V2 – Intro continued, experimental basics

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Institut für Experimentelle Teilchenphysik

Web page and links

Teilchenphysik II: W, Z, Higgs am Collider

Karlsruher Institut für Technologie

Sommersemester 2023 (4022161)

Institut für Experimentelle Teilchenphysik

Die Vorlesung Teilchenphysik II - W, Z, Higgs am Collider ist eine vertiefende Vorlesung im Rahmen des Masterstudiums Physik. Die Vorlesung kann als Teil eines Schwerpunkts- bzw. Ergänzungsfachs im Bereich Experimentelle Teilchenphysik verwendet werden.
Content: Historic introduction, electroweak symmetry breaking in the Standard Model, experimental techniques and modern methods of statistical data analysis, W and Z boson physics at colliders, properties of the Higgs bosons, search for and discovery of the Higgs boson, multi-boson processes, W/Z/Higgs processes in physics beyond the Standard Model.
Basic knowledge from the bachelor lectures â€œModerne Experimentalphysik IIIâ€, â€œModerne Theoretische Physik IIâ€ and â€œRechnernutzung in der Physikâ€ as well as from the master lecture â€œTeilchenphysik Iâ€œ is assumed.

Lecturers: Priv.-Doz. Dr. Klaus Rabbertz, Dr. Nils Faltermann.
Tutors: Dr. Xunwu Zuo, Rufa Rafeek, Ralf Schmieder

Schedule

Lecture: Fridays, 09:45-11:15h, kl Hörsaal B, see also ILIAS for inscription.
Start: 21.04.2023
Exercises: Following agreement during first lecture; inscription via ILIAS.
Inscription mandatory: 17.04.2023 --- 02.05.2023

- **ILIAS page to the lectures:**
  - **https://ilias.studium.kit.edu/goto.php?target=crs_2073357&client_id=produktiv**
- **ILIAS page to the exercises:**
  - **https://ilias.studium.kit.edu/goto.php?target=crs_2073358&client_id=produktiv**

Preliminary schedule

SS2023	Kalender	TBD	U2	Thema	Fr 09.45 kl. HS B	V2	Themen	Anmerkung
1	16	20. April	-		21. April	V K1	Organisation; Historical intro	
2	17	27. April	-		28. April	V K2	Exp. Basics	
3	18	04. May	-		05. May	V K3	Theory basics I	Mo: 1. Mai
4	19	11. May	E1	Calc 1	12. May	V K4	Theory basics II	
5	20	18. May	E2	Calc 2	19. May	V N1	EWK theory	Do: Himmelfahrt
6	21	25. May	P1	W mass	26. May	V N2	Higgs mechanism	no KR
7	22	01. June	-		02. June	-	-	Mo: Pfingstwoche
8	23	08. June	C1	NN	09. June	V K5	Early EWK measurements (GIM,NC)	Do: Fronleichnam
9	24	15. June	-		16. June	V N3	Stat. Tools for discoveries	no KR
10	25	22. June	C2	Stat.	23. June	V K6	W/Z discovery at SPPS & LEP, HERA	
11	26	29. June	P2	Z0	30. June	V K7	W/Z at the LHC	
12	27	06. July	-		07. July	V N4	Higgs search & discovery	
13	28	13. July	P3	H disc.	14. July	V N5	Higgs properties (CP, width)	
14	29	20. July	P4	H prop.	21. July	V N6	Higgs properties (Couplings, EFT)	
15	30	27. July	C3	limits	28. July	V N7	BSM Higgs	

Change of schedule:

The first exercise is now planned for calendar week 19: 08.-12.05.2023

Exercise schedule

The agreed day and time for the exercises is:

To be determined ASAP.

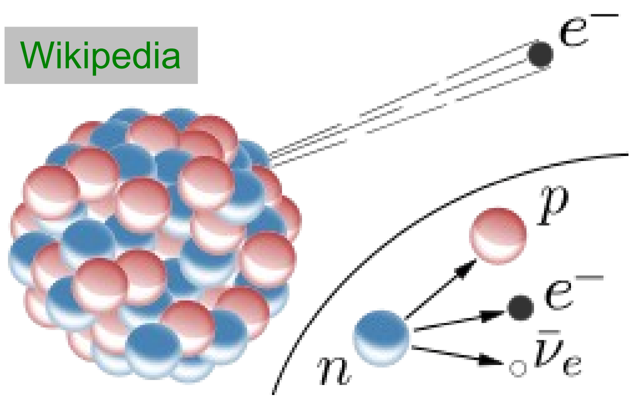
Today: Fast Overview Cont.

- **Weak interaction anthology:**
  - **The path to a unified theory**

Fermi's four-fermion coupling

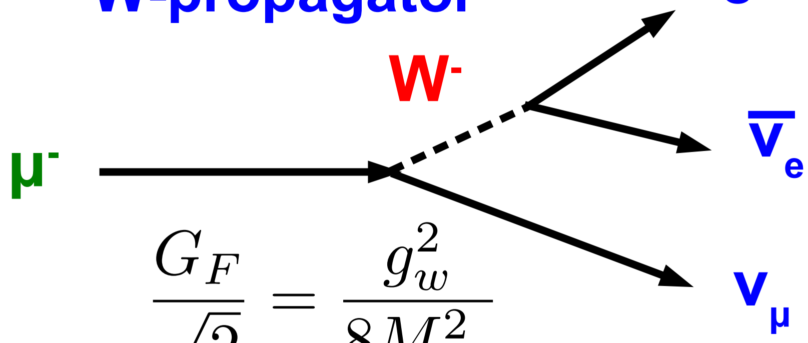
β-decay

Wikipedia



μ-decay

W-propagator



$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$$

Modern interpretation as weak decays!

Versuch einer Theorie der β-Strahlen. I[1]).

Von **E. Fermi** in Rom.

Mit 3 Abbildungen. (Eingegangen am 16. Januar 1934.)

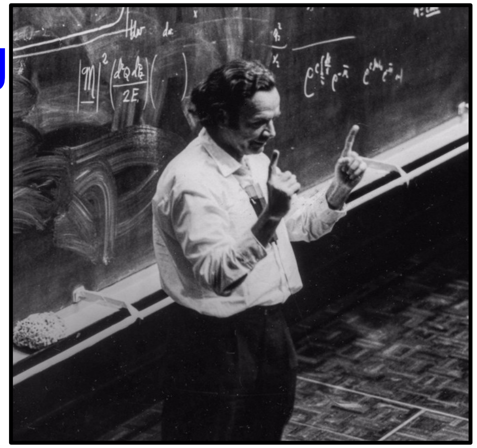
Eine quantitative Theorie des β-Zerfalls wird vorgeschlagen, in welcher man die Existenz des Neutrinos annimmt, und die Emission der Elektronen und Neutrinos aus einem Kern beim β-Zerfall mit einer ähnlichen Methode behandelt, wie die Emission eines Lichtquants aus einem angeregten Atom in der Strahlungstheorie. Formeln für die Lebensdauer und für die Form des emittierten kontinuierlichen β-Strahlenspektrums werden abgeleitet und mit der Erfahrung verglichen.

Fermi, Z. Phys., 1934, 88, 16; Nuovo Cim., 1934, 11, 1

Extension of Fermi theory

- **Extension of Fermi theory to a maximally parity violating theory by R. Feynman & M. Gell-Mann (1958)**

$$\mathcal{H}_{\mathrm{IA}} = \frac{G_{\mathrm{F}}}{2} \int d^3x \, \underbrace{\left(\bar{p}(x)\gamma^{\mu}\left(1-\gamma^5\right) n(x)\right)}_{\text{Proton-Neutron Strom}} \underbrace{\left(\bar{e}(x)\gamma_{\mu}\left(1-\gamma^5\right) \nu(x)\right)}_{\text{Elektron-Neutrino Strom}} + h.c.$$



Richard Feynman

- **Only left-handed fermions and right-handed anti-fermions participate in weak interactions**

- **Unified description of decays**

$$\left.\begin{aligned} n &\to p\,\bar{\nu}_{\mathrm{e}}\,\mathrm{e} \\ \mu^- &\to \mathrm{e}^-\,\bar{\nu}_{\mathrm{e}}\,\nu_{\mu} \\ \pi^- &\to \bar{\nu}_{\mu}\,\mu^- \end{aligned}\right\} \to \text{universality of weak interaction}$$

Explanation of suppressed decay: $\pi^- \to \bar{\nu}_e\,\mathrm{e}^-$

Cabibbo angle

- To explain differences in weak decays, matrix elements are
  - multiplied by sin(θ) for strangeness non-conserving reactions,
  - and by cos(θ) for strangeness conserving ones
  - → one gets: $\vartheta = 13°$

