Karlsruhe Institute of Technology

Institut für Experimentelle Teilchenphysik

Teilchenphysik II - W, Z, Higgs am Collider

Lecture 05: Electroweak Theory

PD Dr. K. Rabbertz, **Dr. Nils Faltermann** | 19. Mai 2023

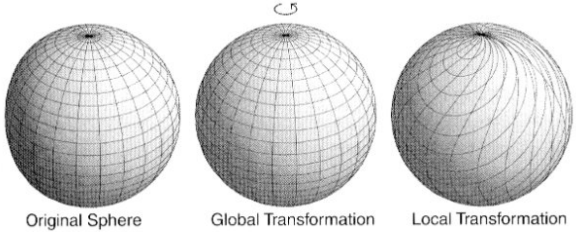
Recap: Symmetries

Noether's theorem:

To each continuous symmetry of a system, there is a conserved quantity.

(Emmy Noether, 1918)

- Standard Model: interactions as consequence of symmetries
- Lagrangian is invariant under global phase transformation
  - BUT: in general **not invariant under local phase transformation**



universe-review.ca

Recap: Lagrangian of QED

$$\mathcal{L}_{\text{QED}} = \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$= \underbrace{\overline{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi}_{\text{free fermion}} - \underbrace{q(\overline{\psi}\gamma^\mu\psi)A_\mu}_{\text{interaction}} - \underbrace{\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{gauge field}}$$

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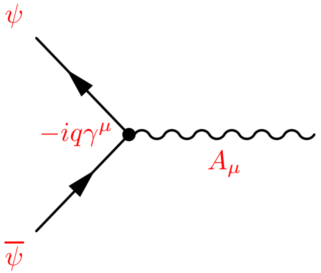
- **Covariant derivative needed for local gauge invariance introduces gauge field A_μ: the photon field**
- Allows arbitrary phase $\alpha(x)$ of $\psi(x)$ at each space-time point x
  - A_μ ‘transports’ this information from point to point (physical: no instantaneous information exchange)
- A_μ couples to property q of spinor field $\psi(x)$
  - q can be identified with electric charge



Yang–Mills Theories

- Extension of the gauge principle to **non-Abelian groups**, i. e. , non-commutative groups
  - Standard Model: in particular SU(2) and SU(3)
- SU(n) transformations $\psi \rightarrow \exp[i\frac{1}{2}g\alpha^a(x)\tau^a]\psi$
  - **$n^2 - 1$ generators τ^a**
  - Non-Abelian algebra $[\tau^a, \tau^b] = if^{abc}\tau^c$ with **structure constants f^{abc}**

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- Analogue to QED: invariance under local SU(n) transformations by introducing covariant derivative and field-strength tensor

$$D_\mu = \partial_\mu + ig\tau^a A^a_\mu$$

with

$$A^a_\mu \to A^a_\mu + \frac{1}{g}\partial_\mu\alpha^a(x) + f^{abc}\alpha^b(x)A^c_\mu$$

$$[D_\mu, D_\nu]^a = igF^a_{\mu\nu}, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

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- **Non-zero structure constants lead to gauge boson self-interaction**
- NB: above relations also hold for U(1)

Example: Invariance Under Local SU(2)

- **Generators: 3 Pauli matrices τ^a** with $f^{abc} = \epsilon^{abc}$
  - Act on isospin doublets, e. g. $\psi = \begin{pmatrix} \nu \\ e \end{pmatrix}$
- 3 vector fields F^a_μ: **3 vector bosons**

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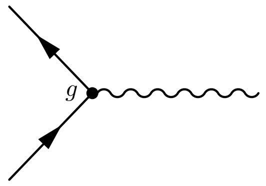
- Lagrangian

$$\mathcal{L}_{\mathrm{SU(2)}} = \overline{\psi}(i\partial\!\!\!/ - m)\psi - \underbrace{g\left(\overline{\psi}\gamma^\mu\tau^a\psi\right)A^a_\mu}_{\text{fermion-boson interaction}}$$



fermion-boson int.

