



V4 – Theoretical basics II, W mass

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- Classical mechanics
 - All information of a physical system is contained in the action

$$\mathbf{S} = \int \mathrm{d}t \, L(\vec{q}, \dot{\vec{q}}, t)$$

 \vec{q} generalised coordinates

Lagrange function L: $L = T - U = (E_{kin} - E_{pot})$

Equations of motion from principle of stationary action, dS = 0:

$$\frac{d}{\mathrm{d}t}\frac{\partial L}{\partial(\partial \dot{q}_i)} - \frac{\partial L}{\partial q_i} = 0$$

Euler-Lagrange equations





- Relativistic quantum mechanics
 - All information is contained in the action integral

 $S = \int dt \int d^3x \mathcal{L}(\phi(x), \partial_{\mu}\phi(x))$ Field $\phi(x)$ here scalar: Separate coordinate at each x (generalisation of canonical coords.) Lagrange density \mathcal{L} $T - U ||_{\delta x} \equiv (E_{\text{kin}} - E_{\text{pot}})|_{\delta x}$ ('Lagrangian'):

Equations of motion from principle of stationary action, dS = 0:

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi(x))} - \frac{\partial \mathcal{L}}{\partial \phi(x)} = 0$$
 Euler-Lagrange equations





Lagrange densities for various fields of particles with mass m

field	Lagrange density \mathcal{L}	equation of motion
scalar field $\phi(x)$ (S = 0)	$rac{1}{2}\left[\left(\partial_{\mu}\phi ight)\left(\partial^{\mu}\phi ight)^{*}-\textit{m}^{2}\phi^{2} ight]$	Klein-Gordon eq.
fermion field $\psi(x)$ (S = $\frac{1}{2}$)	$\overline{\psi}\left(extsf{i}\gamma^{\mu}\partial_{\mu}- extsf{m} ight) \psi$	Dirac eq.
vector field $A_{\mu}(x)$ ($S = 1$)	$-rac{1}{4}m{F}^{\mu u}m{F}_{\mu u}+rac{1}{2}m{m}^2m{A}_\mum{A}^\mu$	Proca eq.

- L has dimension GeV⁻¹
- \mathcal{L} is a Lorentz scalar: Lorentz-invariant without 'free' indices μ



Symmetries



- Global and local symmetries
 - Global: same everywhere
 - Local: varies with x





universe-review.ca



- Example: quantum-mechanical phase
 - Global: physics unchanged for
 - Local: physics unchanged for $\psi(x) \to U(x) \psi(x) = \exp[i\alpha(x)] \psi(x)$
- Connection to group theory
 - Continuous transformations $\psi(x) \rightarrow U\psi(x)$ form Abelian group U(1) under multiplication: Group of unitary transformations
 - Abelian group: U commute, i.e. $[U_i, U_j] = 0$

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 $\psi(x) \to U \,\psi(x) = \exp[i\alpha] \,\psi(x)$

Global phase transformations

- Lagrangians of fermions and bosons are invariant
 - **Global:** The phase α = const is the same for any space-time point x
- Example: Lagrangian of free fermions

$$\psi(x) \to \psi'(x) = e^{i\alpha}\psi(x)$$
$$\overline{\psi}(x) \to \overline{\psi}'(x) = \overline{\psi}(x)e^{-i\alpha}$$

Proof $\begin{aligned} \mathcal{L}' &= \overline{\psi}' (i\gamma^{\mu}\partial_{\mu} - m)\psi' \\ &= \overline{\psi}e^{-i\alpha}(i\gamma^{\mu}\partial_{\mu} - m)e^{i\alpha}\psi \\ &= \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi = \mathcal{L} \end{aligned}$

Local phase transformations



• Let's allow different phases at each point in space-time ... $\alpha = \alpha(x)$?

 \rightarrow physics still should stay invariant!