N. Cabbibo, Phys. Rev. Lett. 10, (1963) 531.

Nicola Cabbibo

Weak eigenstates of quarks

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Mass eigenstates of quarks

- Weak interaction of quarks
  - modifies to $g\,\bar{u}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) d' = g\,\bar{u}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) \left(d\,\cos\vartheta + s\,\sin\vartheta\right)$
  - with Cabibbo mixing angle
  - Weak coupling keeps universality

Problems with weak reactions

- Fermi theory corresponds to contact interaction
  - Coupling constant G_F has dimensions $[G_F] = [E]^{-2}$

$$G_F \approx 1.166 \cdot 10^{-5} \mathrm{GeV}^{-2}$$

  - Cross sections grow beyond all bounds

$$\sigma \sim G_F^2 E_{\mathrm{cms}}^2 = G_F^2 \cdot s$$

- Interaction becomes very weak at large distances (low energies)
- Parity conservation is maximally violated
  - Weak reactions differentiate between left- and right-handed particles
- Particles change charge
- Particles change “flavor”

$\longrightarrow$ **How to describe something so different?**

GWS-Theory

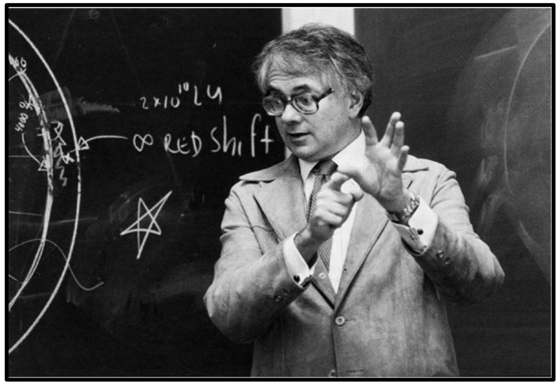
- **Development of unified quantum field theory for weak and electromagnetic interaction**

S. Glashow, Nuc. Phys. B 22, (1961) 579.

S. Weinberg, Phys. Rev. Lett. B 19, (1967) 1264.

A. Salam, Conv. Proc C680915, (1968) 367.

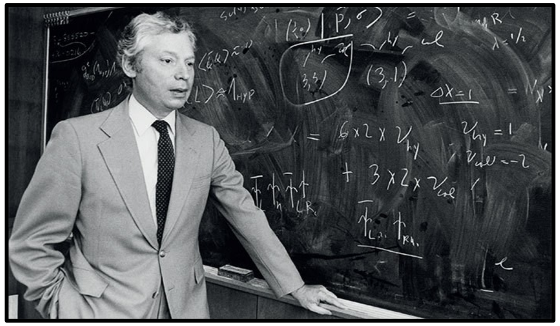
$$SU(2)_L \times U(1)_Y$$



Sheldon Glashow

- **Left-handed electron and neutrino, up and down quarks are isospin partners of of this QFT**
- **Right-handed ones are singletts (but no massless neutrino)**

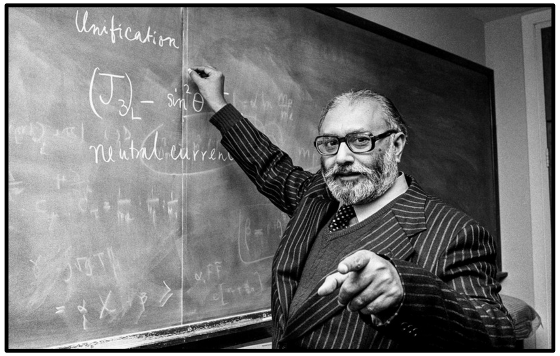
$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L \qquad e_R \qquad \qquad \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R,\, d_R$$



Steven Weinberg

- **In addition to photon fields a triplet of weak exchange fields exist**

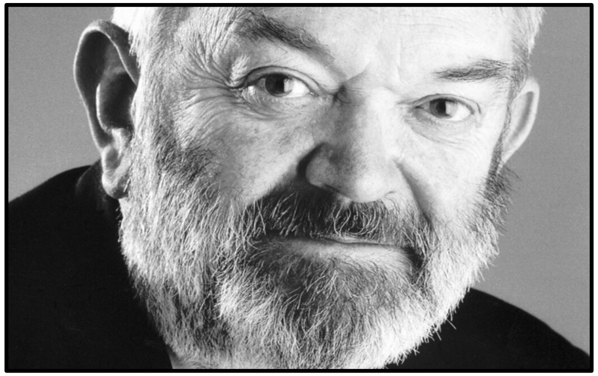
$$B_\mu \qquad \qquad (W_\mu^+, W_\mu^-, W_\mu^{(3)})$$



Abdus Salam

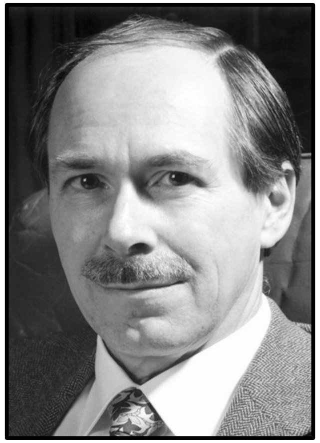
The problem of mass

- Fundamental advantage of gauge bosons
  - QFTs are renormalisable
- Fundamental disadvantage of gauge bosons
  - Bosons must be massless



Martinus Veltman

G. t'Hooft, M. Veltman, Nucl. Phys. B 44, (1972) 189.



Gerard t'Hooft

- Weak exchange bosons must have mass
  - Otherwise must have been observed already
- Assuming equal strengths of elm. and weak interaction
  - Can estimate mass of W boson

$$G_\mathrm{F} \propto \frac{g^2}{m_W^2} \longrightarrow m_W \approx 100\,\mathrm{GeV}$$

Postulate Brout-Englert-Higgs

- Massive exchange particles are introduced into the theory with the help of the Higgs mechanism

F. Englert, R. Brout, Phys. Rev. Lett. 13, (1964) 321.

P. Higgs, Phys. Lett. 12, (1964) 132.

P. Higgs, Phys. Rev. Lett. 13 (1964) 508.

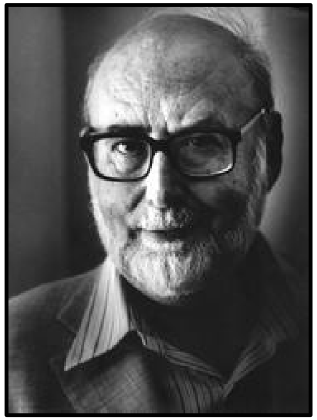
G. Guralnik, et al, Phys. Rev. Lett. 13 (1964) 585..

T. Kibble, Phys. Rev. 155 (1967) 1554.

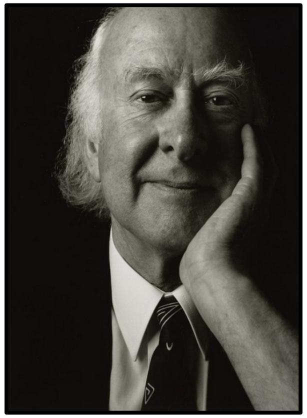
- As a consequence a new field must exist with non-zero vacuum expectation value filling all space
- Particles acquire mass through interaction with this Higgs field
- Excitations of the Higgs field must be visible as a new scalar particle