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- Additional terms in field-strength tensor (from non-zero commutator):

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \boxed{g\epsilon^{abc} A^b_\mu A^c_\nu}$$ → vector boson **self-interaction**

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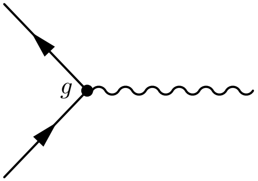

fermion-boson int.

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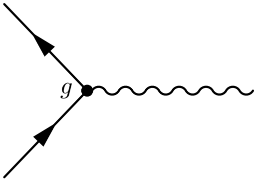
- Lagrangian

fermion-boson interaction; boson propagation and self-interaction

$$\mathcal{L}_{\mathrm{SU(2)}} = \overline{\psi}(i\not{\partial} - m)\psi - \boxed{g\left(\overline{\psi}\gamma^\mu \tau^a \psi\right) A^a_\mu} - \boxed{\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}}$$



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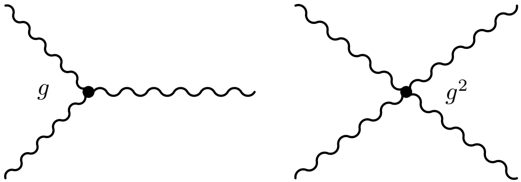

boson self-interaction

Abelian vs. Non-Abelian Gauge Theories

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$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{i\vartheta}\psi(\vec{x},t)$	$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{i\vartheta_a \mathbf{t}_a}\psi(\vec{x},t)$
$\overline{\psi}(\vec{x},t) \to \psi'(\vec{x},t) = \overline{\psi}(\vec{x},t)e^{-i\vartheta}$	$\overline{\psi}(\vec{x},t) \to \psi'(\vec{x},t) = \overline{\psi}(\vec{x},t)e^{-i\vartheta_a \mathbf{t}_a}$
$\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$	$\partial_\mu \to D_\mu = \partial_\mu + igW_{\mu,a}\mathbf{t}_a$
$D_\mu \to D'_\mu = D_\mu - i\partial_\mu\vartheta$	$D_\mu \to D'_\mu = D_\mu + i\left[\vartheta_a \mathbf{t}_a, D_\mu\right]$
$A_\mu \to A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\vartheta$	$W_\mu \to W'_\mu = W_\mu + i\left[\vartheta_a \mathbf{t}_a, W_{\mu,a}\mathbf{t}_a\right] - \frac{1}{g}\partial_\mu\left(\vartheta_a \mathbf{t}_a\right)$
$F_{\mu\nu} \equiv [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu$	$W_{\mu\nu} \equiv [D_\mu, D_\nu] = \partial_\mu W_\nu - \partial_\nu W_\mu + ig\left[W_\mu, W_\nu\right]$
$F_{\mu\nu} \to F'_{\mu\nu} = F_{\mu\nu}$	$W_{\mu\nu} \to W'_{\mu\nu} = W_{\mu\nu} + i\left[\vartheta_a \mathbf{t}_a, W_{\mu\nu}\right]$
$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$	$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{4}W_{a\mu\nu}W^{a\mu\nu}$

The Standard Model

3 (r,g,b) x 6 (L,R)

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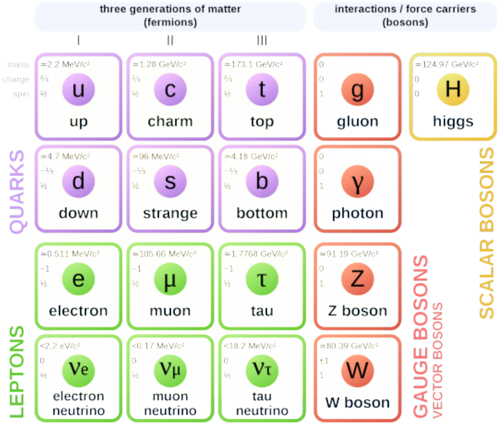
6 (L,R)

