

# Determination of CKM angle $\gamma$ and introduction to Dalitz analysis

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**Flavor Physics Lectures**  
**VII / XII**



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# Reading material and references

**Lecture material based on several textbooks and online lectures/notes.**

**Credits for material and figures include:**

## Literature

- Perkins, Donald H. (2000), *Introduction to High Energy Physics*.
- Griffiths, David J. (2nd edition), *Introduction to Elementary Particles*.
- Stone, Sheldon (2nd edition), *B decays*.

## Online Resources

- Belle/BaBar Collaborations, *The Physics of the B-Factories*.  
<http://arxiv.org/abs/1406.6311>
- Bona, Marcella (University of London), *CP Violation Lecture Notes*,  
<http://pprc.qmul.ac.uk/bona/ulpg/cpv/>
- Richman, Jeremy D. (UCSB), *Heavy Quark Physics and CP Violation*.  
[https://courses.physics.ucsd.edu/2010/Winter/physics222/references/driver\\_houches12.pdf](https://courses.physics.ucsd.edu/2010/Winter/physics222/references/driver_houches12.pdf)
- Thomson, Mark (Cambridge University), *Particle Physics Lecture Handouts*,  
<http://www.hep.phy.cam.ac.uk/thomson/partIIIparticles/welcome.html>
- Grossman, Yuval (Cornell University), *Just a Taste. Lectures on Flavor Physics*,  
<http://www.lepp.cornell.edu/pt267/files/notes/FlavorNotes.pdf>
- Kooijman, P. & Tuning, N., *CP Violation*,  
<https://www.nikhef.nl/h71/Lectures/2015/ppII-cpviolation-29012015.pdf>

# Recap & outline

So far, we:

- Learned how to measure CKM angles  $\beta$  and  $\alpha$  through various channels:
  - We looked at theoretically clean “benchmark” decays (e.g.,  $B \rightarrow J/\psi K_S^0$ ) and also several decays dominated by  $b \rightarrow s$  penguin diagrams where we can probe for new physics.
  - We saw how penguin pollution complicates the extraction of  $\alpha$ . We learned how to exploit isospin relations to help resolve the ambiguity.

Today, we'll:

- Learn now to measure the final CKM angle  $\gamma$ .
- Along the way, we'll introduce the Dalitz analysis technique.

Credits for additional material and figures:

Tim Gershon, Tom Latham, and Brian Lindquist

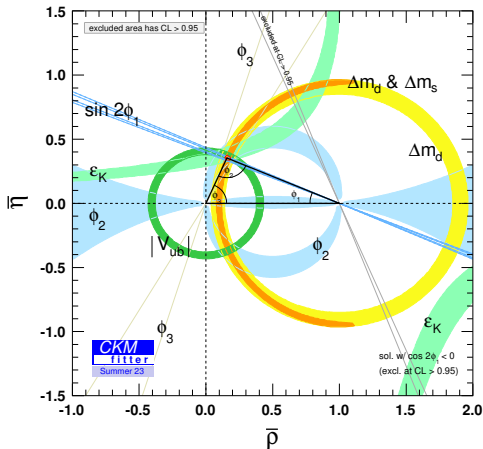
# Measurements of angle $\gamma = \phi_3$

$$\gamma = \phi_3$$

$$\gamma \equiv \arg \left[ -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right] \approx \arg V_{ub}^*$$

- The standard candle along with  $V_{ub}/V_{cb}$ .
- Less precise than  $\phi_2$ .
- Limited by the small branching fractions of the processes used in its measurement.

⇒ *The one place a large experimental gain in UT metrology can be made*



<http://ckmfitter.in2p3.fr>

# Determine $\gamma$ from $CP$ asymmetries in $B \rightarrow DK$ decays

## Tree-level Determination

- $b \rightarrow c\bar{u}s$  and  $b \rightarrow u\bar{c}s$  tree amplitudes in the charged- $B$  meson decays to open-charm final states.

$\Rightarrow$  Interference between same final state for  $D$  and  $\bar{D} \Rightarrow$  possibility of  $DCPV$ .

- \* No penguin contribution (no theoretical uncertainty)

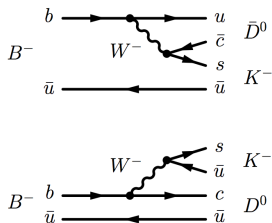
$\Rightarrow$  *All hadronic unknowns obtainable from experiment:*

$r_B$  = magnitude of the ratio of the amplitudes for  $B^- \rightarrow \bar{D}^0 K^-$  and  $B^- \rightarrow D^0 K^-$ .

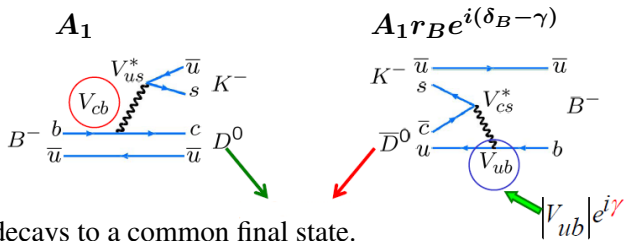
$\delta_B$  = the relative strong phase between these 2 amplitudes.

! Challenging: Small overall  $\mathcal{B}$  from  $5 \times 10^{-6}$  to  $10^{-9}$ .

$\Rightarrow$  *Precise measurement of  $\phi_3$  requires a very large data sample.*



# Determine $\gamma$ from $CP$ asymmetries in $B \rightarrow DK$ decays



$D^0$  and  $\bar{D}^0$  decays to a common final state.

Grouped into 3 categories:

- (1) Flavor eigenstate:  $K^+\pi^-$ ,  $K^+\pi^-\pi^0$
- (2)  $CP$  eigenstate:  $K^+K^-$ ,  $\pi^+\pi^-$ ,  $K_S^0\pi^0$
- (3) 3-body decay:  $K_S^0\pi^+\pi^-$ ,  $K_S^0K^+K^-$

Relative strength of the two  $B$  decay amplitudes important for interference

$$r_B = \left| \frac{A(b \rightarrow u)}{A(b \rightarrow c)} \right| \sim 0.1 - 0.3$$

Large  $r_B \Rightarrow$  large  $CP$  asymmetry

# More on these 3 types of final states

Each category corresponds to a method, whose names reflect the authors who first described them:

The 3 methods according to final state are:

- (1) Cabibbo-favored and double-CS final states ( $K^+\pi^-$ ,  $K^+\pi^-\pi^0$ ) [“ADS Method”]  
Atwood, Dunietz & Soni *Phys. Rev. D* **63**, 036005 (2001)
- (2) Cabibbo-suppressed (CS)  $D$  decays to  $CP$ -eigenstates ( $K^+K^-$ ,  $\pi^+\pi^-$ ,  $K_S^0\pi^0$ ) [“GLW Method”]  
Gronau & London *Phys. Lett. B* **253**, 483 (1991), Gronau & Wyler *Phys. Lett. B* **265**, 172 (1991)
- (3) Dalitz plot distribution of the products of  $D$  decays to multi-body self-conjugate final states ( $K_S^0\pi^+\pi^-$ ,  $K_S^0K^+K^-$ ) [“GGSZ Method”]  
Giri, Grossman, Soffer and Zupan *Phys. Rev. D* **68**, 054018 (2003); Bondar (unpublished)