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Local phase transformations



$$\psi(x) \to \psi'(x) = e^{i\alpha(x)}\psi(x)$$
$$\overline{\psi}(x) \to \overline{\psi}'(x) = \overline{\psi}(x)e^{-i\alpha(x)}$$

Proof:

$$\begin{aligned} \mathcal{L}' &= \overline{\psi}' (i\gamma^{\mu}\partial_{\mu} - m)\psi' \\ &= \overline{\psi}e^{-i\alpha(x)}(i\gamma^{\mu}\partial_{\mu} - m)e^{i\alpha(x)}\psi \\ &= \overline{\psi}(i\gamma^{\mu}\left(\partial_{\mu} + i\partial_{\mu}\alpha(x)\right) - m)\psi \neq \mathcal{L} \end{aligned}$$





Invariance can be restored at the cost of introducing artificially the covariant derivative $\partial_{\mu} \rightarrow D_{\mu} = \partial_{\mu} + iqA_{\mu}$ and an additional gauge field A_{μ} with transformation behaviour:

$$\psi(x) \to \psi'(x) = e^{i\alpha(x)}\psi(x)$$
$$\overline{\psi}(x) \to \overline{\psi}'(x) = \overline{\psi}(x)e^{-i\alpha(x)}$$
$$A_{\mu}(x) \to A_{\mu}'(x) = A_{\mu}(x) - \frac{1}{q}\partial_{\mu}\alpha(x)$$

Proof:

$$\begin{aligned} \mathcal{L}' &= \overline{\psi}'(i\gamma^{\mu}D'_{\mu} - m)\psi' \\ &= \overline{\psi}'(i\gamma^{\mu}\left(\partial_{\mu} + iqA'_{\mu}\right) - m)\psi' \\ &= \overline{\psi}e^{-i\alpha(x)}(i\gamma^{\mu}\left(\partial_{\mu} + iqA_{\mu} - i\partial_{\mu}\alpha(x)\right) - m)e^{i\alpha(x)}\psi \\ &= \overline{\psi}(i\gamma^{\mu}\left(\partial_{\mu} + i\partial_{\mu}\alpha(x) + iqA_{\mu} - i\partial_{\mu}\alpha(x)\right) - m)\psi \\ &= \overline{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi = \mathcal{L} \end{aligned}$$

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The "artificial" gauge field

• Allows arbitrary phase $\alpha(x)$ of $\psi(x)$

Covariant derivative introduces gauge field A





Postulation of local U(1) gauge symmetry — Lagrangian of QED

$$\mathcal{L}_{\text{QED}} = \overline{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$
$$= \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - q(\overline{\psi}\gamma^{\mu}\psi)A_{\mu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

free fermion

interaction photon gauge field

• Euler-Lagrange eq. for
$$\overline{\psi}$$
: $\partial_{\mu} \frac{\partial \mathcal{L}}{\partial(\partial_{\mu} \overline{\psi})} - \frac{\partial \mathcal{L}}{\partial \overline{\psi}} = 0$

Dirac equation for interacting fermion

$$\rightarrow (i\gamma^{\mu}\partial_{\mu} - m)\psi = q\gamma^{\mu}A_{\mu}\psi$$

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Postulation of local U(1) gauge symmetry — Lagrangian of QED

$$\mathcal{L}_{\text{QED}} = \overline{\psi} (i\gamma^{\mu} D_{\mu} - m)\psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$



free fermion

interaction photon gauge field

Euler-Lagrange eq. for
$$A_{\nu}$$
 $\partial_{\mu} \frac{\partial \mathcal{L}}{\partial(\partial_{\mu}A_{\nu})} - \frac{\partial \mathcal{L}}{\partial A_{\nu}} = 0$
 $\partial_{\mu} \frac{\partial \mathcal{L}}{\partial(\partial_{\mu}A_{\nu})} = \partial_{\mu}F^{\mu\nu} = 0$
 $\partial_{\mu} (\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}) = (\partial_{\mu}\partial^{\mu}A^{\nu} - \partial^{\nu}\partial_{\mu}A^{\mu}) = 0$
Lorenz gauge

Proca equation for massless vector boson

$$\rightarrow \left(\partial_{\mu}\partial^{\mu} - 0\right)A^{\nu} = 0$$



• Postulation of local U(1) gauge symmetry \rightarrow Lagrangian of QED

$$\begin{aligned} \mathcal{L}_{\text{QED}} &= \overline{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ &= \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - q(\overline{\psi}\gamma^{\mu}\psi)A_{\mu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ & \text{free fermion} & \text{interaction} & \text{photon gauge field} \end{aligned}$$