3 (L)



8 SU(3)

3+1 $SU(2)_L$ x $U(1)_Y$

45 fermion fields

1 scalar field

12 gauge fields

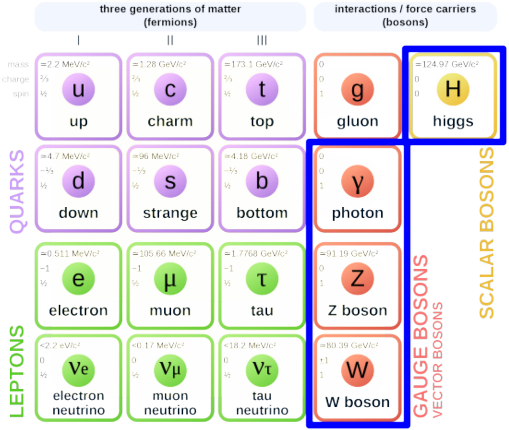
The Standard Model - Electroweak Sector

3 (r,g,b) x 6 (L,R)

3 (r,g,b) x 6 (L,R)

6 (L,R)

3 (L)



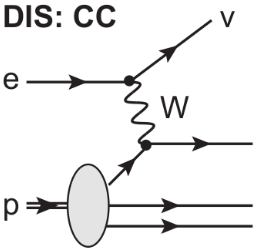
3+1 $SU(2)_L \times U(1)_Y$

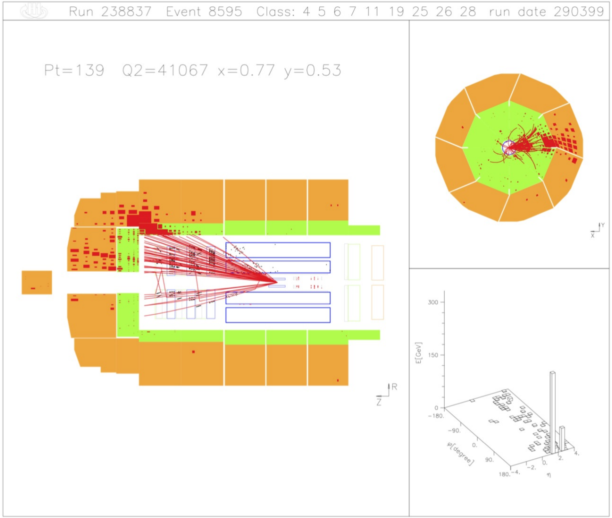
45 fermion fields

1 scalar field

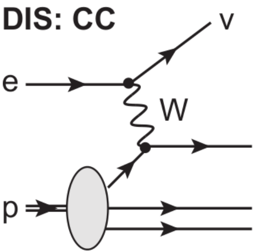
4 gauge fields

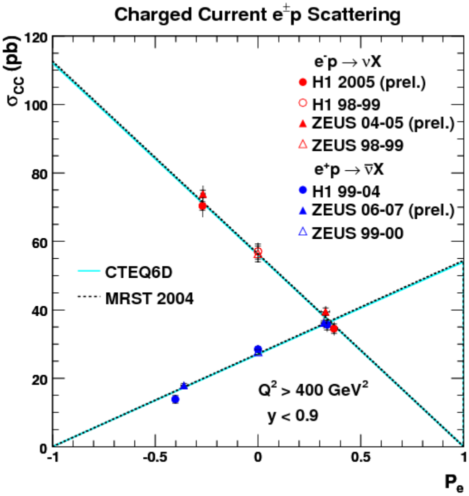
Weak Interaction: Change of Flavour





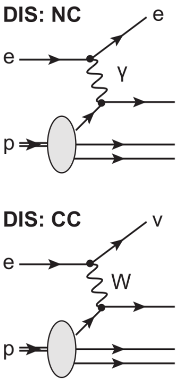
Weak Interaction: Parity Violation





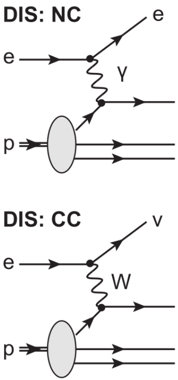
- W bosons couple only to **left-handed particles** (and right-handed antiparticles): weak interaction is **maximally parity violating**
- Also CP violating, e. g. K^0 system