All methods are statistics-limited **but have common  $B$  parameters**

$\Rightarrow$  *Perform a simultaneous fit using the results of all methods.*



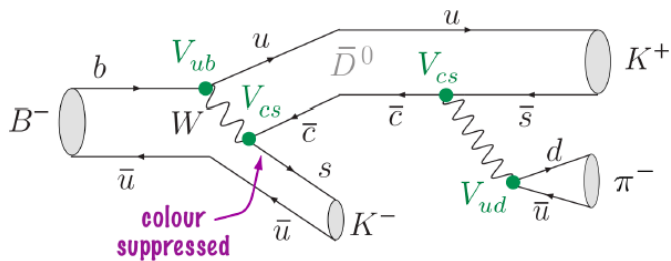
## Aside: Cabbibo suppression

Recall the magnitude of the CKM matrix elements (represented by  $\square$ )

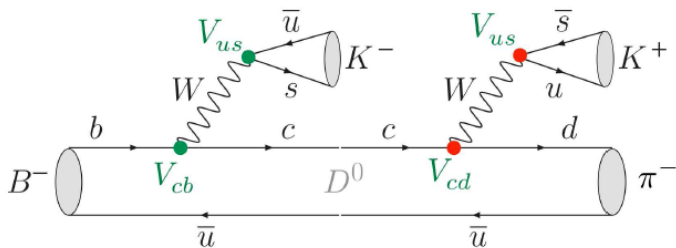
$$\begin{array}{c} \begin{array}{ccc} & d & s & b \end{array} \\ \begin{array}{l} u \\ c \\ t \end{array} \left( \begin{array}{ccc} \square & \square & \cdot \\ \square & \square & \square \\ \cdot & \square & \square \end{array} \right) \end{array}$$

- Weak decays whose amplitudes contain only diagonal CKM elements are referred to as Cabbibo favored.
- Those with one factor of  $V_{us}$ ,  $V_{cb}$ ,  $V_{cd}$ , or  $V_{ts}$  are singly Cabbibo suppressed.
- Decays containing 2 of these factors, or one of  $V_{ub}$  or  $V_{td}$  are doubly Cabbibo suppressed.

# Cabibbo-favored and double-CS final states [ADS]

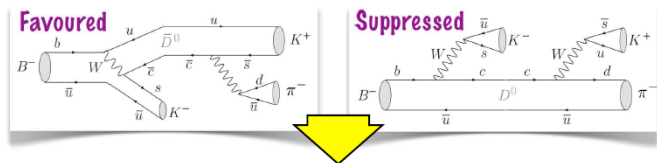


Cabibbo  
favoured  
 $V$  decay



doubly  
Cabibbo  
suppressed  
 $V$  decay

# Cabibbo-favored and double-CS final states [ADS]



**Common**

$$\mathcal{R}_{DK} = \frac{\Gamma([K^+ \pi^-] K^-) + \Gamma([K^- \pi^+] K^+)}{\Gamma([K^- \pi^+] K^-) + \Gamma([K^+ \pi^-] K^+)}$$

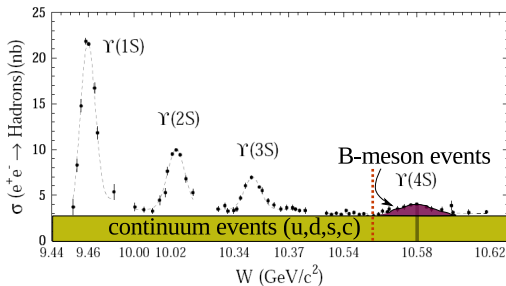
$$= r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos \phi_3$$

$$\mathcal{A}_{DK} = \frac{\Gamma([K^+ \pi^-] K^-) - \Gamma([K^- \pi^+] K^+)}{\Gamma([K^- \pi^+] K^-) + \Gamma([K^+ \pi^-] K^+)}$$

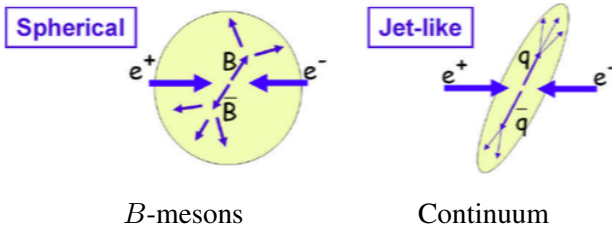
$$= 2r_B r_D \sin(\delta_B + \delta_D) \sin \phi_3 / \mathcal{R}_{DK}$$

where  $r_D = \left| \frac{\mathcal{A}(D^0 \rightarrow K^+ \pi^-)}{\mathcal{A}(\bar{D}^0 \rightarrow K^+ \pi^-)} \right| = 0.0613 \pm 0.0010$

# Recall $e^+e^- \rightarrow q\bar{q}$ continuum production



Different event topologies



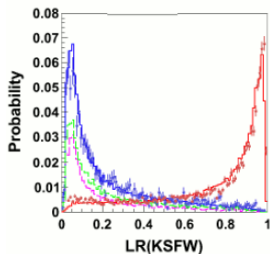
# Continuum suppression in ADS decay $B^- \rightarrow DK^-$ (I)

$$\underline{B^- \rightarrow DK^-, D \rightarrow K^+ \pi^- \text{ ADS}}$$

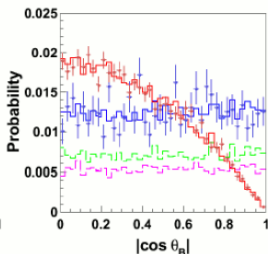
Main background is  $e^+ e^- \rightarrow q\bar{q}$  ( $q=u, d, s, c$ ) continuum  
combine 10 variables with neural network:

- ▶ Variables which have different distributions for signal and  $q\bar{q}$  background are used.

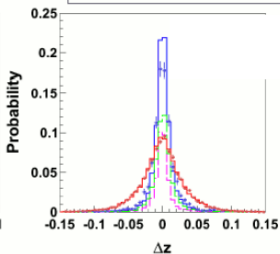
$B^- \rightarrow D\pi^-$   
 $\rightarrow K^-\pi^+$   
 $M_{bc}$ -sideband  
 histogram: MC, dots: data  
 signal  
 $q\bar{q} = \text{charm} + \text{uds}$



Likelihood ratio for KSWF.

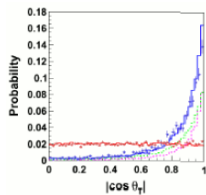


$\theta_B$  is angle between  $B$ -flight direction and beam axis.

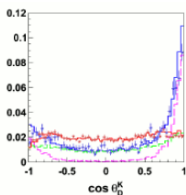


Vertex separation between reconstructed  $B$  and the other  $B$ .

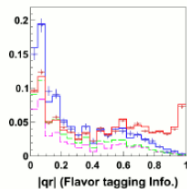
# Continuum suppression in ADS decay $B^- \rightarrow DK^-$ (II)



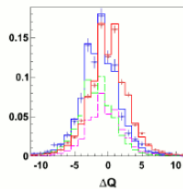
Angle between thrust axes of  $B$  decay and remainder.  
No full correlation to  $LR(KSFV)$ .



