  $CP$  asymmetry using the rates we derived on S7, which includes the effects of interference between mixing and decay, as

$$\begin{aligned} \mathcal{A}_{CP}(t) &= \frac{|\langle f_{CP} | H | B^0(t) \rangle|^2 - |\langle f_{CP} | H | \bar{B}^0(t) \rangle|^2}{|\langle f_{CP} | H | B^0(t) \rangle|^2 + |\langle f_{CP} | H | \bar{B}^0(t) \rangle|^2} \\ &= \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \cos(\Delta Mt) - \frac{2\text{Im}\lambda}{1 + |\lambda|^2} \sin(\Delta Mt) \\ &= A_f \cos(\Delta Mt) - S_f \sin(\Delta Mt) \end{aligned}$$

$$A_f \equiv \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \quad S_f \equiv \frac{2\text{Im}\lambda}{1 + |\lambda|^2}$$

*sometimes called  $A_{CP}$  and  $S_{CP}$ , or just  $A$  and  $S$*

# How do we calculate $\lambda_f$ ?

**For a given final state  $f$ , the magnitude and phase of  $\lambda_f$  fully describe the decay and oscillation of the  $B^0$ - and the  $\bar{B}^0$ -meson.**

Our job is to search for decays which we can use to extract and *measure* a meaningful asymmetry.

On slide 10 we looked at  $\frac{q}{p}$  and defined the  $B$  mixing angle  $\phi_M$ .

Now lets study the decays of  $B^0 \rightarrow f_{CP}$  and  $\bar{B}^0 \rightarrow f_{CP}$ .

Recall from our discussion of  $CPV$  in decay, that the total amplitude for the  $B^0 \rightarrow f_{CP}$  decay is written as:

$$A \equiv A(B^0 \rightarrow f_{CP}) = \langle f_{CP} | H | B^0 \rangle = \sum_j A_j = \sum_j a_j e^{i\phi_j} = \sum_j |a_j| e^{i\delta_j} e^{i\phi_j}$$

where  $\phi_j$  is the weak phase which changes sign under  $CP$ , and  $\delta_j$  is the  $CP$ -conserving strong phase.

# How do we calculate $\lambda_f$ ?

Now let's look at a special case:

*When the direct decay to  $f_{CP}$  is dominated by a single amplitude.*

The  $B^0 \rightarrow f_{CP}$  simplifies to one term:

$$\langle f_{CP} | H | B^0 \rangle = |a| e^{i(\delta + \phi)}$$

For the  $\bar{B}^0 \rightarrow f_{CP}$  decay, we have

$$\langle f_{CP} | H | \bar{B}^0 \rangle = \eta_{CP}(f) e^{-2i\theta_{CP}(B)} |a| e^{i(\delta - \phi)}$$

**What are these extra terms?**

- $e^{-2i\theta_{CP}(B)}$  should ring a bell. It's the *intrinsic CP* phase factor associated with  $B$ . (Recall **Lecture III S5** when we derived the condition for direct *CPV*).
- $\eta_{CP}(f)$  is the *CP* eigenvalue of the final state.

Recall how we determined this in **Lecture II S6-7**, for Kaon decays to  $2\pi, 3\pi$ :

the 2 pion system has eigenvalue  $\eta_{CP}(f) = +1$  [ $\mathcal{CP}|\pi^0\pi^0\rangle = +|\pi^0\pi^0\rangle$ ],

while the 3 pion system has  $\eta_{CP}(f) = -1$  [ $\mathcal{CP}|\pi^0\pi^0\pi^0\rangle = -|\pi^0\pi^0\pi^0\rangle$ ]

# How do we calculate $\lambda_f$ ?

## Lets put the pieces together

Start with our definition:

$$\lambda_f \equiv \frac{q}{p} \frac{\langle f_{CP} | H | \bar{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle} = \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}} \frac{\langle f_{CP} | H | \bar{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle}$$

Substitute in our result from  $B^0 - \bar{B}^0$  oscillation

$$\lambda_f = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} e^{2i\theta_{CP}(B^0)} \frac{\langle f_{CP} | H | \bar{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle}$$

recalling how we defined the mixing phase  $\phi_M(B^0)$

$$\lambda_f = e^{2i\phi_M(B^0)} e^{2i\theta_{CP}(B^0)} \frac{\langle f_{CP} | H | \bar{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle}$$

and including our result from the direct decays  $B^0 \rightarrow f_{CP}$  &  $\bar{B}^0 \rightarrow f_{CP}$

$$\lambda_f = e^{2i\phi_M(B^0)} e^{2i\theta_{CP}(B^0)} \frac{\eta_{CP}(f) e^{-2i\theta_{CP}(B)} |a| e^{i(\delta - \phi)}}{|a| e^{i(\delta + \phi)}}$$