- For decades no trace of the Higgs boson was found

R. Brout



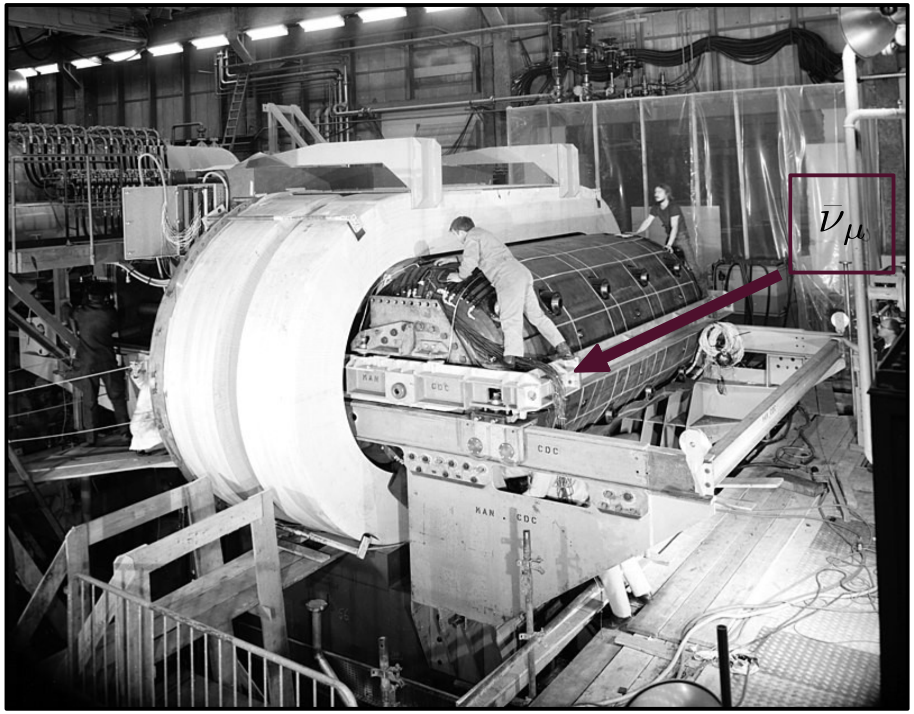
F. Englert



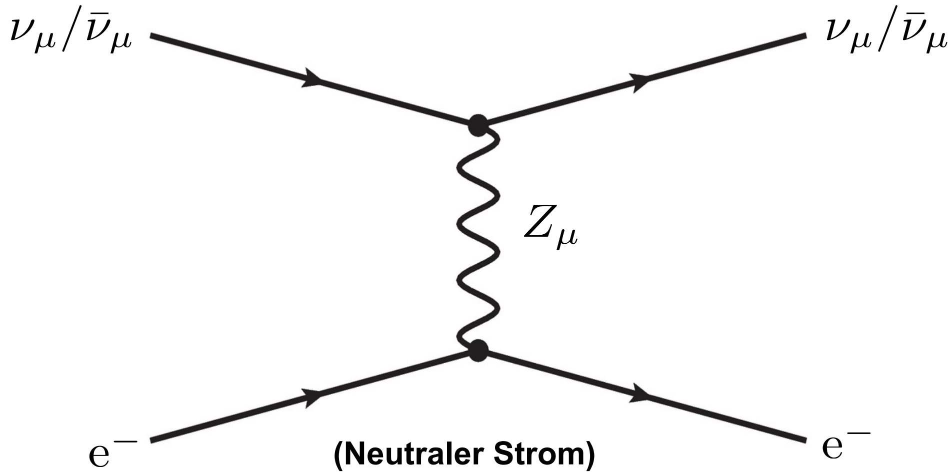
P. Higgs

Discovery of neutral currents

- **Dedicated search with PS at CERN**



Gargamelle Blasenkammer

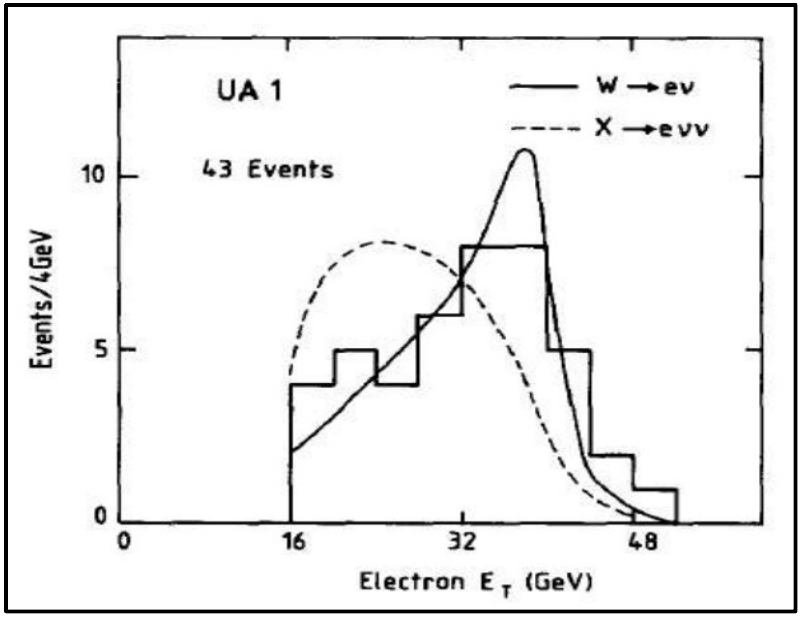


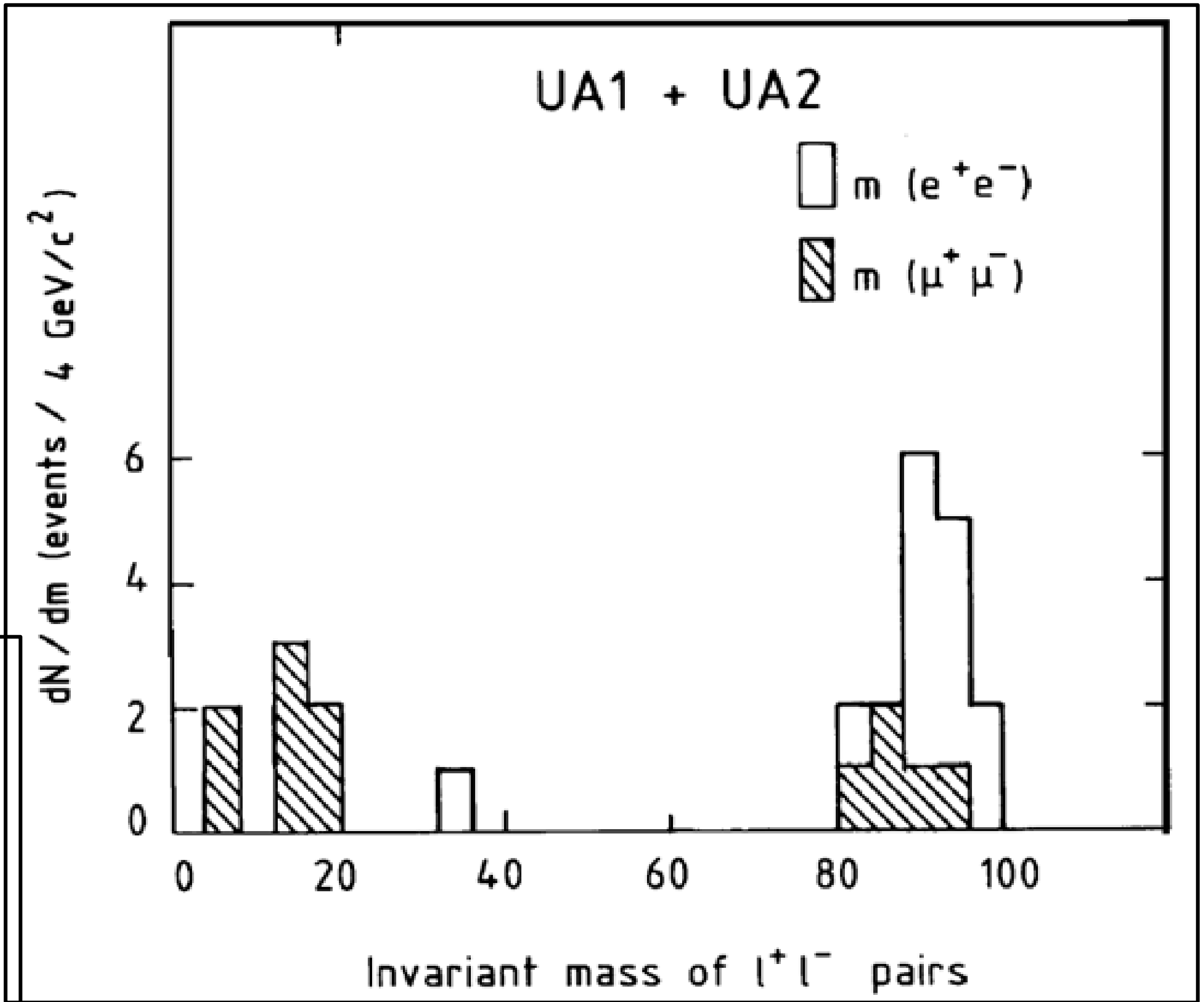
(Neutraler Strom)

Phys. Lett. B 46, (1973) 121.

Phys. Lett. B 46, (1973) 138.

Discovery of W± and Z bosons





- Gefundene Ereignisse:

UA1		UA2	
$W \to e\nu_e$	43	$W \to e\nu_e$	33
$W \to \mu\nu_\nu$	14		
$Z \to ee$	5	$Z \to ee$	4
$Z \to \mu\mu$	5		

$$m_W = 82.1 \pm 1.7 \text{ GeV}$$

$$m_Z = 93.0 \pm 1.7 \text{ GeV}$$

Phys. Lett. B 122 (**1983**) 103.