- Electromagnetic interaction derived as a consequence of local gauge invariance
- Lagrangian must not have boson mass term $m^2 A_\mu A^\mu$
 - massive gauge bosons break local gauge invariance



Process rate given by (cf. Fermi's gold rule)

dN _	matrix element ²	, phase space
d <i>t</i> _	flux of incoming particles	· phase space

- Dynamics of process encoded in matrix element
 - Element of scattering matrix that transforms initial state into outgoing state ('scattered wave')
- From Lagrange density: rules how to compute matrix element in perturbation theory \rightarrow *Feynman rules*
 - Graphical representation: Feynman graphs



Perturbative series



Process dynamics given by matrix element

$$\mathcal{M}_{\mathit{fi}} = \psi_{\mathit{f}}^{\dagger}\psi_{\mathsf{scat}} = \psi_{\mathit{f}}^{\dagger}\mathcal{S}\psi_{\mathit{i}}$$

- ψ_f : final state after scattering of initial state ψ_i
- E.g. fermion scattering: $\psi_{\rm scat}$ given by solution of inhomogeneous Dirac equation

$$(i\gamma^\mu\partial_\mu-m)\psi_{
m scat}=-e\gamma^\mu A_\mu\psi_{
m scat}$$

Cannot be solved analytically!

Possible to *expand solution* in orders of coupling constant $\alpha \equiv e^2 \ll 1$

$$\psi_{\mathsf{scat}} = \mathcal{S}\psi_i = \left[\sum_{n=0}^{\infty} lpha^n \mathcal{S}_n\right]\psi_i$$

Each term in *perturbation series* associated with distinct process
 S_n can be computed with *Feynman rules*

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- Elements of Feynman rules
 - External lines: incoming/outgoing particles
 - Vertices: coupling between particles
 - Propagators (=internal lines): exchange of virtual particles during scattering process (Green's function of free field equation in momentum space)

For example, $e^+e^-
ightarrow \mu^+\mu^-$ scattering

- → fermion
- → anti-fermion
- √√√ vector boson: y,W,Z
- 00000 gluon
- – - scalar: Higgs
 - vertex









- Symmetries are a basic principle of physics
- Principle of local gauge invariance
 - Postulate of local gauge invariance of the Lagrange density
 → leads to interaction terms with gauge bosons as mediators
- QED: Symmetry under U(1) gauge transformation → photon exchange
- Cross section from Lagrange density
 - Fermi's golden rule: matrix element squared X phase space → cross section
 - Feynman rules:
 - set of rules how to calculate matrix elements
 - can be read off Lagrange density (at leading order ...)
 - represented by Feynman graphs

Solution Control Control of Cont



Modern interpretation as weak decays!

Versuch einer Theorie der β -Strahlen. I¹).

Von E. Fermi in Rom.

Mit 3 Abbildungen. (Eingegangen am 16. Januar 1934.)

Eine quantitative Theorie des β -Zerfalls wird vorgeschlagen, in welcher man die Existenz des Neutrinos annimmt, und die Emission der Elektronen und Neutrinos aus einem Kern beim β -Zerfall mit einer ähnlichen Methode behandelt, wie die Emission eines Lichtquants aus einem angeregten Atom in der Strahlungstheorie. Formeln für die Lebensdauer und für die Form des emittierten kontinuierlichen β -Strahlenspektrums werden abgeleitet und mit der Erfahrung verglichen.