We arrive at the final expression:

$$\lambda_f = \eta_{CP}(f) e^{2i(\phi_M(B^0) - \phi)}$$

# $\mathcal{A}_{CP}(t)$ for a single weak phase in the decay amplitude

What can we obtain from this result?

$$\lambda_f = \eta_{CP}(f) e^{2i(\phi_M(B^0) - \phi)}$$

Immediately we see that  $|\lambda_f| = 1$

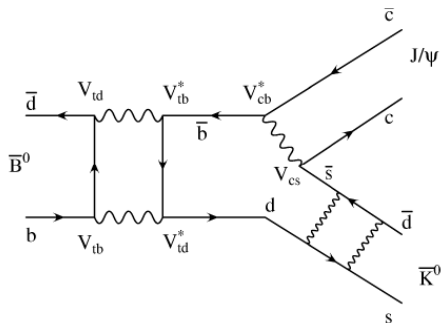
How can we take advantage of this?

It simplifies our  $\mathcal{A}_{CP}(t)$  considerably

$$\begin{aligned}\mathcal{A}_{CP}(t) &= \frac{|\langle f_{CP} | H | B^0(t) \rangle|^2 - |\langle f_{CP} | H | \bar{B}^0(t) \rangle|^2}{|\langle f_{CP} | H | B^0(t) \rangle|^2 + |\langle f_{CP} | H | \bar{B}^0(t) \rangle|^2} \\ &= \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \cos(\Delta M t) - \frac{2 \operatorname{Im} \lambda}{1 + |\lambda|^2} \sin(\Delta M t) \\ &= -\operatorname{Im} \lambda \sin(\Delta M t)\end{aligned}$$

$\Rightarrow$  Now our task is to find a decay where we can exploit this nice result

# The golden mode: $B^0 \rightarrow J/\psi K_S^0$



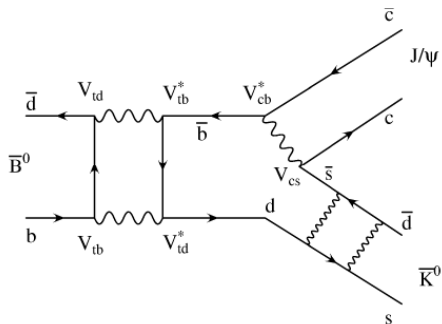
Two important criteria:

- The  $J/\psi K_S^0$  is a  $CP$  eigenstate accessible to **both**  $B^0$  and  $\bar{B}^0$ .
- This is the only major diagram contributing to this decay, so there is only one weak phase  $\phi$ .

$\Rightarrow$  now lets study this diagram in detail



# The golden mode: $B^0 \rightarrow J/\psi K_S^0$



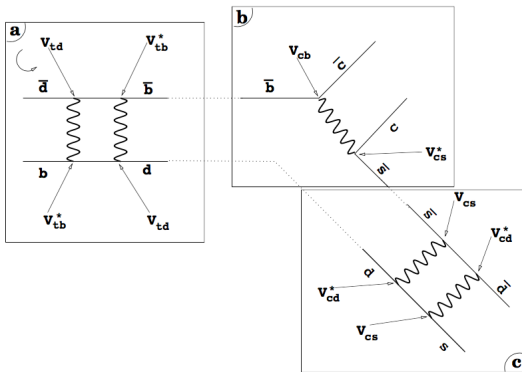
$$\lambda_{J/\psi K_S^0} = \left(\frac{q}{p}\right)_{B^0} \left( \eta_{J/\psi K_S^0} \frac{\bar{A}_{J/\psi K_S^0}}{A_{J/\psi K_S^0}} \right) = - \left(\frac{q}{p}\right)_{B^0} \left( \frac{\bar{A}_{J/\psi \bar{K}^0}}{A_{J/\psi K^0}} \right) \left(\frac{p}{q}\right)_K$$

**Do you see why we need the  $K^0$ - $\bar{K}^0$  mixing?**

$B^0 \rightarrow J/\psi K^0$  but  $\bar{B}^0 \rightarrow J/\psi \bar{K}^0$ , so the  $K^0$ 's must mix in order to reach the same final state  $f$  for the  $B^0$  and  $\bar{B}^0$  decay (recall  $|K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$ ).