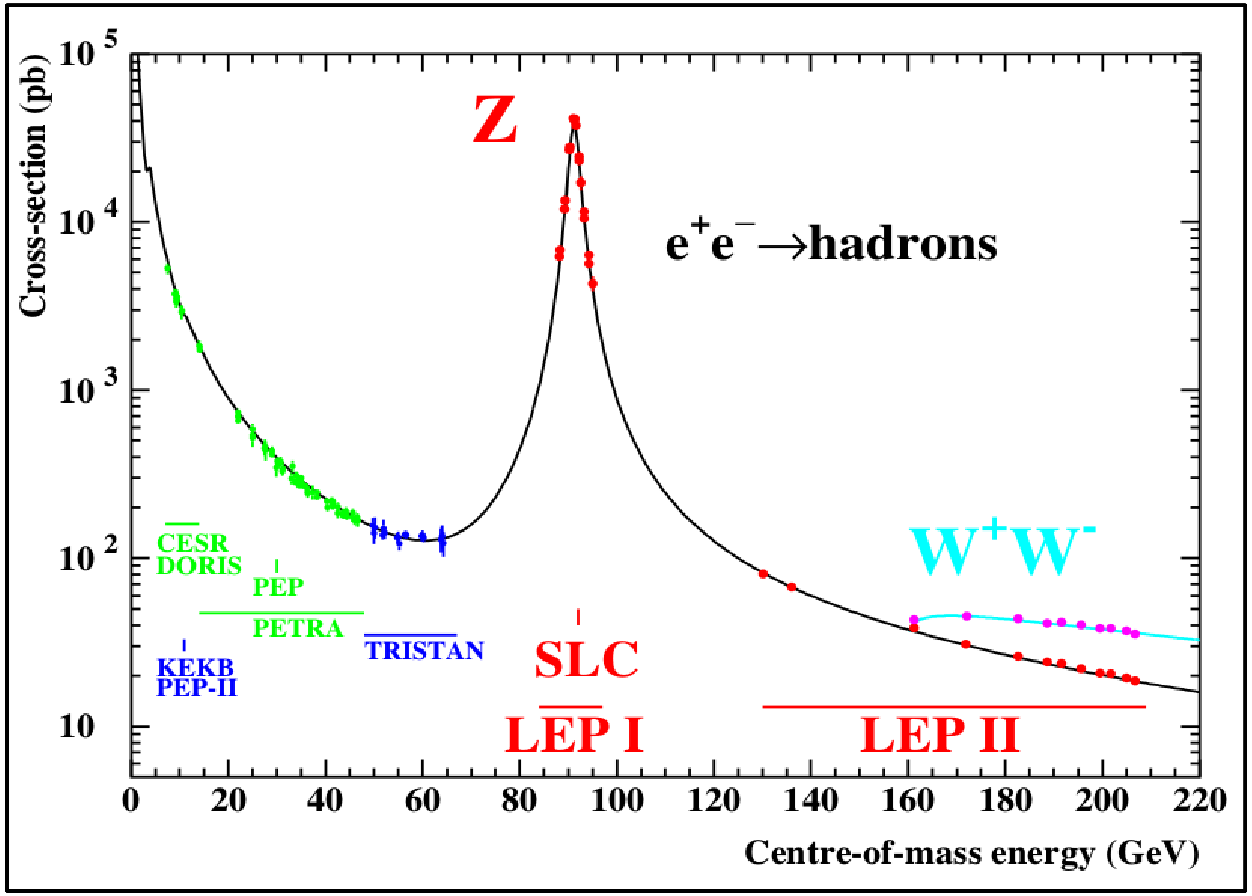
Phys. Lett. B 122 (**1983**) 476.

Phys. Lett. B 126 (**1983**) 398.

Phys. Lett. B 129 (**1983**) 130.

Precision measurements at LEP

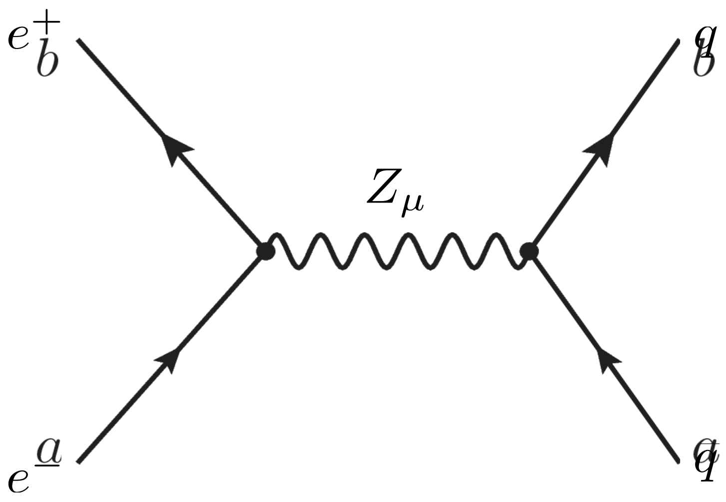
- e+e- collisions at $\sqrt{s} \approx M_Z$ during LEP-I data taking period from 1989 - 1995



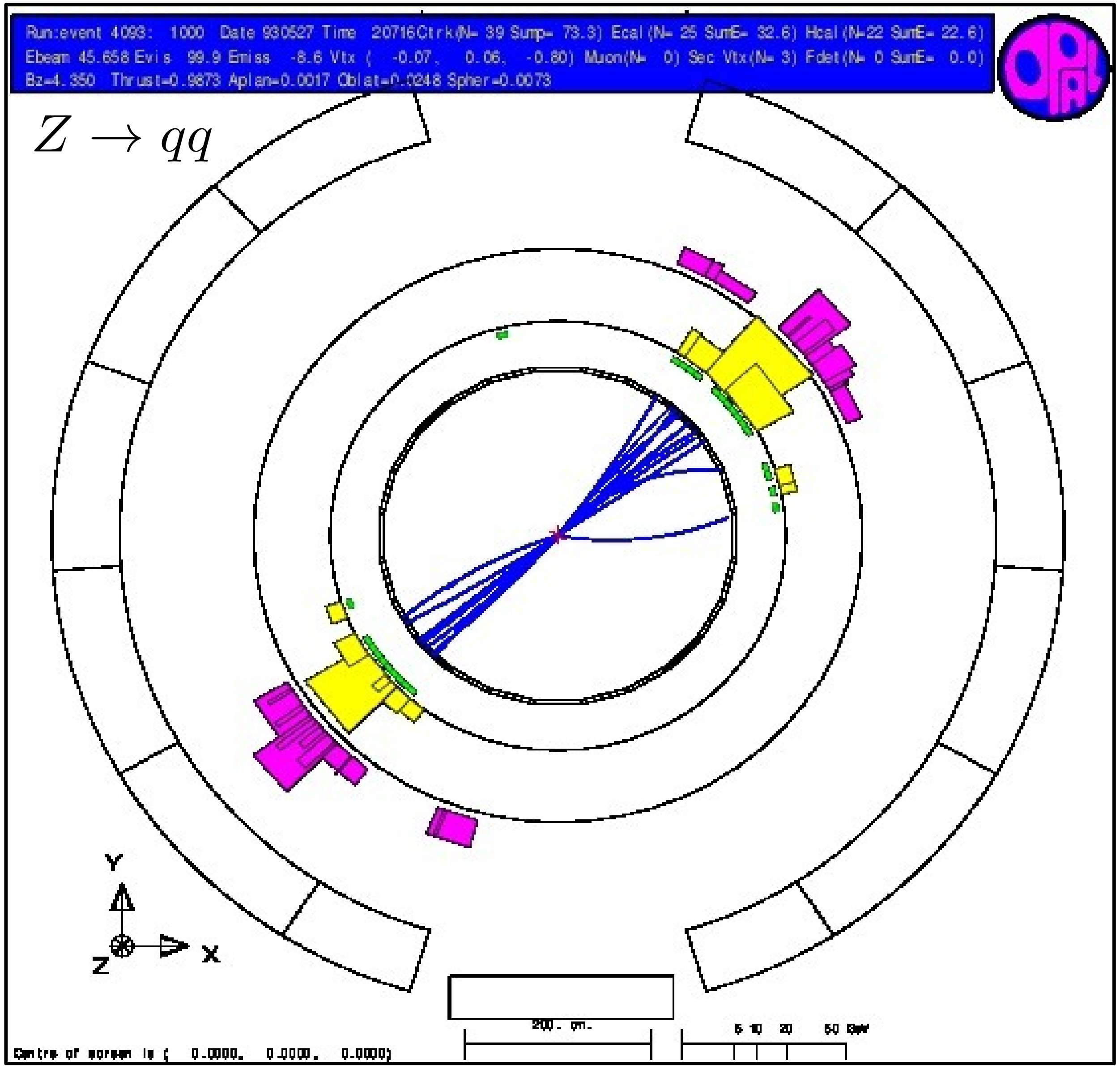
Year	Centre-of-mass energy range [GeV]	Integrated luminosity [pb^{-1}]
1989	88.2 – 94.2	1.7
1990	88.2 – 94.2	8.6
1991	88.5 – 93.7	18.9
1992	91.3	28.6
1993	89.4, 91.2, 93.0	40.0
1994	91.2	64.5
1995	89.4, 91.3, 93.0	39.8
		$\boxed{202.1}$

- $15 \cdot 10^6$ $Z \rightarrow qq$ events
- $1.7 \cdot 10^6$ $Z \rightarrow \ell\ell$ events