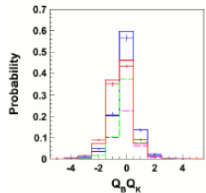
Decay angle of  $D \rightarrow K\pi$ .



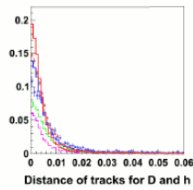
Flavor tagging Info. by MDLH. (NB possible.)



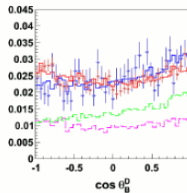
Difference of charges in  $D$  hemisphere and opposite hemisphere.



Product of charge of  $B$  and sum of charges for  $K$  not used in  $B$  reconstruction.



Distance of tracks for  $D$  and  $K$ .



Decay angle of  $B \rightarrow DK$

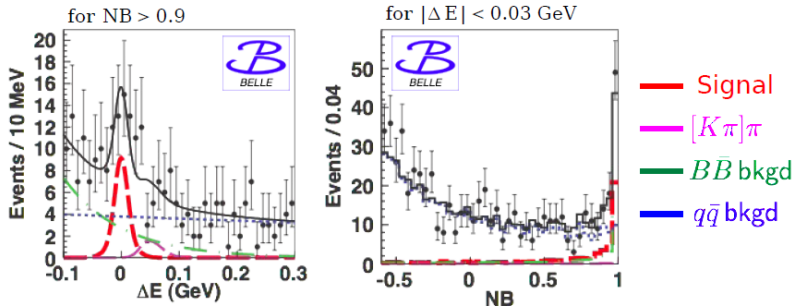
10 variables combined to obtain a single NN output (NB)

**for example,**  
**at 99% bckg rej.**  
**signal eff. = 42%**  
**now becomes 60%**

# Belle result for ADS decay $B^- \rightarrow [K^+\pi^-]_D K^-$

Yields for the ADS mode  $B^- \rightarrow [K^+\pi^-]_D K^-$  from 772 million  $B\bar{B}$  events  
PRL 106, 231803 (2011)

Fit  $\Delta E$  and NB distributions together to extract signal



$$56.0^{+15.1}_{-14.2} \text{ events}$$

$$R_{DK} = (1.63^{+0.44+0.07}_{-0.41-0.13}) \times 10^{-2}$$

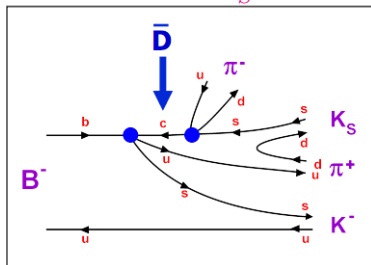
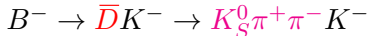
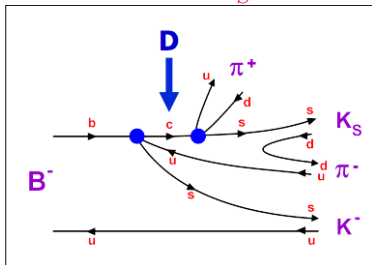
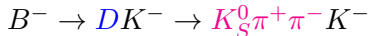
$$A_{DK} = -0.39^{+0.26+0.04}_{-0.28-0.03}$$

First evidence obtained  
with a significance of  $4.1\sigma$   
(including syst.)

# Measurement of $\gamma$ using Dalitz plot analysis [GGSZ]

The basic idea of this method is to use final states accessible to both  $D^0$  and  $\bar{D}^0$  and to measure the phase of the interference between them in the decay of  $D$  mesons produced in  $B^\pm \rightarrow DK^\pm$  transitions

The most convenient decay for this type of measurement is  $D \rightarrow K_S^0 \pi^+ \pi^-$



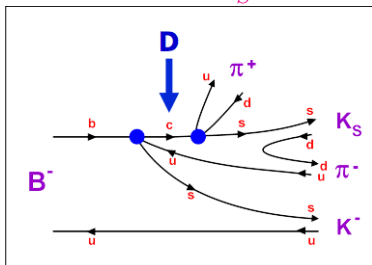


$$D \rightarrow K_S^0 \pi^+ \pi^-$$

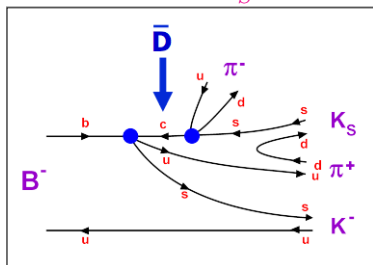
Unique combination of 3 advantages.

- 1) Large branching fraction.
- 2) Significant overlap of  $D \rightarrow K_S^0 \pi^+ \pi^-$  and  $\bar{D} \rightarrow K_S^0 \pi^+ \pi^-$  amplitudes which gives a large interference term sensitive to  $\gamma$ .
- 3) Rich resonant structure which provides large variations of the strong phase in  $D$  decays and results in sensitivity to  $\gamma$  that is only weakly dependent on the values of  $\gamma$  and strong phase  $\delta_B$ .

$$B^- \rightarrow DK^- \rightarrow K_S^0 \pi^+ \pi^- K^-$$



$$B^- \rightarrow \bar{D}K^- \rightarrow K_S^0 \pi^+ \pi^- K^-$$



# Intermission - Dalitz plot formalism

# What is a Dalitz plot?

- Visual representation of
  - the phase-space of a three-body decay
    - involving only spin-0 particles
    - (term often abused to refer to phase-space of any multibody decay)
  - Named after it's inventor, Richard Dalitz (1925–2006):
  - “On the analysis of tau-meson data and the nature of the tau-meson.”
    - R.H. Dalitz, Phil. Mag. 44 (1953) 1068
    - (historical reminder: tau meson = charged kaon)
  - For scientific obituary, see
    - I.J.R. Aitchison, F.E. Close, A. Gal, D.J. Millener,
    - Nucl.Phys.A771:8-25,2006

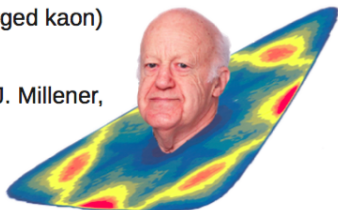
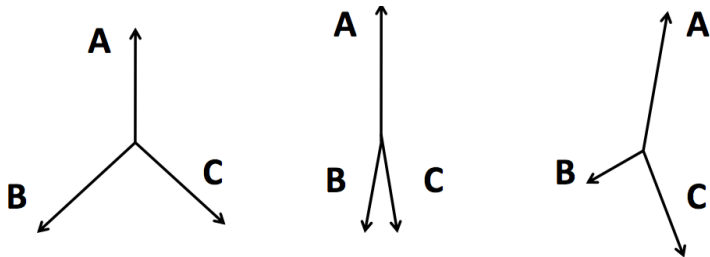


Image credit: Mike Pennington

## 3-body decays

For 2-body decays,  $M \rightarrow ab$ ,  $p_a$  and  $p_b$  are completely determined by  $E$ ,  $p$  conservation.