**Do you see why the  $CP$  eigenvalue of  $J/\psi K_S^0$  is -1?**

# The golden mode: $B^0 \rightarrow J/\psi K_S^0$



## In terms of the CKM matrix elements

(a)  $B^0$ - $\bar{B}^0$  mixing:

$$\begin{pmatrix} q \\ p \end{pmatrix}_{B^0} = \frac{V_{tb}^* V_{td}}{V_{td} V_{tb}^*}$$

(b)  $b \rightarrow c$  decay

$$\begin{pmatrix} \bar{A} \\ A \end{pmatrix} = \frac{V_{cb} V_{cs}^*}{V_{cs}^* V_{cb}}$$

(c)  $K^0$ - $\bar{K}^0$  mixing:

$$\begin{pmatrix} p \\ q \end{pmatrix}_K = \frac{V_{cs} V_{cd}^*}{V_{cd}^* V_{cs}}$$

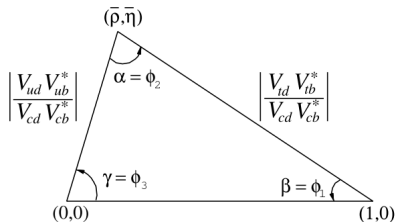
# The golden mode: $B^0 \rightarrow J/\psi K_S^0$

## Lets put the pieces together

$$\lambda_{J/\psi K_S^0} = (-1) \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \cdot \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \cdot \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} = (-1) \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \cdot \frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}}$$

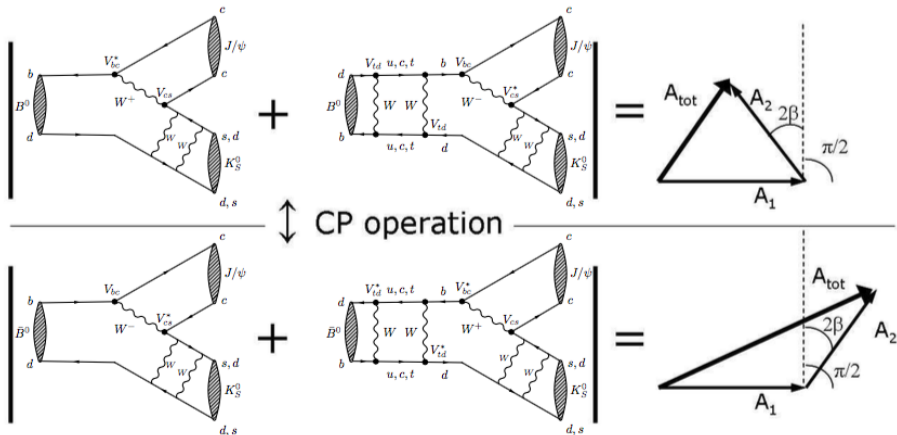
$$\text{Im} \lambda_{J/\psi K_S^0} = -\sin \left\{ \arg \left( \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}} \right) \right\} = -\sin \left\{ 2 \arg \left( \frac{V_{cb} V_{cd}^*}{V_{tb} V_{td}^*} \right) \right\} \equiv \sin(2\beta)$$

$$\mathcal{A}_{CP}(t) = -\sin(2\beta) \sin(\Delta M t)$$



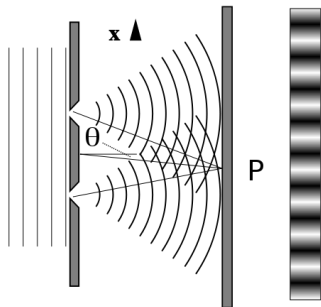
# How can we visualize the total amplitude?

Adding two amplitudes results in a  $A_{tot}$  with a different magnitude under  $CP$ .



# Recall your QM basics

The difference in decay rates arises from a different interference term for the matter vs. antimatter process. Analogy to double-slit experiment:



Classical double-slit experiment

Relative phase variation due to different path lengths leads to interference pattern in space.

## $B$ meson system $\Rightarrow$ *Experiment*

### **Observing $CP$ violation with $B$ -mesons is much more difficult than with Kaons**

- $B_L$  and  $B_H$  have essentially the same lifetimes. No way to get a beam of  $B_L$  and look for “forbidden” decay modes.
- It is much harder to produce large quantities of  $B$ -mesons than Kaons.