Typical Z → qq event at LEP

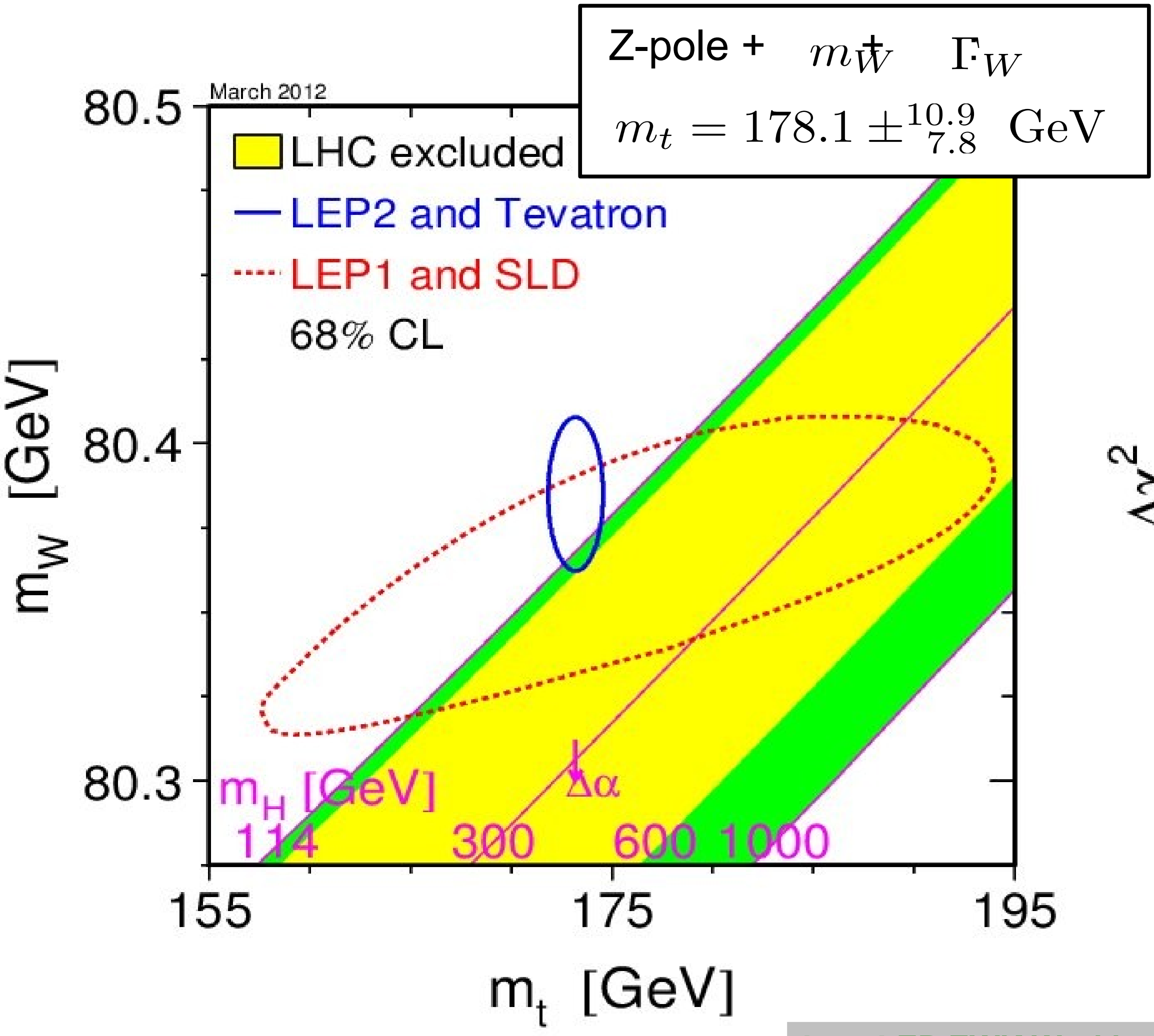


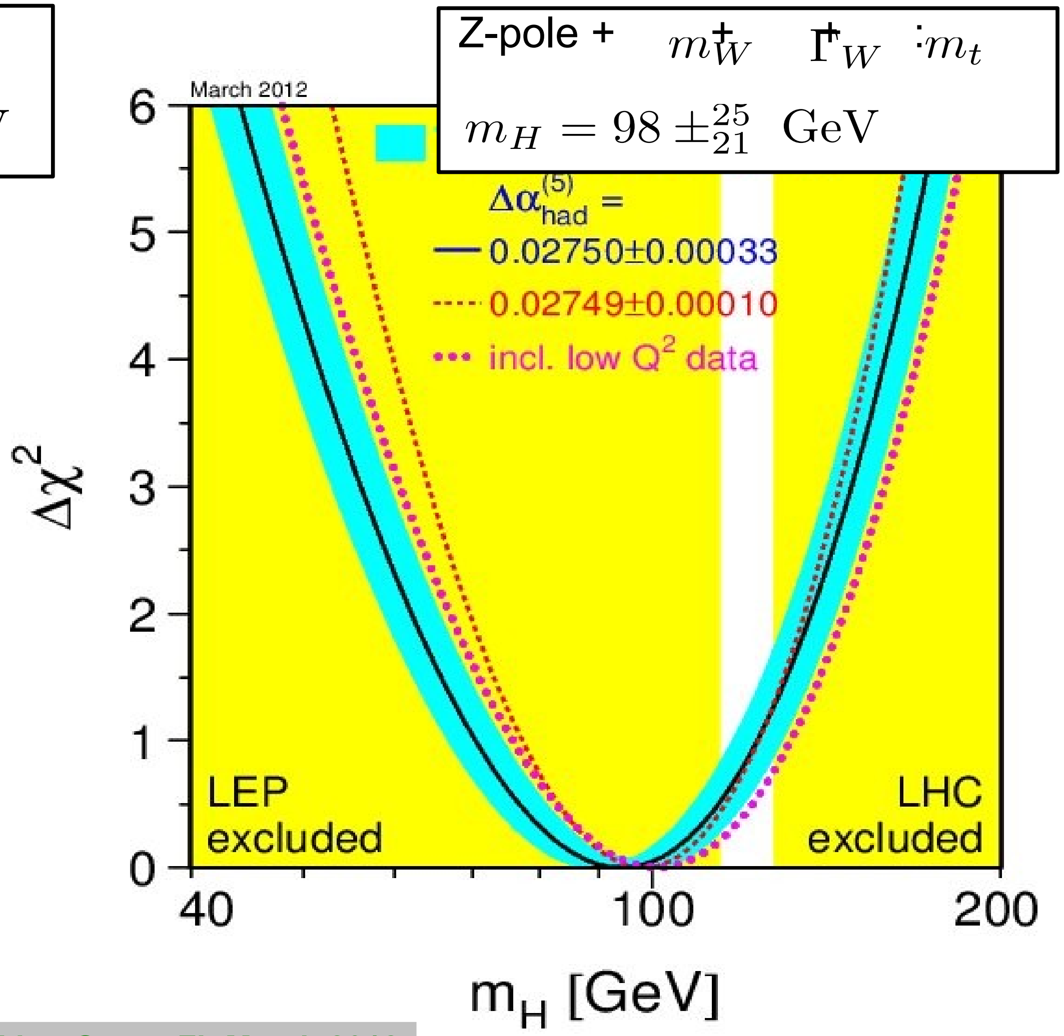
$$Z \to q\bar{q}$$



Constraints on top and H mass

- **Even before discovery, fits of predictions including radiative corrections to LEP-I precision measurements constrained the Higgs mass**

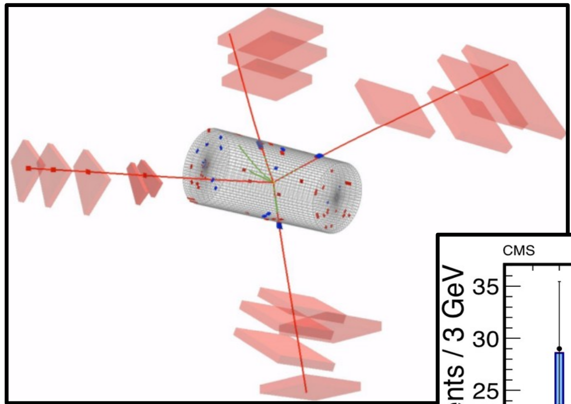




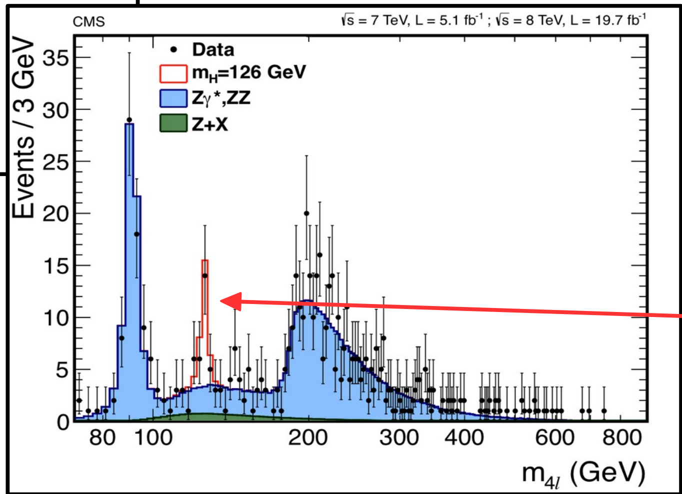
Last LEP EWK Working Group Fit March 2012