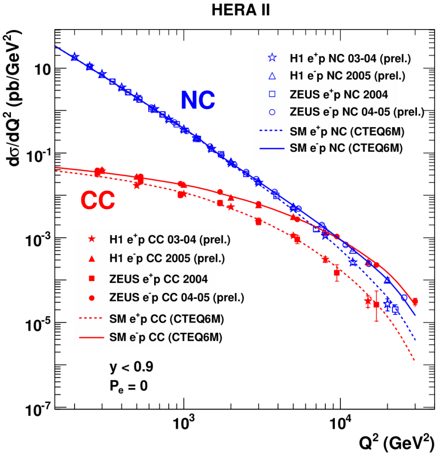
Weak Interaction: Heavy Mediators



- **Heavy mediators**: short range/weakness of interaction
  - Propagator suppressed by large mass in denominator
  - Resolves divergencies in 4-point contact-interaction model (Fermi theory)

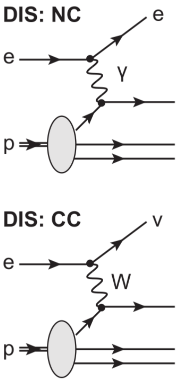
Electroweak Unification

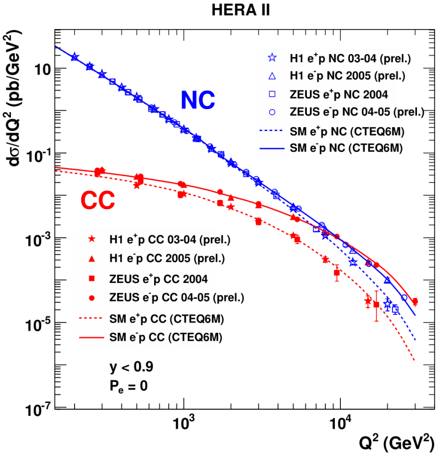




- **Electroweak unification**: same coupling at high energies
  - Also: resolves divergencies in $e^+e^- \rightarrow$ WW by contributions from triple-gauge couplings γWW, ZWW (prediction of Z boson!)

Electroweak Gauge Group





- Simplest combination of gauge-symmetry groups for unified electroweak interaction: $SU(2)_L \times U(1)_Y$
  - $SU(2)_L$: **weak isospin** acts on left-handed particles only
  - $U(1)_Y$: **hypercharge** acts on all particles ($\neq$ U(1) gauge group of QED!)

Electroweak Gauge Group

- **Particle content**: distinguish left-handed and right-handed particles
  - **Left-handed particles**: weak isospin **doublets** ($I = 1/2$, $I_3 = \pm 1/2$)

$$\psi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_\mathrm{L}, \ldots, \begin{pmatrix} u \\ d \end{pmatrix}_\mathrm{L}, \ldots$$

  - **Right-handed particles**: weak isospin **singlets** ($I = I_3 = 0$)

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- Left- and right-handed (chirality!) components of fermions can be projected with

$$\psi_{L/R} \equiv \frac{1}{2}\left(1 \mp \gamma^5\right)\psi \quad \Rightarrow \quad \psi = \psi_L + \psi_R \qquad \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$$

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  - Important equality: scalar bilinear form $\overline{\psi}\psi = \overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L$