$\Rightarrow$  *Kobayashi-Maskawa mechanism quantitatively predicts large  $CP$  violating asymmetries in the decays of the  $B$  meson system. Worthwhile to try to measure!*

# Time evolution of $B^0\bar{B}^0$ pairs at $B$ -factories

Production mechanism:

$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^0\bar{B}^0$$

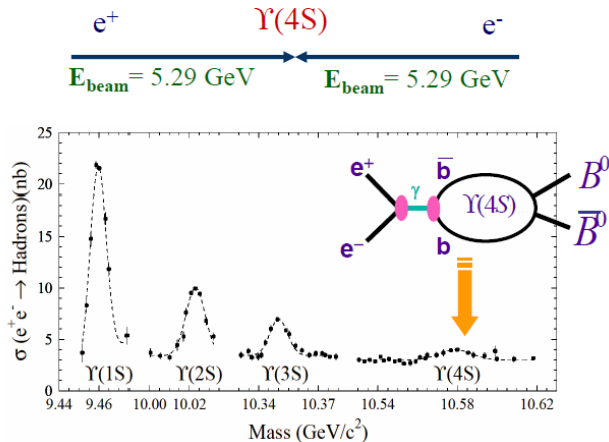
The wave function of the  $B$  meson pair in a coherent  $P$  – *wave* state:

$$\Psi = \frac{1}{\sqrt{2}} [ |B^0\rangle|\bar{B}^0\rangle - |\bar{B}^0\rangle|B^0\rangle ]$$

- At all times one  $B^0$  and one  $\bar{B}^0$  meson, until one of them decays.
- The remaining un-decayed  $B$  meson will continue to propagate through space-time and mix until it decays.

# $B$ mesons produced in $\Upsilon(4S)$ decays

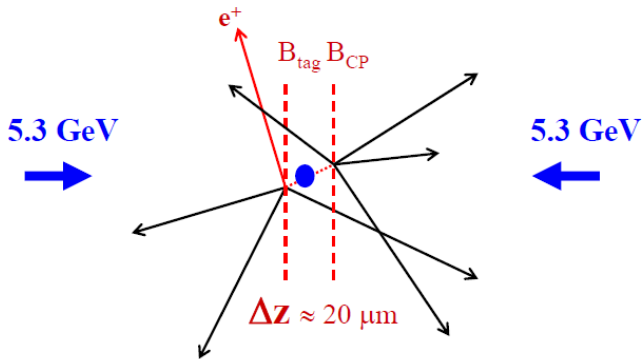
What can we say about the  $B$ -mesons produced from beams where  $E(e^+) = E(e^-)$ ?



- Enough energy to barely produce 2  $B$  mesons, nothing else!
- $B$ -mesons produced with  $\sim 300 \text{ MeV}$  momentum



# $B$ mesons produced in $\Upsilon(4S)$ decays



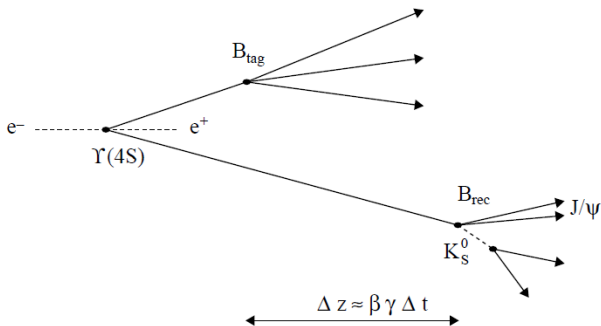
- Experimentally the decay time is measured by measuring the decay length
- Distances of a few  $10 \mu\text{m}$  are too small to measure

# Asymmetric energy $B$ -factories

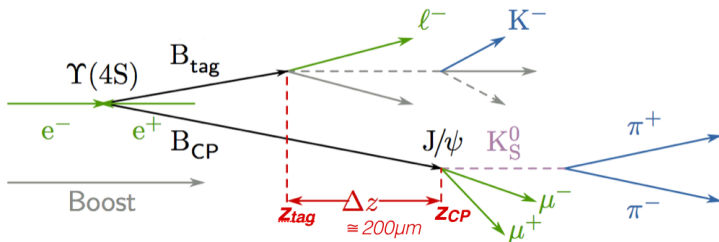
Solution:

- Boost  $\Upsilon(4S)$  in laboratory frame by colliding beams of unequal energy but same  $CM$  energy

$$E_{CM}^2 = 4E_{LER}E_{HER} = m_{\Upsilon(4S)}^2$$



# Asymmetric energy $B$ -factories



- Decay of first  $B$  (as  $B^0$ ) at  $t_{\text{tag}}$  ensures the other  $B$  is  $\bar{B}^0$ .