Direct search of the Higgs boson

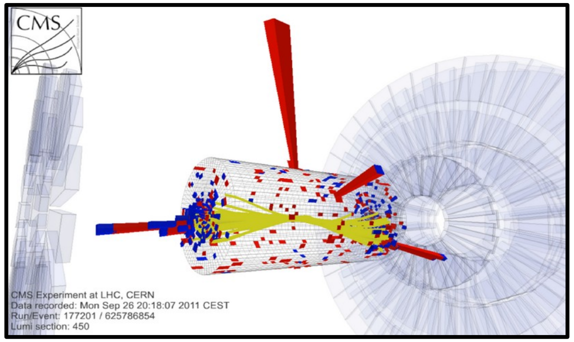
- **Direct searches at LEP (1996-2000) and Tevatron (2001-2012) without success**
- **Discovery of the Higgs remained for LHC 2012**

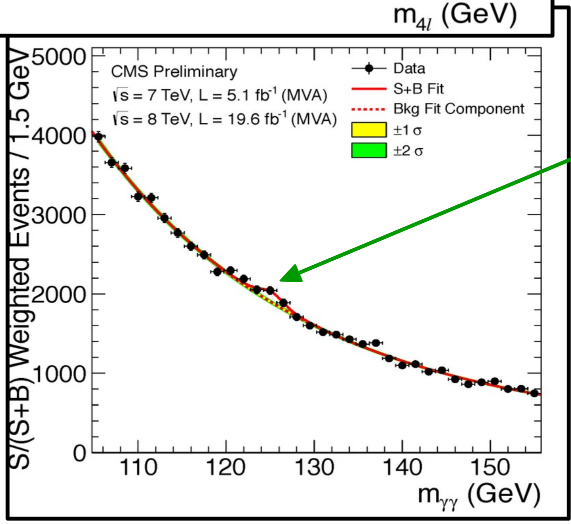


$H \to ZZ^* \to 4\ell$



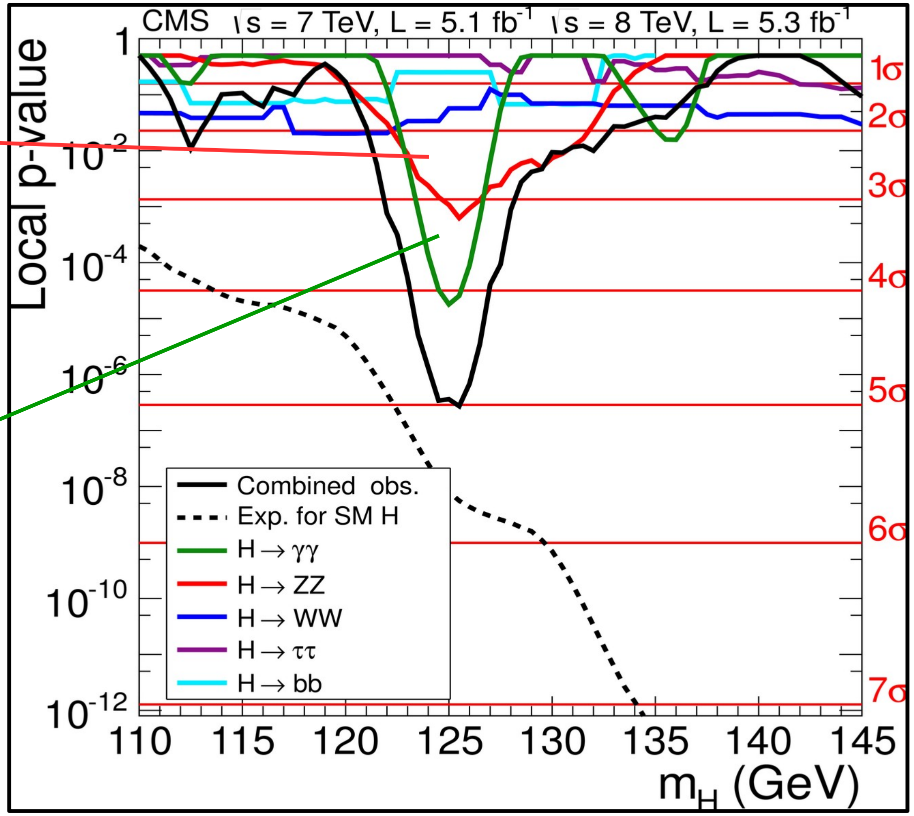
$H \to \gamma\gamma$





PLB 716 (2012) 30

4 Juli 2012

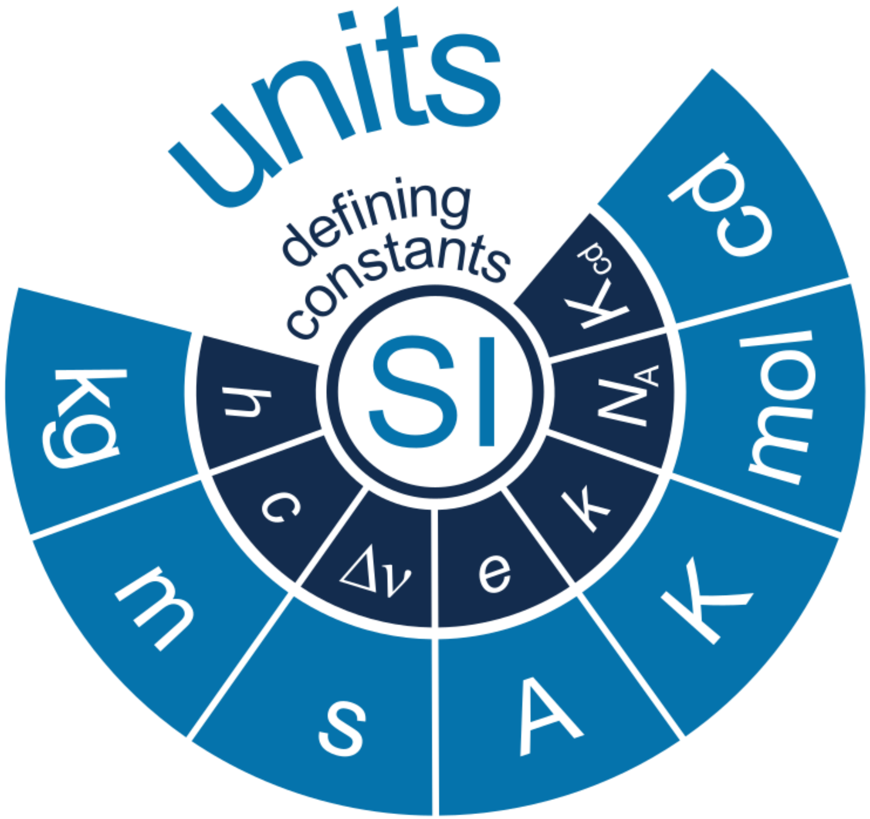


Experimental basics

Definition of SI base units

Like for the “`second`” or “`meter`” all other base units are redefined by fixing the numerical values of seven universally valid constants of nature:
→ No dependence any more of
- local noble persons
- an artifact deposed in Paris
- measures of earth or our solar system

Has been decided in November 2018 in Versailles and is officially in force since World Metrology Day (20. May) 2019.