Fermi, Z. Phys., 1934, 88, 16; Nuovo Cim., 1934, 11, 1

Divergence #1: From Fermi's 4-fermion point-coupling

- Coupling constant G_F has dimensions of 1/GeV² $G_F = 1.16639(2) \cdot 10^{-5} \text{GeV}^{-2}$
- Predicted cross section grows as $\sigma \propto G_F^2 E_{
 m cms}^2 = G_F^2 \cdot s$
- Violation of Froissart bound $\sigma < \text{const} \cdot \ln^2(s/[s])$

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- Solution: Exchange of massive vector bosons

$$m_V = O\left(1/\sqrt{G_F}\right) \approx 100 \,\mathrm{GeV}$$



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- Divergence #2: WW pair production in V-A theory
 - Again predicted cross section grows too fast
 - Violation of unitarity (more WW produced than incoming ee flux)



t-channel v exchange

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 - Again predicted cross section grows too fast
 - Violation of unitarity (more WW produced than incoming ee flux)
- Solution 1: Trilinear coupling among W+, W-, and photon γ
 - Triple gauge coupling (TGC) γW⁺W⁻
 - But not sufficient!



- Divergence #2: WW pair production in V-A theory
 - Again predicted cross section grows too fast
 - Violation of unitarity (more WW produced than incoming ee flux)
- Solution 1: Trilinear coupling among W+, W-, and photon γ
 - Triple gauge coupling (TGC) γW⁺W⁻
 - But not sufficient!
- Solution 2: Trilinear coupling also with neutral massive boson Z
 - Triple gauge coupling ZW⁺W⁻
 - Again weak neutral currents!





- **Divergence #3: Longitudinal scattering W_LW_L \rightarrow W_LW_L**
 - Again cross section grows too fast ...





WW quartic and triple gauge boson couplings

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- Divergence #3: Longitudinal scattering $W_LW_L \rightarrow W_LW_L$
 - Again cross section grows too fast ...
- Solution: Additional scattering amplitudes with SCALAR boson
 - Indirect hint to existence of Higgs boson









WW quartic and triple gauge boson couplings

Scalar boson exchanges

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Search for neutral currents (NC) ETE

- Strategy: interactions of neutrinos
 - No electromagnetic processes
 - $\circ~$ Charged-particle scattering: huge background of electromagnetic processes \rightarrow impossible
- o CERN neutrino beam (since 1970)
 - Protons from CERN PS on target: **muon neutrinos**, e.g. $\pi^+ \rightarrow \mu^+ \nu_{\mu}$
 - Detection: bubble chamber experiment Gargamelle



Search for neutral currents (NC) ETE

- Historical perspective nicely described in article to celebrate the 50th anniversary of CERN
 - Theory developments until 1972: Great theory candidate for electroweak interactions → weak neutral currents (NC) must exist!
 - Previous v experiments no weak neutral currents!
 - Two v experiments active but unprepared for NC:
 - Gargamelle at CERN (7 european labs incl. Aachen)
 - HPWF at Fermilab
 - Major challenge: Not designed for NC, events to be found in "background" category B of original measurement plan
 - Upstream v reactions create neutrons entering chamber
 - Versus v entering directly creating NC reaction
 - Discovery reported in 1973
 - HPWF: Disappearance of their signal after changing setup
 - Months of hard work to prove n background estimation ...

D. Haidt in CERN Courier Vol. 44 no. 8, October 2004.

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1954-2004

CERN

Gargamelle bubble chamber





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NC "background" in background ETE

Table 1				
	v-exposure	v-exposure		
No. of neutral-current candidates	102	64		
No. of charged-current candidates	428	148		

Background modeling of neutron-initiated hadronic cascade tested with incident proton beam → proton-initiated hadronic cascades → established overshoot of NC events

D. Haidt in CERN Courier Vol. 44 no. 8, October 2004.



Fig. 3. A 7 GeV proton enters the Gargamelle bubble chamber from below and induces a three-step neutron cascade.

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- **Neutral-current** interaction: scattered electron → electromagnetic shower (bremsstrahlung and e^+e^- production)
- $\circ~$ Charged-current interaction: muon in final state \rightarrow long track

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ve neutral currents





No Nobel prize for experiment





"for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current"

[nobelprize.org]

W and Z boson production



- Production process: lepton production in hadronic interactions ("Drell–Yan process")
- Requirement to produce real W and Z bosons at accelerators: E_{cms} of initial-state fermions at expected boson mass ($\approx 100 \text{ GeV}$)