$\Rightarrow$  *End of quantum entanglement!*

Defines a reference time

- At  $t > t_{\text{tag}}$ ,  $B^0$  can mix to  $\bar{B}^0$  before it decays.
- Possible outcomes:

$$B^0 B^0, B^0 \bar{B}^0, \bar{B}^0 \bar{B}^0$$

# Asymmetry as a function of $\Delta t$

In our derivation earlier, we simply used time  $t$ , but now it's clear we need to consider  $\Delta t$ , defined as  $t_{CP} - t_{tag}$

Our general time-dependent asymmetry as a function of  $\Delta t = t_{CP} - t_{tag}$  is:

$$\mathcal{A}_{CP}(\Delta t) = \frac{f_{B_{tag=B^0}}(\Delta t) - f_{B_{tag=\bar{B}^0}}(\Delta t)}{f_{B_{tag=B^0}}(\Delta t) + f_{B_{tag=\bar{B}^0}}(\Delta t)} = \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \cos(\Delta M \Delta t) - \frac{2 \text{Im}[\lambda]}{1 + |\lambda|^2} \sin(\Delta M \Delta t)$$

$\Rightarrow$  Need to know the flavor of the  $B_{tag}$

# Flavor tagging

$B^0$  or  $\bar{B}^0$  flavor identified from the decay products.

Several different categories of tagging, e.g.,

- ① Lepton tag
- ② Kaon tag
- ③ Pion tag
- ④ Lambda tag
- ...

# Lepton tag

Primary leptons originate directly from  $B$  mesons in semileptonic decays. These modes are due to leading order weak interaction mediated by a charged  $W^\pm$  boson. The charge of

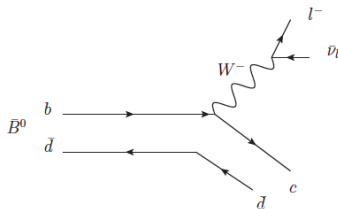


FIG. 3: Production process for direct leptons.

direct leptons is associated with the flavor of its mother particle (Fig. 3). In a  $b \rightarrow c l^- \bar{\nu}_l$  semileptonic decay a positively (negatively) charged lepton indicates a  $B^0$  ( $\bar{B}^0$ ) decay

$$\bar{B}^0 \rightarrow X l^- \bar{\nu}_l, \quad (3)$$

where  $X$  indicates another hadronic particle.

Secondary leptons descend from semileptonic decaying  $D$  mesons via  $b \rightarrow c \rightarrow s$  transitions. In this cascade decay the charge of the lepton corresponding to the  $B$  meson is reversed: a negatively (positively) charged lepton indicates a  $B^0$  ( $\bar{B}^0$ )

$$\bar{B}^0 \rightarrow DX \rightarrow X' l^+ \nu_l. \quad (4)$$

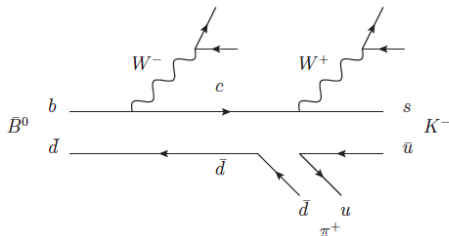
# Kaon tag

## 3. Kaons

Kaons give the most powerful flavor identification and also have a high occurrence in  $B^0$  decays. They mainly originate from  $b \rightarrow c \rightarrow s$  cascade decays

$$\begin{aligned} \bar{B}^0 &\rightarrow DX \\ &\rightarrow K^- X'. \end{aligned} \quad (7)$$

A positively (negatively) charged kaon indicates a  $B^0$  ( $\bar{B}^0$ ), as illustrated in Fig. 7. Since the mother of the kaon is unclear, it can also originate from charm decay or  $s\bar{s}$  quark pair popping out of the vacuum, therefore combining the total charge is important. Emerging  $K_S^0$  can indicate a kaon from  $s\bar{s}$  quark pair popping. In addition to kinematic variables like  $p_{cms}$  and  $\theta_{lab}$ , the charge and PID can help to identify candidates.



# Pion tag

Pions are the most common final state particles. Slow pions originate from a  $D^{*\pm}$  decay, where a negatively (positively) charged pion indicates a  $B^0$  ( $\bar{B}^0$ )

$$\begin{aligned}\bar{B}^0 &\rightarrow D^{*+} X \\ &\rightarrow D^0 \pi^+.\end{aligned}\tag{5}$$

Because of the low mass difference between the  $D^{*+}$  and the  $D^0$ , slow pions are produced nearly at rest in the  $D^{*+}$  frame. The pion moves nearly in the same direction as the  $D^0$  in the  $B_{tag}$  frame. On the contrary, pions coming from the hadronization of a  $W$  boson have

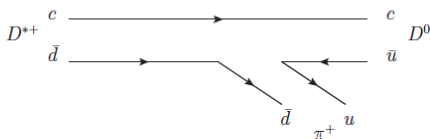


FIG. 5: Production process for slow pions.