3-body decays have additional degrees of freedom; different values of  $p_a$ ,  $p_b$  and  $p_c$  are possible, depending on the decay configuration.



# Degrees of freedom (d.o.f)

For 3-body decays,  $M \rightarrow abc$ , where  $a$ ,  $b$  and  $c$  are spin-0, the final state can be described by three 4-vectors:  $p_a^\mu$ ,  $p_b^\mu$  and  $p_c^\mu$ .

There are 12 parameters in total, but not all are independent:

- Set  $p_{i,z} = 0$  since  $a$ ,  $b$  and  $c$  all decay in the same plane; removes 3 d.o.f.
- Remove 3 d.o.f. by  $E_i = \sqrt{m_i^2 + p_i^2}$ , ( $i = a, b, c$ )
- Remove 3 d.o.f. by  $\vec{p}_M = \vec{p}_a + \vec{p}_b + \vec{p}_c$  and  $E_M = E_a + E_b + E_c$ .
- Can rotate entire system in  $x - y$  plane without effect; removes 1 d.o.f.

$\Rightarrow$  Only 2 d.o.f. left.

What should we use?!

To answer this, let's look at the differential decay probability of  $M \rightarrow abc$ .

# Kinematic constraints

For a particle of mass  $M$  decaying into 3 particles denoted as  $a$ ,  $b$  and  $c$ , the differential decay probability is:

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M^3} |A|^2 dm_{ab}^2 dm_{bc}^2$$

where  $m_{ab}$  and  $m_{bc}$  are the invariant masses of the pairs of particles  $ab$  and  $bc$ , respectively (i.e.,  $m_{ab}^2 = (p_a^\mu + p_b^\mu)^2$ ).

**Use  $m_{ab}$  and  $m_{bc}$  (or  $m_{ab}$  &  $m_{ac}$ ) as our 2 d.o.f.**

The invariant masses of pairs of final-state particles are related by the linear dependence:

$$m_{ab}^2 + m_{bc}^2 + m_{ac}^2 = M^2 + m_a^2 + m_b^2 + m_c^2$$

# Kinematic constraints

The range of invariant masses  $m_{bc}^2$  can be written in terms of either one of the other squared invariant masses (e.g., for  $m_{ab}^2$ ):

$$(m_{bc}^2)_{\max} = (E_b^* + E_c^*)^2 - (p_b^* - p_c^*)^2$$
$$(m_{bc}^2)_{\min} = (E_b^* + E_c^*)^2 - (p_b^* + p_c^*)^2$$

where the energies of the particles  $b$  and  $c$  in the  $ab$  rest frame are

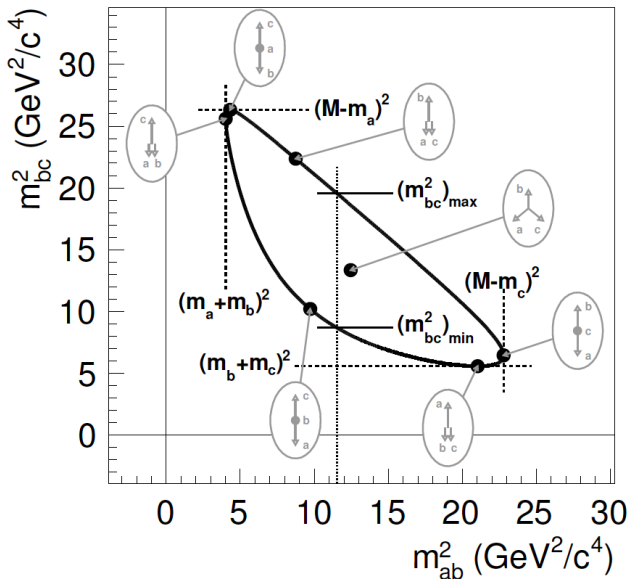
$$E_b^* = \frac{m_{ab}^2 - m_a^2 + m_b^2}{2m_{ab}}, \quad E_c^* = \frac{M^2 - m_{ab}^2 + m_c^2}{2m_{ab}}$$

and their corresponding momenta are:

$$p_b^* = \sqrt{E_b^* - m_b^2}, \quad p_c^* = \sqrt{E_c^* - m_c^2}$$

[Click here for a PDF containing a complete derivation of the kinematic limits \(i.e., boundary curve on next slide\) of the Dalitz plot \[eqn 39\]](#) (Uploaded to ILIAS)

# Kinematic boundaries of the 3-body decay phase space

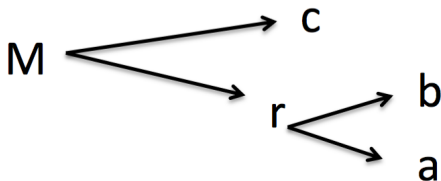




# Decay of $M$ via resonances

Sometimes  $M$  will decay directly to  $a, b, c$ ; this is called non-resonant (NR) or “phase space” decay.

The majority of times  $M$  will decay through intermediate particles (or “resonances”)  $r$



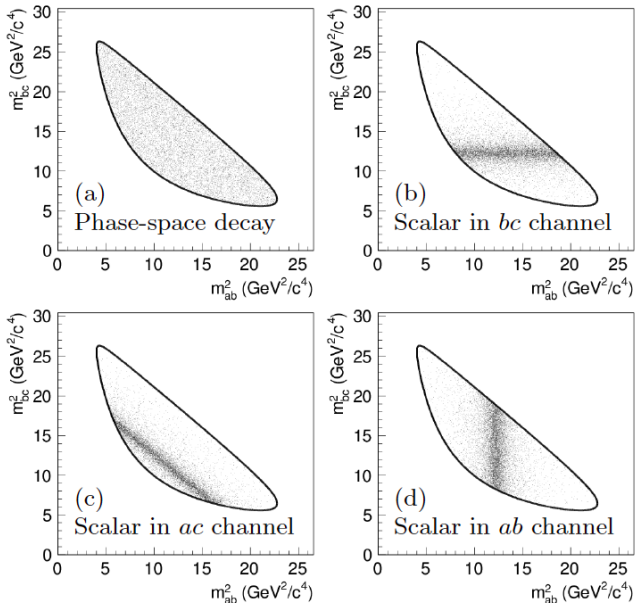
$r$  is typically very short lived  $\Rightarrow$  *can't observe directly*.

But can study  $r$  with a Dalitz plot.