Electroweak Gauge Group

- Gauge transformation of $SU(2)_L$: $U(x) = \exp[i\frac{g}{2}\alpha^a(x)\tau^a]$
  - Coupling constant g
  - Acts on **isospin doublets**
  - 3 generators: Pauli matrices $\tau^a = \sigma_a \rightarrow$ **3 gauge bosons $\mathbf{W}^i_\mu$**

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- Gauge transformation of $\mathrm{U}(1)_\mathrm{Y}$: $U(x) = \exp[i\frac{g'}{2}Y\alpha(x)]$
  - Coupling constant g'
  - Weak hypercharge Y (additive quantum number)
  - Acts on **isospin doublets and singlets**
  - **Single gauge boson** $\mathbf{B}_\mu$

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  - **Single gauge boson** $\mathbf{B}_\mu$
- Require that $\mathrm{SU}(2)_\mathrm{L}$ doublets are $\mathrm{U}(1)_\mathrm{Y}$ singlets

$\rightarrow$ **Gell-Mann–Nishijima formula** $I_3 = Q - \frac{1}{2}Y$

Particles and Quantum Numbers

Fermion	Chirality	Isospin (I, I_3)	Hypercharge Y	Charge Q (e)
Neutrinos: ν_e, ν_μ, ν_τ	*L*	(1/2, +1/2)	–1	0
	R	Not part of the standard model		
Charged leptons: e, μ, τ	*L*	(1/2, –1/2)	–1	–1
	R	(0, 0)	–2	–1
up-type quarks: u, c, t	*L*	(1/2, +1/2)	+1/3	+2/3
	R	(0, 0)	+4/3	+2/3
down-type quarks: d, s, b	*L*	(1/2, –1/2)	+1/3	–1/3
	R	(0, 0)	–2/3	–1/3

Electroweak Lagrangian (without gauge-boson mass terms)

$$\mathcal{L}_{\text{EWK}} = \boxed{i\overline{\psi}_L\gamma^\mu D_\mu\psi_L + i\overline{\psi}_R\gamma^\mu D_\mu\psi_R}$$

Electroweak Lagrangian (without gauge-boson mass terms)

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1. Covariant derivatives

$$\begin{aligned} D_\mu\psi_L &= \left(\partial_\mu - i\frac{g}{2}\tau^a W^a_\mu - i\frac{g'}{2}Y_L \mathbb{1}_2 B_\mu\right)\psi_L \\ D_\mu\psi_R &= \left(\partial_\mu - i\frac{g'}{2}Y_R B_\mu\right)\psi_R \end{aligned}$$

Covariant Derivative of $\mathbf{SU}(2)_L \times \mathbf{U}(1)_Y$

$$\boxed{\mathcal{L} = i\overline{\psi}_L \gamma^\mu \left[\partial_\mu + \ i\frac{g}{2}\tau^a \mathrm{W}^a_\mu \ + \ i\frac{g'}{2} Y_L \mathrm{B}_\mu \ \right] \psi_L + i\overline{\psi}_R \gamma^\mu \left[\partial_\mu + \ i\frac{g'}{2} Y_R \mathrm{B}_\mu \ \right] \psi_R}$$

Covariant Derivative of SU$(2)_L \times$ U$(1)_Y$

Covariant derivative corresponding to SU(2) acts on isospin doublet only

$$\mathcal{L} = i\overline{\psi}_L\gamma^\mu\left[\partial_\mu + i\frac{g}{2}\tau^a W^a_\mu + i\frac{g'}{2}Y_L B_\mu\right]\psi_L + i\overline{\psi}_R\gamma^\mu\left[\partial_\mu + i\frac{g'}{2}Y_R B_\mu\right]\psi_R$$

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$$\tau^a W^a_\mu = \sqrt{2}\left(\tau^+ W^+_\mu + \tau^- W^-_\mu\right) + \tau^3 W^3_\mu$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu), \qquad \tau^+ \equiv \frac{1}{2}(\tau^1 + i\tau^2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$ (ascending operator)

$$\tau^- \equiv \frac{1}{2}(\tau^1 - i\tau^2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$ (descending operator)

$$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Charged Currents [Example: 1. Generation Leptons]

$$\mathcal{L}_{\text{CC}} = i\overline{\psi}_L\gamma^\mu \left[i\frac{g}{2}\tau^1 W_\mu^1 + i\frac{g}{2}\tau^2 W_\mu^2\right]\psi_L$$