a higher momentum, e.g., in the decay

$$\bar{B}^0 \rightarrow D^{*+} \pi^- X.\tag{6}$$



# Lambda tag

## 4. Lambdas

Lambdas are not directly measured as final state particles but have to be reconstructed from protons and pions. They can arise through a  $b \rightarrow c \rightarrow s$  cascade decay, such as

$$\begin{aligned}\bar{B}^0 &\rightarrow \bar{\Lambda}_c^+ X \\ &\rightarrow \bar{\Lambda}_0 X'.\end{aligned}\tag{8}$$

Despite their very low occurrence in  $B$  meson events, they are valuable for flavor determination. A lambda (anti-lambda) indicates a  $B^0$  ( $\bar{B}^0$ ). The quality of a lambda candidate depends on a correct reconstruction, therefore the quality of the lambda vertex is of interest. The angle between the lambda momentum, its vertex and the interaction point  $\theta_\Lambda$  can also give information about a good candidate, as well as its mass  $M_\Lambda$ .

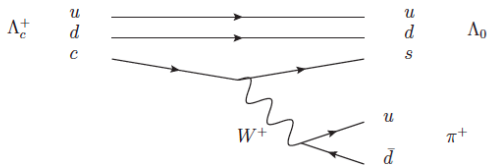
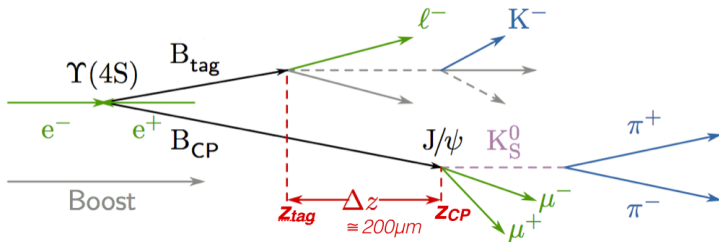


FIG. 8: Production process for lambdas.

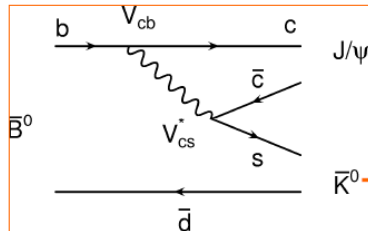
# B-meson reconstruction

How do we build our  $B$ -mesons?



$\Rightarrow$  Work backwards from the final-state particles, which we can identify in our detector

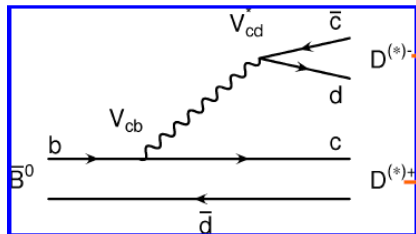
# Start with the final-state particles ( $e, \mu, \pi, K, \gamma$ )



$J/\psi \rightarrow \mu^+ \mu^-$  or  $e^+ e^-$

$\bar{K}^0 \rightarrow \pi^+ \pi^-$  displaced vertex ( $c\tau=2.7\text{cm}$ )

⇒ Work Backwards



$D^{(*)-} \rightarrow D^0 \pi^+$   
 $\searrow$   
 $K^- \pi^+$

$D^{(*)+} \rightarrow D^- \pi^0$   
 $\searrow \quad \searrow \gamma\gamma$   
 $K^+ \pi^- \pi^-$

# Use energy-momentum to build the intermediate states

- Reconstruct  $J/\psi$  (first measurement @ Belle with  $5.5\text{fb}^{-1}$ )

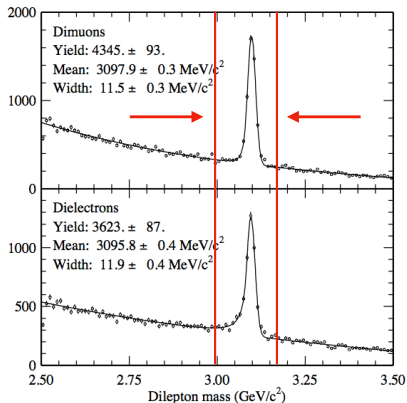


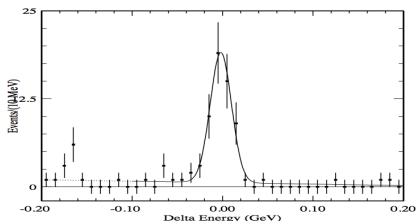
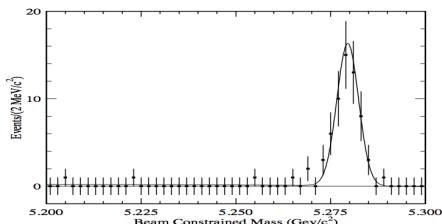
Figure 2: The  $J/\psi$  invariant mass distributions for the  $5.5 \text{ fb}^{-1}$  inclusive  $J/\psi$  data.