$E$  and  $p$  conservation imply that if  $r \rightarrow ab$ , then  $m_{ab}^2 = m_r^2$

These resonances show up as bands on the Dalitz plot

# Dalitz plot for NR decay and for spin-0 resonances



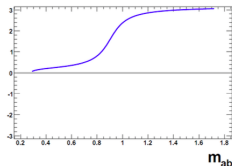
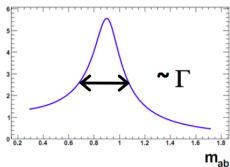
# Resonance lifetimes I

- Recall  $\Delta E \Delta t \sim \hbar$
- Short-lived resonances have broad peaks.
- Commonly described using a relativistic Breit-Wigner (RBW) parameterization with mass-dependent width.

$$A_{\text{RBW}} = \frac{1}{m_r^2 - m_{ab}^2 - im_r \Gamma_{ab}}$$

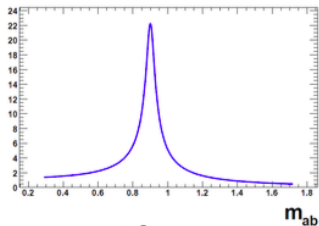
where the width is inversely proportional to the lifetime  $\Gamma = \frac{\hbar}{\tau}$

- Plot of magnitude and phase of  $A_{\text{RBW}} = |A|e^{i\phi}$

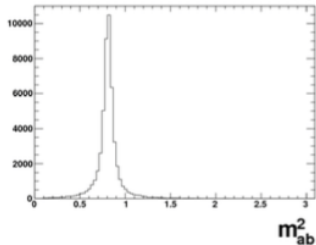
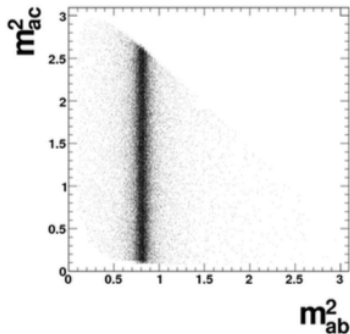
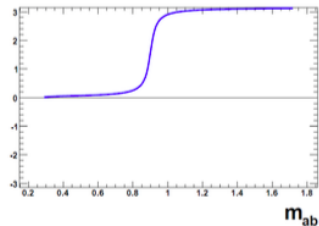


# Resonance lifetimes II

## Magnitude

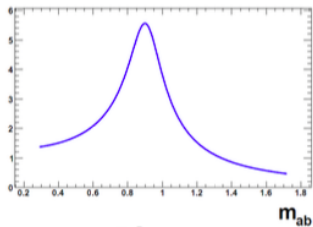


## Phase

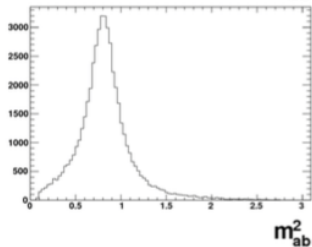
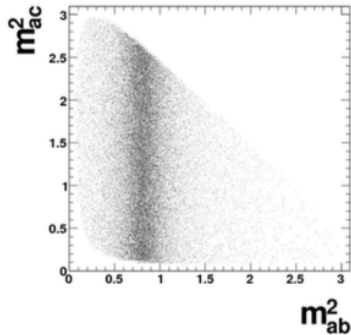
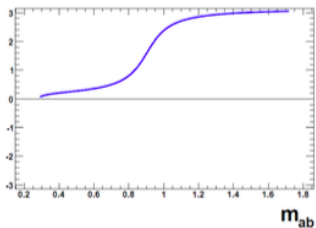


# Resonance lifetimes III

## Magnitude



## Phase



# Resonance spins

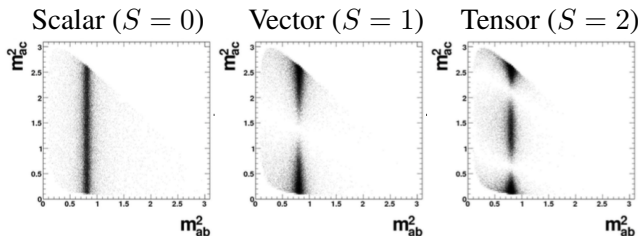
- If the **resonance has spin  $S$** , and  $M$ ,  $a$ ,  $b$ , and  $c$  are  $S = 0$ , the decay amplitude is proportional to Legendre polynomials:

$$A \propto A_{\text{RBW}}(m_{ab}) P_S(\cos \theta)$$

$$P_0(\cos \theta) = 1$$

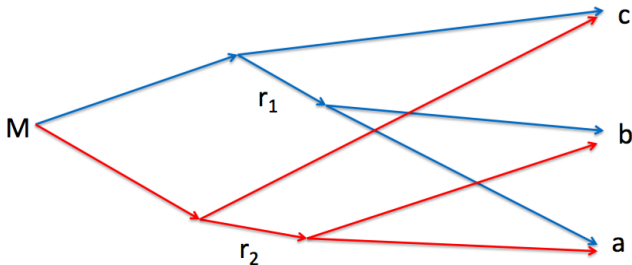
$$P_1(\cos \theta) = \cos \theta$$

$$P_2(\cos \theta) = \frac{1}{2} (3 \cos^2 \theta - 1)$$



# Multiple resonances

- Typically  $M$  can decay to the same final state through multiple resonances.
- Results in interference as in Young's double slit experiment.



# Isobar model

- The usual strategy is to model the total decay amplitude as a **sum of individual resonances** plus a non-resonant term:

$$A = \sum_r a_r e^{i\phi_r} A_r + a_{\text{NR}} e^{i\phi_{\text{NR}}} A_{\text{NR}}$$

where  $A_r(m_{ab}^2, m_{ac}^2)$  are the Dalitz plot dependent amplitudes which are of the form

$$A_r = F_P \times \mathcal{F}_r \times A_{\text{RBW}} \times W_r$$

and  $a_r$  and  $\phi_r$  can be measured in a maximum-likelihood fit.

- Here,  $A_{\text{RBW}} \times W_r$  is the resonance propagator, where  $W_r$  describes the angular distribution of the decay (and recall we used a relativistic BW as the dynamical function of the resonance:  $A_{\text{RBW}}$ )<sup>1</sup>.
- $F_P$  and  $F_r$  are the transition form factors of the parent particle and resonance, respectively.

*⇒ now lets look at an example of 2 interfering amplitudes*

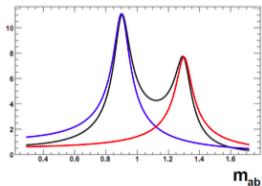
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<sup>1</sup>Strictly speaking the RBW works well only in the case of narrow states. The use of the mass-dependent width results in the amplitude becoming a non-analytic function. An alternative parametrization proposed by Gounaris and Sakurai (GS) recovers the analyticity of the amplitude and provides a better description for broad vector resonances.

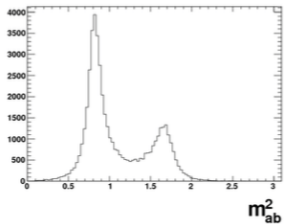
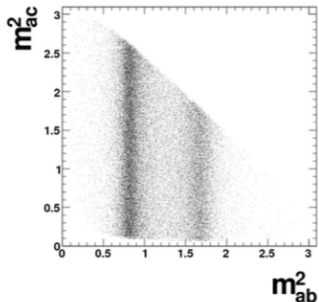
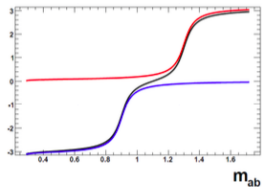