Charged Currents [Example: 1. Generation Leptons]

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$$= -\tfrac{g}{\sqrt{2}}\overline{\psi}_L\gamma^\mu\left[\tau^+ W^+_\mu + \tau^- W^-_\mu\right]\psi_L$$

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$$= -\frac{g}{\sqrt{2}}\overline{\psi}_L\gamma^\mu\left[\tau^+ W^+_\mu + \tau^- W^-_\mu\right]\psi_L$$

$$= -\frac{g}{\sqrt{2}}(\overline{\nu}_e\ \overline{e}_L)\gamma^\mu\left[\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}W^+_\mu + \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}W^-_\mu\right]\begin{pmatrix}\nu\\ e_L\end{pmatrix}$$

Charged Currents [Example: 1. Generation Leptons]

$$\mathcal{L}_{\mathrm{CC}} = i\overline{\psi}_L\gamma^\mu \left[i\tfrac{g}{2}\tau^1 \mathrm{W}^1_\mu + i\tfrac{g}{2}\tau^2 \mathrm{W}^2_\mu\right]\psi_L$$

$$= -\frac{g}{\sqrt{2}}\overline{\psi}_L\gamma^\mu \left[\tau^+ \mathrm{W}^+_\mu + \tau^- \mathrm{W}^-_\mu\right]\psi_L$$

$$= -\frac{g}{\sqrt{2}}(\overline{\nu}_e\ \overline{e}_L)\gamma^\mu \left[\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\mathrm{W}^+_\mu + \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\mathrm{W}^-_\mu\right]\begin{pmatrix} \nu \\ e_L \end{pmatrix}$$

$$= -\frac{g}{\sqrt{2}}(\overline{\nu}_e\ \overline{e}_L)\gamma^\mu \left[\begin{pmatrix} e_L \\ 0 \end{pmatrix}\mathrm{W}^+_\mu + \begin{pmatrix} 0 \\ \nu \end{pmatrix}\mathrm{W}^-_\mu\right]$$

Charged Currents [Example: 1. Generation Leptons]

$$
\begin{aligned}
\mathcal{L}_{CC} &= i\overline{\psi}_L\gamma^\mu\left[i\tfrac{g}{2}\tau^1 W^1_\mu + i\tfrac{g}{2}\tau^2 W^2_\mu\right]\psi_L \\
&= -\tfrac{g}{\sqrt{2}}\overline{\psi}_L\gamma^\mu\left[\tau^+ W^+_\mu + \tau^- W^-_\mu\right]\psi_L \\
&= -\tfrac{g}{\sqrt{2}}(\overline{\nu}_e\ \overline{e}_L)\gamma^\mu\left[\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix} W^+_\mu + \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix} W^-_\mu\right]\begin{pmatrix}\nu\\ e_L\end{pmatrix} \\
&= -\tfrac{g}{\sqrt{2}}(\overline{\nu}_e\ \overline{e}_L)\gamma^\mu\left[\begin{pmatrix}e_L\\ 0\end{pmatrix} W^+_\mu + \begin{pmatrix}0\\ \nu\end{pmatrix} W^-_\mu\right] \\
&= -\tfrac{g}{\sqrt{2}}\left[(\overline{\nu}_e\gamma^\mu e_L)W^+_\mu + (\overline{e}_L\gamma^\mu\nu)W^-_\mu\right]
\end{aligned}
$$

Charged Currents [Example: 1. Generation Leptons]