Make “cuts” (vertical lines) to remove obvious background-dominated regions

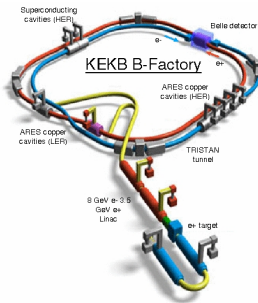
# Finally arrive at your $B$ mesons

Main variables:

- Energy difference:  $\Delta E = E_B^* - E_{\text{beam}}^*$
- Beam constrained mass:  $M_{bc} = \sqrt{(E_{\text{beam}}^*)^2 + (p_B^*)^2}$

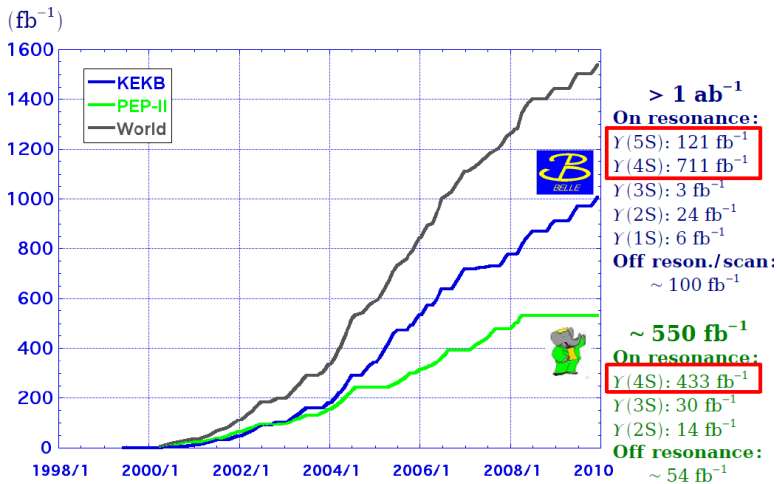


# Belle Experiment @ KEKB accelerator



- Asymmetric energy collider:  $e^+(3.5\text{GeV}) \rightarrow \leftarrow e^-(8\text{GeV})$
- Energy released in collisions:  $\sqrt{s} = 10.58 \text{ GeV} \approx M_{\Upsilon(4S)}$

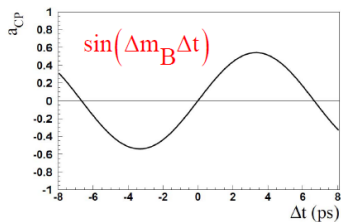
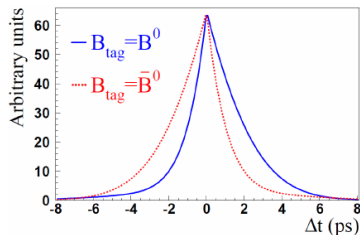
# Total datasets from Belle and BaBar ( $B$ -factory at SLAC)



$\sim 770 \text{ MB}\bar{B}$  for Belle,  $\sim 470 \text{ MB}\bar{B}$  for BaBar  
 $\sim 14\text{M } B_s$  also! ( $Y(5S)$  runs)

# Perfect detector

For the case of  $B^0 \rightarrow J/\psi K_S^0$ , where  $A_f = 0$

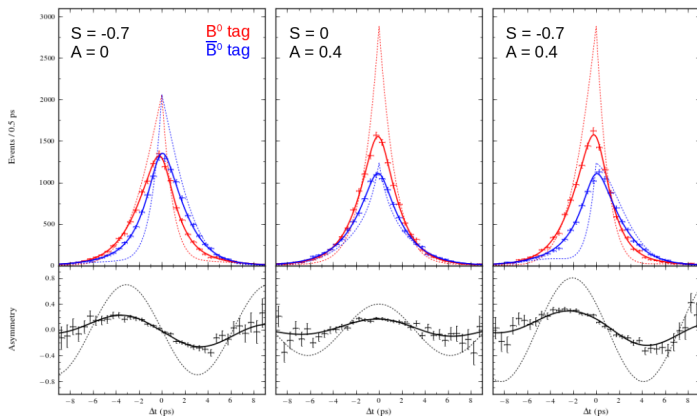


$$\mathcal{A}_{CP}(\Delta t) = \frac{2\text{Im}\lambda}{1 + |\lambda|^2} \sin(\Delta M \Delta t)$$