# Constructive interference

## Magnitude

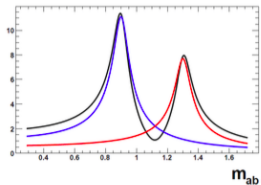


## Phase

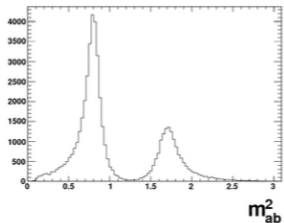
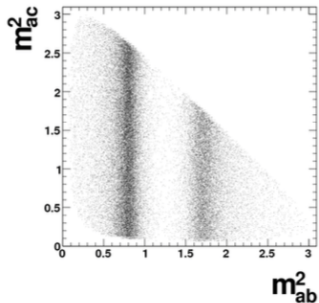
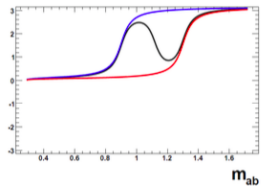


# Destructive interference

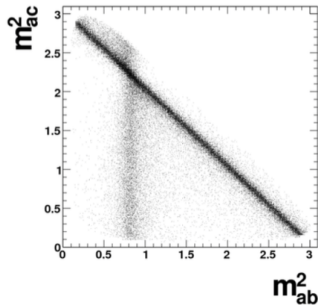
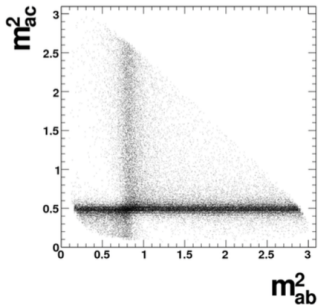
## Magnitude



## Phase

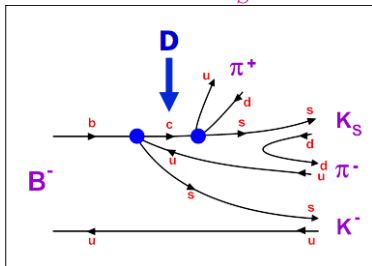


# Cross-channel interference

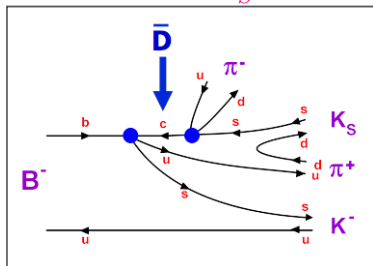


# Back to measuring $\gamma$ with $B \rightarrow DK$ decays

$$B^- \rightarrow DK^- \rightarrow K_S^0 \pi^+ \pi^- K^-$$



$$B^- \rightarrow \bar{D}K^- \rightarrow K_S^0 \pi^+ \pi^- K^-$$

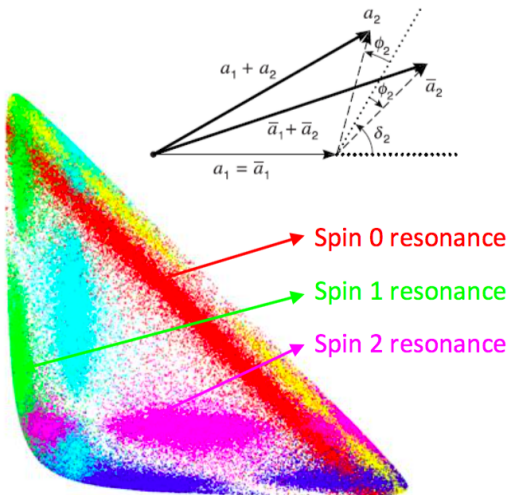


# MC simulation

Resonance structure in  
 $D \rightarrow K_S^0 \pi^+ \pi^-$  Dalitz plane

- **Green & blue:**  $K^{*0}(892)$  [vector]
- **Cyan & magenta:**  $K_2^*(1430)$  [tensor]
- **yellow:**  $\rho(770)$  [vector]
- **red:**  $f_0(980)$  [scalar]
- ... even more which are not simulated here (fit result table)

*Main advantage of Dalitz plots is the ability to exploit the interference between different resonances.*



- Define the 2 dalitz plot variables as

$$m_+^2 \equiv m_{K_S^0 \pi^+}^2$$

$$m_-^2 \equiv m_{K_S^0 \pi^-}^2$$

- The amplitude for the  $B^\pm \rightarrow DK^\pm$  decays are

$$A_{B^+}(m_+^2, m_-^2) = \bar{A}_D + r_B e^{i(\delta_B + \gamma)} A_D$$

$$A_{B^-}(m_+^2, m_-^2) = A_D + r_B e^{i(\delta_B - \gamma)} \bar{A}_D$$

where

$A_D = A_D(m_+^2, m_-^2)$  is the complex amplitude of  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$

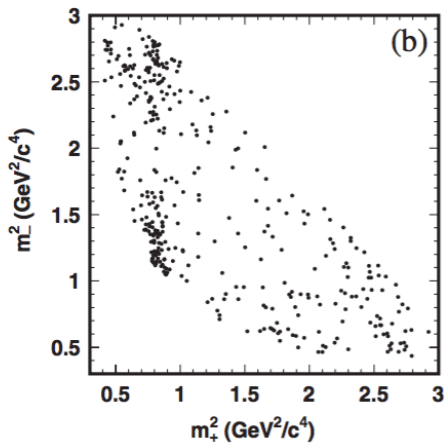
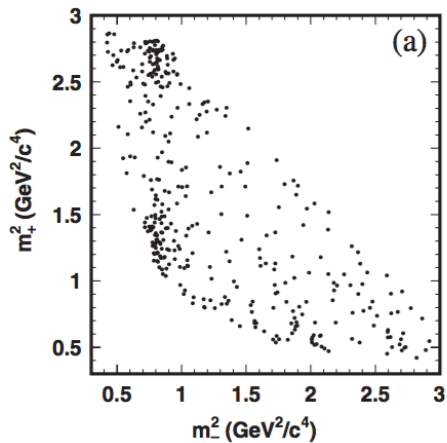
$\bar{A}_D = \bar{A}_D(m_+^2, m_-^2)$  is the complex amplitude of  $\bar{D}^0 \rightarrow K_S^0 \pi^+ \pi^-$

Recall:

$r_B =$  magnitude of the ratio of the amplitudes for  $B^- \rightarrow \bar{D}^0 K^-$  and  $B^- \rightarrow D^0 K^-$ .

$\delta_B =$  the relative strong phase between these 2 amplitudes.