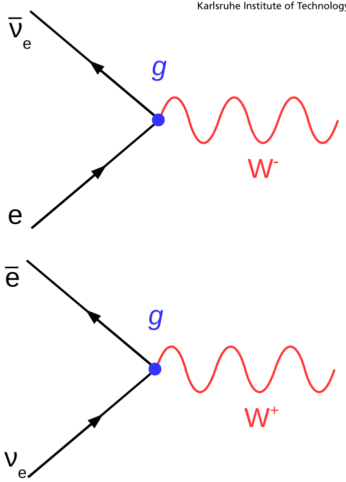
$$\mathcal{L}_{\text{CC}} = -\frac{g}{\sqrt{2}}\Big[\underbrace{(\overline{\nu}_e \gamma^\mu e_L)}_{J^{\mu,+}_{CC}} \mathrm{W}^+_\mu + \underbrace{(\overline{e}_L \gamma^\mu \nu_e)}_{J^{\mu,-}_{CC}} \mathrm{W}^-_\mu \Big]$$

$$= -\frac{g}{\sqrt{2}}\Big[\big(\overline{\nu}_e \underbrace{\gamma^\mu \tfrac{1}{2}(1-\gamma_5)}_{\text{V-A}} e\big)\mathrm{W}^+_\mu + \big(\overline{e} \underbrace{\gamma^\mu \tfrac{1}{2}(1-\gamma_5)}_{\text{V-A}} \nu_e\big)\mathrm{W}^-_\mu \Big]$$

Charged Currents [Example: 1. Generation Leptons]

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\Big[\underbrace{(\overline{\nu}_e\gamma^\mu e_L)}_{J^{\mu,+}_{CC}}\mathrm{W}^+_\mu + \underbrace{(\overline{e}_L\gamma^\mu \nu_e)}_{J^{\mu,-}_{CC}}\mathrm{W}^-_\mu\Big]$$

$$= -\frac{g}{\sqrt{2}}\Big[(\overline{\nu}_e\underbrace{\gamma^\mu\tfrac{1}{2}(1-\gamma_5)}_{\text{V-A}}e)\mathrm{W}^+_\mu + (\overline{e}\underbrace{\gamma^\mu\tfrac{1}{2}(1-\gamma_5)}_{\text{V-A}}\nu_e)\mathrm{W}^-_\mu\Big]$$



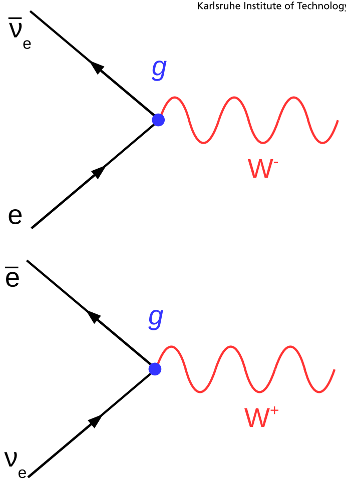
Operator W^+_μ:
annihilates W^+ or creates W^-

Charged Currents [Example: 1. Generation Leptons]

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\left[\underbrace{(\overline{\nu}_e \gamma^\mu e_L)}_{J^{\mu,+}_{CC}} W^+_\mu + \underbrace{(\overline{e}_L \gamma^\mu \nu_e)}_{J^{\mu,-}_{CC}} W^-_\mu \right]$$

$$= -\frac{g}{\sqrt{2}}\left[(\overline{\nu}_e \underbrace{\gamma^\mu \tfrac{1}{2}(1-\gamma_5)}_{\text{V-A}} e) W^+_\mu + (\overline{e} \underbrace{\gamma^\mu \tfrac{1}{2}(1-\gamma_5)}_{\text{V-A}} \nu_e) W^-_\mu \right]$$

- Transitions within isospin doublets
  - Simultaneous **change of charge** (by $\pm e$) and **flavour** ($e \leftrightarrow \nu_e$)
- **Parity violation**: W boson couples only to left-handed particles
  - Only left-handed particles carry "weak isospin charge" under I_3
- **V-A interaction** ("vector minus axial vector current")