Bureau International des Poids et Mesures

https://www.bipm.org/

Definition of SI base units

All definitions now based on constants of nature

Sekunde (s)
$1\,\mathrm{s} = 9\,192\,631\,770/\Delta\nu$

Meter (m)
$1\,\mathrm{m} = (c/299\,792\,458)\,\mathrm{s} = 30{,}663\,318\ldots\,c/\Delta\nu$

Kilogramm (kg)
$1\,\mathrm{kg} = (h/6{,}626\,070\,15 \cdot 10^{-34})\,\mathrm{m}^{-2}\,\mathrm{s} = 1{,}475\,521\ldots \cdot 10^{40}\,h\,\Delta\nu/c^2$

Ampere (A)
$1\,\mathrm{A} = e/(1{,}602\,176\,634 \cdot 10^{-19})\,\mathrm{s}^{-1} = 6{,}789\,686\ldots \cdot 10^{8}\,\Delta\nu\,e$

Kelvin (K)
$1\,\mathrm{K} = (1{,}380\,649 \cdot 10^{-23}/k)\,\mathrm{kg}\,\mathrm{m}^2\,\mathrm{s}^{-2} = 2{,}266\,665\ldots\,\Delta\nu\,h/k$

Mol (mol)
$1\,\mathrm{mol} = 6{,}022\,140\,76 \cdot 10^{23}/N_A$

Candela (cd)
$1\,\mathrm{cd} = (K_{cd}/683)\,\mathrm{kg}\,\mathrm{m}^2\,\mathrm{s}^{-3}\,\mathrm{sr}^{-1} = 2{,}614\,830\ldots \cdot 10^{10}\,(\Delta\nu)^2\,h\,K_{cd}$

"Natural" units: $\hbar = c = 1$

Energy, momentum, mass have the same unit, e.g. elektronvolt eV:

$$[E] = [p] = [m] = \mathrm{eV}$$

$$1\mathrm{eV} = e \cdot 1\mathrm{Volt} = 1.6 \cdot 10^{-19}\mathrm{J}$$

The rel. energy-momentum relation simply gets:

$$\boxed{E^2 = p^2 + m^2}$$

Length and time can be expressed in units of 1/eV:

$$\hbar\, c = 1 \approx 200\,\mathrm{MeV} \cdot \mathrm{fm}$$

$$E = \hbar\omega \to \omega \qquad \vec{p} = \hbar\vec{k} \to \vec{k}$$

$$\Rightarrow \Delta E \cdot \Delta t = 1 \Rightarrow [t] = \mathrm{eV}^{-1}$$

$$\Rightarrow \Delta p \cdot \Delta x = 1 \Rightarrow [x] = \mathrm{eV}^{-1}$$

Areas correspond to units in 1/eV²:

$$(\hbar\, c)^2 = 1 \approx 40000\,\mathrm{MeV}^2 \cdot \mathrm{fm}^2$$

Proton radius squared

$$1\mathrm{fm}^2 \approx \frac{1}{40000\,\mathrm{MeV}^2} = \frac{25}{\mathrm{GeV}^2}$$

One exception!

"Natural" units: $\hbar = c = 1$

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Areas correspond to units in 1/eV²:

$$(\hbar\, c)^2 = 1 \approx 40000\,\mathrm{MeV}^2 \cdot \mathrm{fm}^2$$

$$1\mathrm{b} = 10^{-28} m^2$$

Proton radius squared $$1\mathrm{fm}^2 \approx \frac{1}{40000\,\mathrm{MeV}^2} = \frac{25}{\mathrm{GeV}^2} = 0.01\mathrm{barn}$$

Barn = Scheune

$$\frac{1}{\mathrm{GeV}^2} \approx 0.4\mathrm{mb}$$

Secret name for nuclear unit in 2nd World War! → Los Alamos
Also proverb: "... couldn't hit the broad side of a barn ..."
→ "... könnte nicht mal'n Scheunentor treffen ..."

Natural units

Previously:

- Useful ***identities***

$$\hbar = 6.6 \cdot 10^{-25}\,\text{GeV s} \quad \rightarrow \quad 1\,\text{GeV}^{-1} \approx 6.6 \cdot 10^{-25}\,\text{s}$$

$$\hbar c = 197\,\text{MeV fm} \quad \rightarrow \quad 1\,\text{fm} \approx 5\,\text{GeV}^{-1}$$

Also: Rydberg energy (ionisation energy of hydrogen):

$$E = -\frac{m_e e^4}{2(4\pi\epsilon_0)^2\hbar^2} = -\frac{1}{2}m_e\alpha^2 = -\frac{1}{2}511\text{keV}/137^2 = -13.6\,\text{eV}$$

Here, instead of elementary charge e resp. e^2 the dimensionless fine structure constant α appears that later becomes the (running) coupling constant of the elektromagnetic interaction:

$$\alpha = e^2/4\pi\epsilon_0\hbar c$$

Similarly, we'll encounter the strong coupling constant! α_S

Natural units: $\hbar = c = k = 1$

System comparison:

$[k] = J\,/\,K$

Quantity	nuclear/particle physics unit	natural unit
$\hbar$	GeV·s	1
c	[m/s]	1
energy	GeV	GeV
mass	GeV/c^2	GeV
temperature	K	GeV
time	s	GeV^{-1}
length	m	GeV^{-1}

Notations

- **Vectors** **3-vector:** $x^i = \vec{x}$ $(i = 1, 2, 3)$

  4-vector: $x^\mu = (t, \vec{x})$ $(\mu = 0, 1, 2, 3)$

- **Contravariant x^μ and covariant x_μ representation connected via metric tensor:**

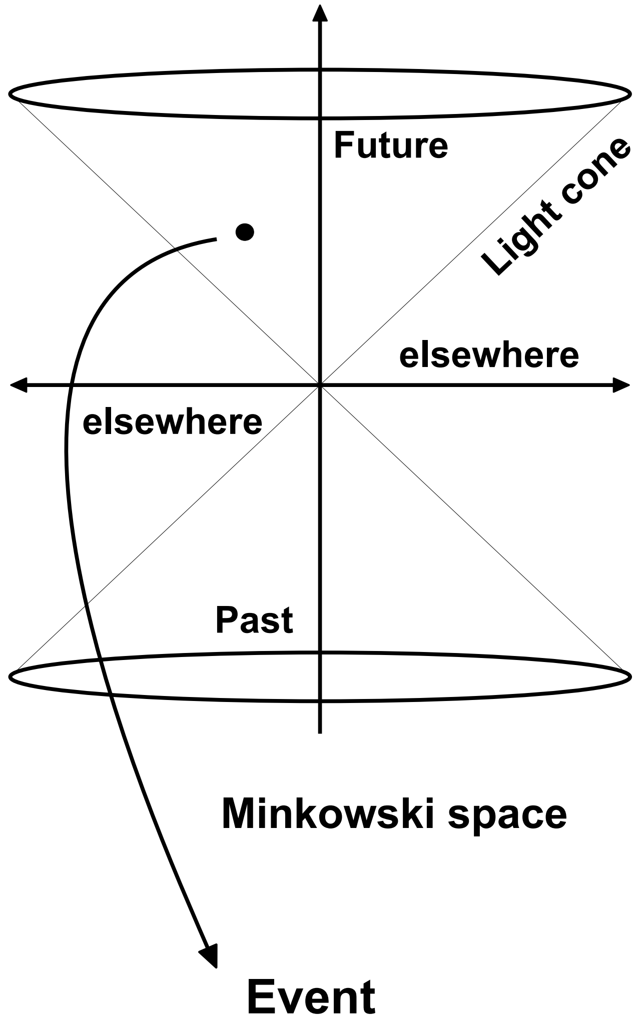
$$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad x_\mu = g_{\mu\nu} x^\nu \equiv \sum_{\nu=0}^{3} g_{\mu\nu} x^\nu$$

- **Einstein convention: Summation over identical indices is implied!**
  - **Lowercase Roman indices:** **a, b, c, … = 1, 2, 3**
  - **Lowercase Greek indices:** **μ, ν, ρ, … = 0, 1, 2, 3**
  - **Uppercase Roman indices:** **A, B, C, … = 1, 2, … , 8**

Special relativity is based on the observation that c = constant in all inertial systems

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}\vec{x}^2 = \mathrm{d}t'^2 - \mathrm{d}\vec{x}'^2 = const$$

Distance between two events



Space and time are grouped into one 4-vector (Lorentz-vector):

$$x_\mu = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \qquad g_{\mu\nu} = g^{\mu\nu} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\mathrm{d}s^2 = \mathrm{d}x_\mu \mathrm{d}x^\mu = \mathrm{d}x_\mu g^{\mu\nu} \mathrm{d}x_\nu$$

Minkowski metric

Lorentz-Transformation

Lorentz-Transformation, e.g. boost along the z axis from S → S'

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \cdot \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma\, t - \gamma\beta\, z \\ x \\ y \\ \gamma\, z - \gamma\beta\, t \end{pmatrix}$$

$$\beta \in [0,\, 1]\, (\text{velocity} S' \text{ relative to } S)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

The distance of an event to the origin is invariant under Lorentz-transformations:

$$x'_\mu x^{\mu\prime} = \text{?}$$

Lorentz-Transformation, e.g. boost along the z axis from S → S'

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The distance of an event to the origin is invariant under Lorentz-transformations:

$$\begin{aligned} x'_\mu x^{\mu\prime} &= \\ &= \gamma^2 \left(t^2 - 2\, t\, \beta\, z + \beta^2 z^2\right) - x^2 - y^2 - \gamma^2 \left(z^2 - 2\, t\, \beta\, z + \beta^2 t^2\right) \\ &= \gamma^2 \left(1 - \beta^2\right) \left(t^2 - z^2\right) - x^2 - y^2 = t^2 - x^2 - y^2 - z^2 \\ &= x_\mu x^\mu \end{aligned}$$