- $\circ~$ Fixed-target setup: $\sqrt{s}=\sqrt{2m_{\rm p}E}\rightarrow E=5\,{\rm TeV}\rightarrow$ unrealistic
- $\circ~$ Electron-positron collider: $\sqrt{s}=E_1+E_2 \rightarrow 50\,\text{GeV}$ beam energy
 - \rightarrow technically feasible only from the 1990s, only Z production
- Proton-antiproton collider: W/Z production by annihilation of valence quark in proton and valence antiquark in antiproton



SppS collider at CERN



- pp collider reach
 - Momentum fractions of colliding valence (anti)quarks $x_1 \approx x_2 \approx 0.2$
 - \rightarrow estimated $E_{\rm cms}$: $\sqrt{\hat{s}} \approx \sqrt{x_1 x_2 s} \stackrel{!}{=} 100 \,{\rm GeV} \rightarrow \sqrt{s} = 500 \,{\rm GeV}$
- SPS (Super Proton Synchrotron) at CERN
 - o 6.9 km circumference
 - 400 GeV protons

Idea (C. Rubbia, 1976): upgrade to a $p\overline{p}$ collider

SppS

- $\circ E_{cms} = 540 \, GeV$ initially later upgrade to 630 GeV
- UA1 and UA2 experiments data taking from 1981



UA2 experiment



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SppS collider at CERN



• Antiprotons

- p beam on target: approximately
 1 p in 10⁹ final-state particles
- Very large emittance of p beam: reduction without violating Liouville's theorem?
- Solution (S. van der Meer, 1968):
 stochastic cooling¹ with pick-up and kicker
- Further challenges
 - \circ p and \overline{p} share same beam pipe
 - Hermetic 4π detectors for the first time at hadron colliders



¹Details e.g. Nucl. Instrum. Meth. A532 (2004) 11

Solution Of the Wooson (1983)

• Process $p\overline{p} \rightarrow W \rightarrow l\nu \ (l = e, \mu)$



- O Analysis strategy: charged lepton + ∉_T
 - Charged lepton: clean detector signature
 - Neutrino: missing transverse momentum



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 - **2-body decay** W → $I\nu$: lepton and neutrino ($\not{\!\!E}_T$) back-to-back in W-boson rest frame
 - QCD jet production **background**: no preferred relative directions of lepton and ∉_T



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 - QCD jet production background: no preferred relative directions of lepton and ∉_T



40th anniversary of W discovery

- Again nice nistorical perspective in article to celebrate this year the 40th anniversary of W discovery at CERN
 - Must find charged and neutral weak bosons
 - C. Rubbia initiated SppS with second "p" really an anti-proton!
 - Two experiments: UA1 and UA2; January 1983: 4 and 6 candidates
 - No imaginable background



Striking A Wevent recorded by UA1 in late 1982, with a high transverse-momentum electron (pink arrow) and only soft particles in opposite directions to it, as expected if an undetected neutrino balances the electron's transverse momentum.

L. Di Lella in CERN Courier Vol. 63 no. 1, Jan/Feb 2023.

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Solution Discovery of the Z boson (1983) ETE

◦ Process p \overline{p} → Z → I^+I^- ($I = e, \mu$)

Analysis strategy: charged leptons

 l^+

- Invariant mass of lepton pair
- Depends on lepton momenta and opening angle (lepton masses neglected)



$$m_{\mathsf{Z}}^2 pprox \mathbf{2} \cdot |ec{p}_{I^+}| \cdot |ec{p}_{I^-}| \cdot (\mathbf{1} - \cos \phi_{I^+I^-})$$



Candidate event with e⁺e⁻



42





Candidate event with e⁺e⁻



43





Nobel prize 1984





"for their decisive contributions to the large project, which led to the discovery of the field particles W and Z, communicators of weak interaction"

[nobelprize.org]



Summary



- The predicted neutral weak currents have been found in myonneutrino scattering on electrons in the Gargamelle bubble chamber at CERN (1973)
- The discovery of the weak bosons W[±] and Z at the SppS protonantiproton collider at CERN by the UA1 and UA2 experiments has been crowned with a Nobel prize for S. van der Meer and C. Rubbia
- Their masses still need to be explained ...