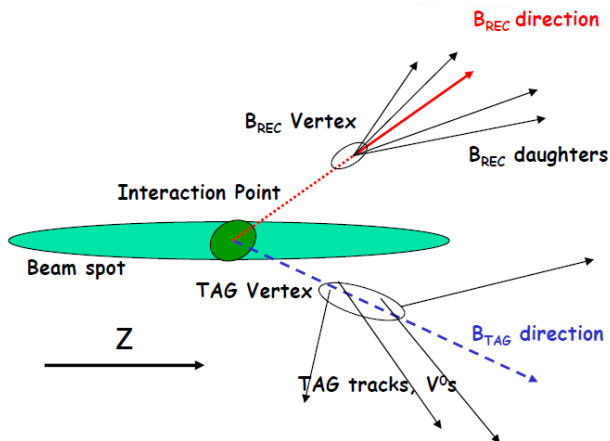
# Perfect detector vs. reality

See the effect of different values of  $A_f$  and  $S_f$ :

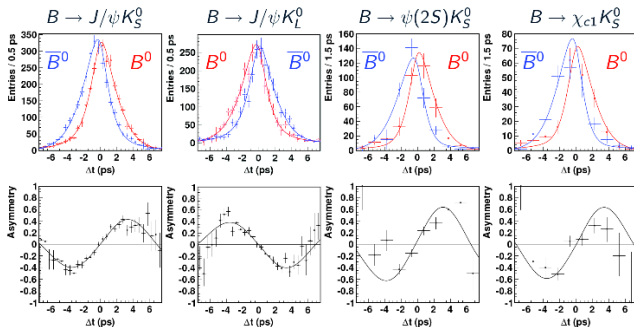
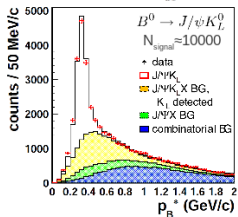
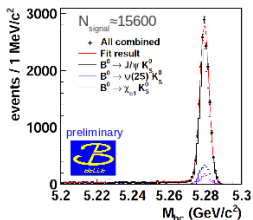


$\Rightarrow$  Need to take into account mis-tagging and  $\Delta t$  resolution,  
which smear the  $\Delta t$  distribution

# Vertex and $\Delta t$ reconstruction



# $CP$ violation in $B^0 \rightarrow J/\psi K_S^0, J/\psi K_L^0, \psi(2S)K_S^0, \chi K_S^0$



Belle with  $772 \times 10^6$  BB:

$$\mathcal{A} = 0.007 \pm 0.016 (\text{stat}) \pm 0.013 (\text{syst})$$

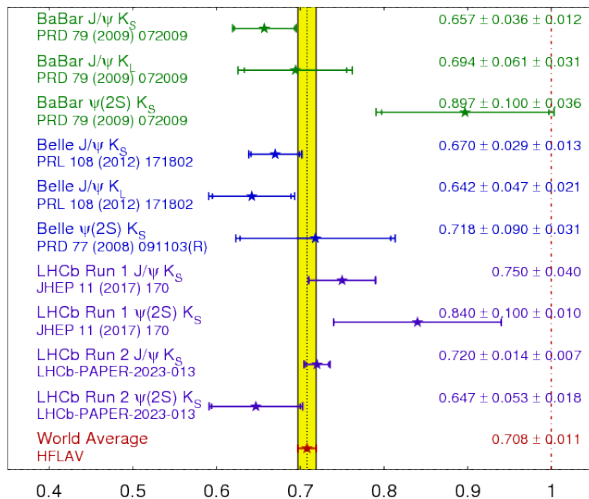
$$\sin(2\phi_1) = 0.668 \pm 0.023 \quad \pm 0.013$$

preliminary

# Combination of results

$$\sin(2\beta) \equiv \sin(2\phi_1)$$

**HFLAV**  
Summer 2023  
PRELIMINARY



## To summarize:

To measure  $CP$  violation at the  $\Upsilon(4S)$  using the interference between mixing and decay, one must:

- 1 Determine the flavor of one of the neutral  $B$  mesons directly from its decay products (e.g., from semileptonic decays).
- 2 Reconstruct the other  $B$  meson in a state that both  $B^0$  and  $\bar{B}^0$  can decay into.
- 3 Measure the time difference,  $\Delta t = t_1 - t_2$ , between the decays. Requires precise vertexing information to measure  $\Delta z$ .

- Richman, Jeremy D. (UCSB), *Heavy Quark Physics and CP Violation*.  
[http://physics.ucsd.edu/students/courses/winter2010/physics222/references/driver\\_houches12.pdf](http://physics.ucsd.edu/students/courses/winter2010/physics222/references/driver_houches12.pdf)  
Pages 199-220.
- Kooijman, P. & Tuning, N., *CP Violation*,  
<http://master.particles.nl/LectureNotes/2011-CP.pdf>  
Pages 38-46.