# Belle data for $D \rightarrow K_S^0 \pi^+ \pi^-$

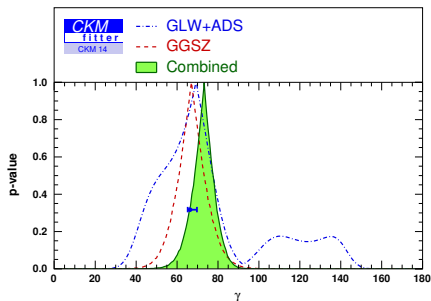


# Fit result to Belle data for $D \rightarrow K_S^0 \pi^+ \pi^-$

Intermediate state	Amplitude	Phase ( $^\circ$ )	Fit fraction (%)
$K_S^0 \sigma_1$	$1.56 \pm 0.06$	$214 \pm 3$	$11.0 \pm 0.7$
$K_S^0 f_0(980)$	$0.385 \pm 0.006$	$207.3 \pm 2.3$	$4.72 \pm 0.05$
$K_S^0 \sigma_2$	$0.20 \pm 0.02$	$212 \pm 12$	$0.54 \pm 0.10$
$K_S^0 f_0(1370)$	$1.56 \pm 0.12$	$110 \pm 4$	$1.9 \pm 0.3$
$K_S^0 \rho(770)^0$	1.0 (fixed)	0 (fixed)	$21.2 \pm 0.5$
$K_S^0 \omega(782)$	$0.0343 \pm 0.0008$	$112.0 \pm 1.3$	$0.526 \pm 0.014$
$K_S^0 f_2(1270)$	$1.44 \pm 0.04$	$342.9 \pm 1.7$	$1.82 \pm 0.05$
$K_S^0 \rho^0(1450)$	$0.49 \pm 0.08$	$64 \pm 11$	$0.11 \pm 0.04$
$K_0^*(1430)^- \pi^+$	$2.21 \pm 0.04$	$358.9 \pm 1.1$	$7.93 \pm 0.09$
$K_0^*(1430)^+ \pi^-$	$0.36 \pm 0.03$	$87 \pm 4$	$0.22 \pm 0.04$
$K^*(892)^- \pi^+$	$1.638 \pm 0.010$	$133.2 \pm 0.4$	$62.9 \pm 0.8$
$K^*(892)^+ \pi^-$	$0.149 \pm 0.004$	$325.4 \pm 1.3$	$0.526 \pm 0.016$
$K^*(1410)^- \pi^+$	$0.65 \pm 0.05$	$120 \pm 4$	$0.49 \pm 0.07$
$K^*(1410)^+ \pi^-$	$0.42 \pm 0.04$	$253 \pm 5$	$0.21 \pm 0.03$
$K_2^*(1430)^- \pi^+$	$0.89 \pm 0.03$	$314.8 \pm 1.1$	$1.40 \pm 0.06$
$K_2^*(1430)^+ \pi^-$	$0.23 \pm 0.02$	$275 \pm 6$	$0.093 \pm 0.014$
$K^*(1680)^- \pi^+$	$0.88 \pm 0.27$	$82 \pm 17$	$0.06 \pm 0.04$
$K^*(1680)^+ \pi^-$	$2.1 \pm 0.2$	$130 \pm 6$	$0.30 \pm 0.07$
non-resonant	$2.7 \pm 0.3$	$160 \pm 5$	$5.0 \pm 1.0$



# Combination of results from 3 methods



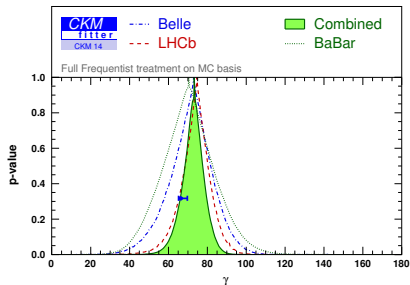
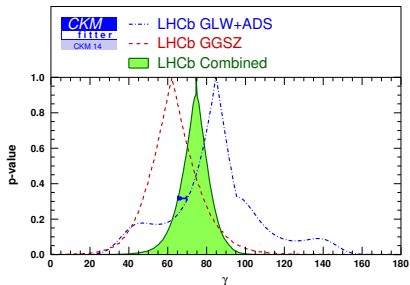
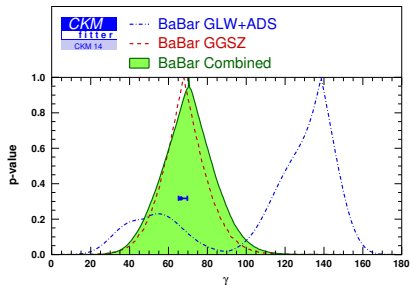
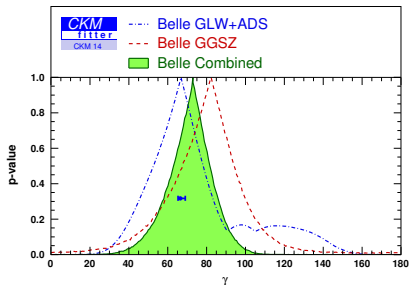
Constraints on  $\gamma$  from world average  $B^\pm \rightarrow D^{(*)}K^{(*)\pm}$  decays (GLW+ADS) and Dalitz analyses (GGSZ)

$$\gamma(\text{combined}) = (73.2^{+6.3}_{-7.0})^\circ$$

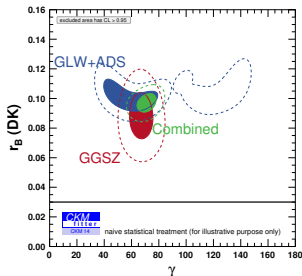
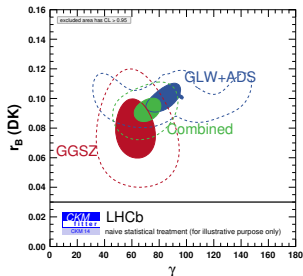
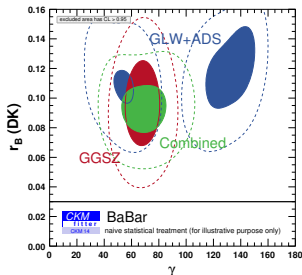
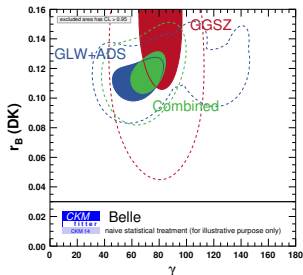
Compared to the prediction from the global CKM fit (not including these measurements):

$$\gamma(\text{fit}) = (66.9^{+1.0}_{-3.7})^\circ$$

# Separated by experiment



# Results for $r_B$ vs. $\gamma$



$$\gamma = (73.2^{+6.3}_{-7.0})^\circ$$

$$r_B = (0.097 \pm 0.006)$$

$$\delta_B = (125.4^{+7.0}_{-7.8})$$

# Prospects @ Belle II

All methods reproducible at Belle II

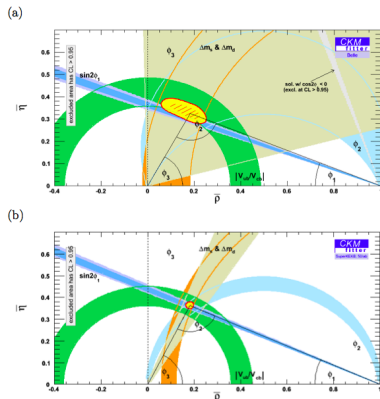
- Improvements in PID and  $q\bar{q}$  suppression using neural networks *Nucl. Instrum. Meth. A654: 432 (2011)*
- Systematic errors from peaking charmless background, and PDFs from  $D\pi$  and sidebands will decrease with statistics.
- Elimination of  $D$  model uncertainty using samples of neutral  $D$  mesons decaying into  $CP$  eigenstates from charm factories CLEO-c and BESIII (via  $\psi(3770) \rightarrow DD$ ).

⇒ *Naive scaling of combination with ADS and GLW yields an error of  $1.5^\circ$ .*

Physics at Super B Factory, arXiv:1002.5012 (2010)

Much more!