Further Lorentz-scalars & -vectors

Lorentz-scalars

Eigen time:

$$\mathrm{d}\tau = \mathrm{d}s' = \sqrt{\mathrm{d}t'^2 - \mathrm{d}\vec{x}'^2} = \frac{\mathrm{d}t'}{\gamma}$$

Mass:

$$p'_\mu p^{\mu\prime} = \gamma^2 \left(m^2 - \vec{p}^2\right) = E^2 - p^2$$

Mass in rest frame:

$$p'_\mu p^{\mu\prime} = E^2 - p^2 = m^2$$

Lorentz-vectors

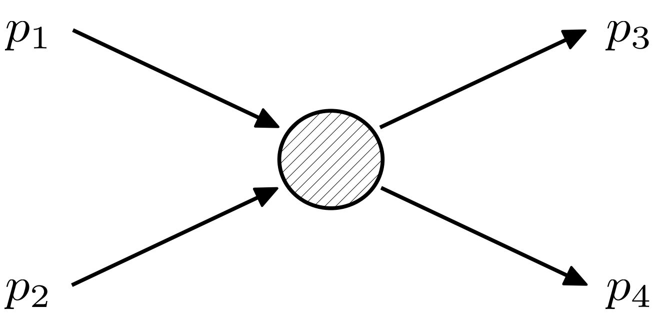
4-velocity:

$$u_\mu = \frac{\mathrm{d}x_\mu}{\mathrm{d}\tau} = \gamma \begin{pmatrix} 1 \\ v_x \\ v_y \\ v_z \end{pmatrix}$$

4-momentum:

$$p_\mu = m\, u_\mu = \gamma \begin{pmatrix} m \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

Mandelstam variables



Elastic scattering between two particles is fully determined by three independent variables, e.g. center-of-mass energy, polar and azimuthal scattering angle in center-of-mass system.

Elastic scatter can also be described by Lorentz-invariant quantities:

$$p_1^\mu + p_2^\mu = p_3^\mu + p_4^\mu$$ **(4-momentum conservation)**

$$s = (p_1^\mu + p_2^\mu)^2 = (p_3^\mu + p_4^\mu)^2$$ **(Squared center-of-mass energy)**

$$t = (p_1^\mu - p_3^\mu)^2 = (p_4^\mu - p_2^\mu)^2$$ **(Squared 4-momentum transfer)**

$$u = (p_1^\mu - p_4^\mu)^2 = (p_3^\mu - p_2^\mu)^2$$

Mandelstam variables

Which one is “right” ...

$$E = m\,c^2$$

$$E_0 = m\,c^2$$

... in the spirit of relativity?

$$E = m_0\,c^2$$

$$E_0 = m_0\,c^2$$

The index 0 indicates the rest frame.

Which one is “right” …

Questionnaire in other course:

$$E = m\,c^2$$

$$E_0 = m\,c^2$$

participants: **20**

Options:

- 5 25% E = m c^2
- 5 25% E_0 = m c^2
- 4 20% E = m_0 c^2
- 6 30% E_0 = m_0 c^2

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As a 4-vector product the mass is an invariant → index 0 makes no sense!

The energy is one component of a 4-vector and is not invariant → E_0

$$E = m_0\,c^2$$

$$E_0 = m_0\,c^2$$

The index 0 indicates the rest frame.

Center-of-mass energy

Two particle scattering:

$$s = \left(p_1^\mu + p_2^\mu\right)^2 = p_1^2 + p_2^2 + 2p_1^\mu p_{\mu,2} \approx 2p_1^\mu p_{\mu,2}$$

Center-of-mass energy

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$$\begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \qquad p \qquad p \qquad \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix}$$

7 TeV 7 TeV

$$s = 2 \begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix} = 4E^2$$

$$E_{\mathrm{cms}} = \sqrt{s} = \sqrt{4E^2} = 2E = 14\,\mathrm{TeV}$$

cms="center of mass frame"

Collider: LHC

Center-of-mass energy

Two particle scattering:

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$$\begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \quad \xrightarrow{7\ \mathrm{TeV}} p \qquad p \xleftarrow{7\ \mathrm{TeV}} \quad \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix}$$

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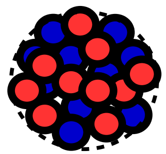
$$\begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \quad \xrightarrow{10^{19}\ \mathrm{eV}} p$$



$${}^{16}_{8}\mathrm{O} \quad \begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$E_{\mathrm{cms}} =$ **?**

$\gamma =$ **?** $\qquad \beta =$ **?**

Collider: LHC

High-energy cosmic rays: 10 Exa eV = 10^{19} eV

Center-of-mass energy

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$$\begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \Rightarrow p \qquad p \Leftarrow \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix}$$

7 TeV 7 TeV

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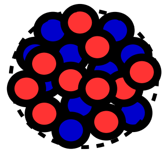
10^{19} eV



$${}^{16}_{8}\mathrm{O} \quad \begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$s = 2 \begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix} = 2EM$$

$$E_{\mathrm{cms}} = \sqrt{s} = \sqrt{2EM} \approx 567\,\mathrm{TeV}$$

$$\sqrt{\frac{EM}{2}} = \gamma M \rightarrow \gamma = \sqrt{\frac{E}{2M}} \approx 17\,678$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \rightarrow \beta = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \approx 0.9999999984$$

Collider: LHC

High-energy cosmic rays: 10 Exa eV = 10^{19} eV

Which collider?

- **Hadron collider (pp or $p\bar{p}$)**
  - Unknown initial state (partons), dense event environment
  - High energies for production of new particles but $\mathcal{O}(10^{-10})$ fraction of signal events over difficult backgrounds $\rightarrow$ discovery machine
- **Lepton collider (e^+e^-)**
  - Known initial state (leptons), clean reconstruction $\rightarrow$ precision meas.
  - Small total cross section, but process of interest with large fraction
  - Limited centre-of-mass energy

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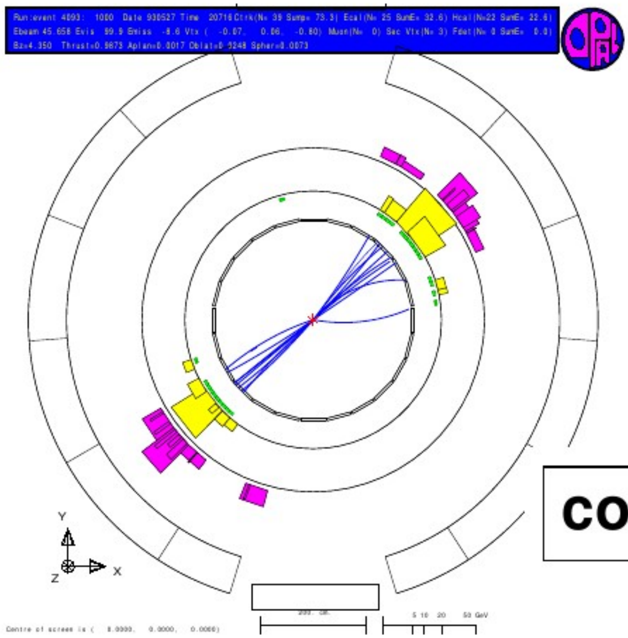

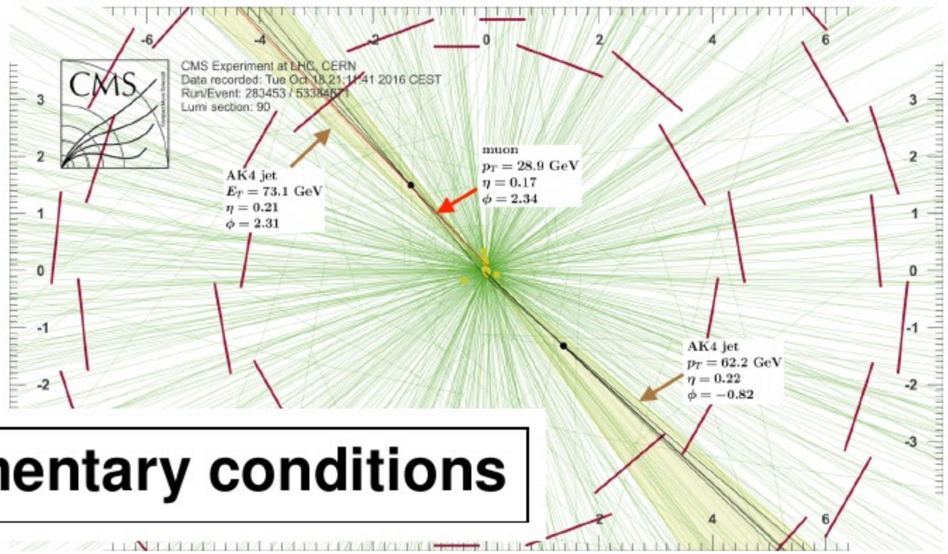

complementary conditions