- Statistical error will be dominant and can be improved by including  $D$  decays to, e.g.,  $K_S^0 K^+ K^-$ ,  $\pi^+ \pi^- \pi^0$ ,  $K_S^0 \pi^+ \pi^- \pi^0$  ( $2 * \mathcal{B}(K_S^0 \pi^+ \pi^-)$ !).
- Use  $D\pi$  in addition to  $DK$



(a) Belle at  $0.5 \text{ ab}^{-1}$  and (b) Belle II at  $50 \text{ ab}^{-1}$

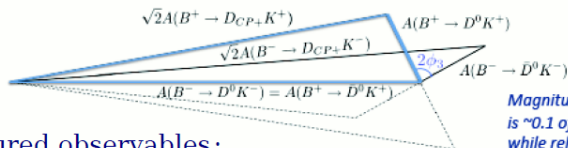
## Extra material - GLW Method

# Cabibbo-suppressed $D$ decays to $CP$ -eigenstates (GLW)

## GLW with $D_{CP}^{(*)}K$

## D decays to CP eigenstates

➤ Amplitude triangle:



*Magnitude of one side is  $\sim 0.1$  of the others while relative magnitude of the others help  $\phi_3$  constraint.*

measured observables:

$$R_{CP\pm} \equiv \frac{\text{Br}(B^- \rightarrow D_{CP\pm} K^-) + \text{Br}(B^+ \rightarrow D_{CP\pm} K^+)}{\text{Br}(B^- \rightarrow D^0 K^-) + \text{Br}(B^+ \rightarrow \bar{D}^0 K^+)}$$

$$A_{CP\pm} \equiv \frac{\text{Br}(B^- \rightarrow D_{CP\pm} K^-) - \text{Br}(B^+ \rightarrow D_{CP\pm} K^+)}{\text{Br}(B^- \rightarrow D_{CP\pm} K^-) + \text{Br}(B^+ \rightarrow D_{CP\pm} K^+)}$$

Relation between  $(A_{CP+}, A_{CP-}, R_{CP+}, R_{CP-})$  and  $(\gamma, r_B, \delta_B)$

$$A_{CP+} = \frac{+2r_B \sin \delta_B \sin \gamma}{1 + r_B^2 + 2r_B \cos \delta_B \cos \gamma}$$

$$A_{CP-} = \frac{-2r_B \sin \delta_B \sin \gamma}{1 + r_B^2 - 2r_B \cos \delta_B \cos \gamma}$$

$$R_{CP+} = 1 + r_B^2 + 2r_B \cos \delta_B \cos \gamma$$

$$R_{CP-} = 1 + r_B^2 - 2r_B \cos \delta_B \cos \gamma$$

⇒ look for  $R_{CP\pm} \neq 1$  and  $A_{CP\pm} \neq 0$

⇒  $\neq CP$ ,  $\neq$  sign of asymmetry

# Cabibbo-suppressed $D$ decays to $CP$ -eigenstates (GLW)

$$\underline{B \rightarrow Dh, D \rightarrow K^+ K^-, \pi^+ \pi^- \rightarrow R_+}$$

Preliminary  
LP 2011

(772 MB $\bar{B}$ )

$B \rightarrow D\pi$

$B \rightarrow DK$

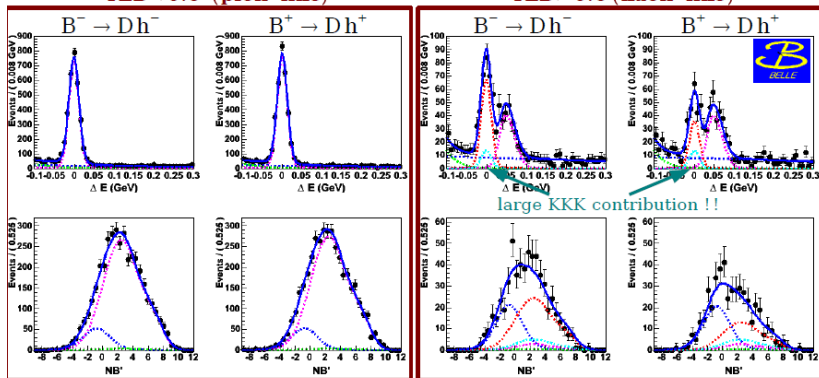
$B\bar{B}$

continuum

Yields	$B \rightarrow D\pi$	$B \rightarrow DK$	$\Rightarrow R_{D_{CP^+}} = (7.32 \pm 0.16)\%$
$D \rightarrow K\pi$	$50432 \pm 243$	$3692 \pm 83$	$A(DK) = (1.4 \pm 2.0)\%$
$D \rightarrow KK, \pi\pi$	$7696 \pm 106$	$582 \pm 40$	

**KID < 0.6 (pion-like)**

**KID > 0.6 (kaon-like)**



$$\Rightarrow R_{D_{CP^+}} = (7.56 \pm 0.51)\%, A_{D_{CP^+}} = (28.7 \pm 6.0)\%$$

large asymmetry !!

# Cabibbo-suppressed $D$ decays to $CP$ -eigenstates (GLW)

$$\underline{B \rightarrow Dh, D \rightarrow K_S \pi^0, K_S \eta \rightarrow R_-}$$

Preliminary  
LP 2011

(772 MB $\bar{B}$ )

$B \rightarrow D\pi$

$B \rightarrow DK$

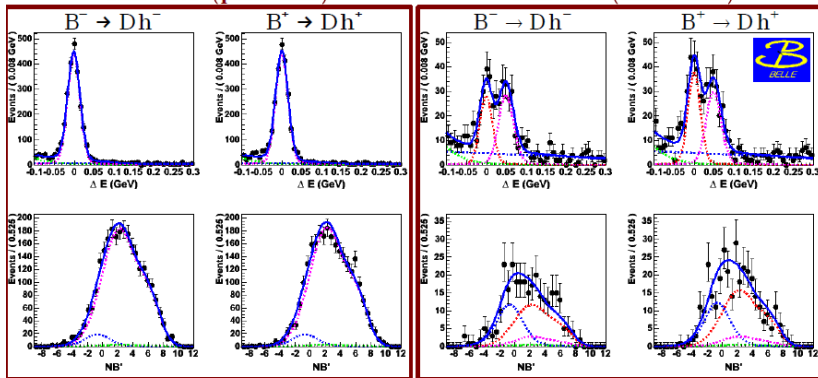
$B\bar{B}$

continuum

Yields  $B \rightarrow D\pi$   $B \rightarrow DK$   
 $D \rightarrow K_S \pi^0, K_S \eta$   $5745 \pm 91$   $476 \pm 37$

KID < 0.6 (pion-like)

KID > 0.6 (kaon-like)



$$\Rightarrow R_{D_{CP^-}} = (8.29 \pm 0.63)\%, \quad A_{D_{CP^-}} = (-12.4 \pm 6.4)\%$$

opposite asymmetry !!



# Summary of GLW results

## GLW Results

Preliminary (LP 2011)

$$R_{CP+} = 1.03 \pm 0.07 \pm 0.03$$

$$R_{CP-} = 1.13 \pm 0.09 \pm 0.05$$

$$A_{CP+} = +0.29 \pm 0.06 \pm 0.02$$

$$A_{CP-} = -0.12 \pm 0.06 \pm 0.01$$

CP-odd observables  
only available at B-factories

(systematics dominated by peaking background, double ratio approximation)