Operator W^+_μ:
annihilates W^+ or creates W^-

Covariant Derivative of $\mathbf{SU}(2)_L \times \mathbf{U}(1)_Y$

Covariant derivative corresponding to SU(2) acts on isospin doublet only

$$\mathcal{L} = i\overline{\psi}_L \gamma^\mu \left[\partial_\mu + i\frac{g}{2}\tau^a W^a_\mu + i\frac{g'}{2} Y_L B_\mu\right]\psi_L + i\overline{\psi}_R \gamma^\mu \left[\partial_\mu + i\frac{g'}{2} Y_R B_\mu\right]\psi_R$$

$$\tau^a W^a_\mu = \sqrt{2}\left(\tau^+ W^+_\mu + \tau^- W^-_\mu\right) + \tau^3 W^3_\mu$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu), \qquad \tau^+ \equiv \frac{1}{2}(\tau^1 + i\tau^2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$ (ascending operator)

$$\tau^- \equiv \frac{1}{2}(\tau^1 - i\tau^2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$ (descending operator)

$$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Covariant Derivative of SU$(2)_L \times$ U$(1)_Y$

$$\mathcal{L} = i\overline{\psi}_L\gamma^\mu\left[\partial_\mu + i\frac{g}{2}\tau^a W^a_\mu + i\frac{g'}{2}Y_L B_\mu\right]\psi_L + i\overline{\psi}_R\gamma^\mu\left[\partial_\mu + i\frac{g'}{2}Y_R B_\mu\right]\psi_R$$

Covariant derivative corresponding to U(1) acts on isospin doublet and on isospin singlet

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Covariant derivative corresponding to U(1) acts on isospin doublet and on isospin singlet

	$Y_{L/R}$	I_3	Q
ν	-1	$+1/2$	0
e_L	-1	$-1/2$	-1
e_R	-2	0	-1

$Q = I_3 + \frac{Y}{2}$ (Gell-Mann–Nishijima)

Neutral Currents [Example: 1. Generation Leptons]

$$\mathcal{L}_{\mathrm{NC}} = -\left[\underbrace{\frac{g}{2}W_\mu^3 - \frac{g'}{2}B_\mu}\right](\overline{\nu}\gamma^\mu\nu) + \left[\underbrace{\frac{g}{2}W_\mu^3 + \frac{g'}{2}B_\mu}\right](\overline{e}_L\gamma^\mu e_L) + \underbrace{g'B_\mu}(\overline{e}_R\gamma^\mu e_R)$$

Neutral Currents [Example: 1. Generation Leptons]

$$\mathcal{L}_{NC} = -\Big[\underbrace{\tfrac{g}{2}W^3_\mu - \tfrac{g'}{2}B_\mu}_{-c_1 Z_\mu}\Big](\overline{\nu}\gamma^\mu\nu) + \Big[\underbrace{\tfrac{g}{2}W^3_\mu + \tfrac{g'}{2}B_\mu}_{[c_2 Z_\mu + c_4 A_\mu]}\Big](\overline{e}_L\gamma^\mu e_L) + \underbrace{g'B_\mu}_{[c_3 Z_\mu + c_4 A_\mu]}(\overline{e}_R\gamma^\mu e_R)$$

Weinberg rotation:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$$

$$\sin\theta_W \equiv \frac{g'}{\sqrt{g^2+g'^2}}\,, \qquad \cos\theta_W \equiv \frac{g}{\sqrt{g^2+g'^2}}$$

Neutral Currents [Example: 1. Generation Leptons]

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$$\sin\theta_W \equiv \frac{g'}{\sqrt{g^2+g'^2}}, \qquad \cos\theta_W \equiv \frac{g}{\sqrt{g^2+g'^2}}$$

$$\begin{aligned}
\mathcal{L}_{NC} = & -\frac{\sqrt{g^2+g'^2}}{2} Z_\mu (\overline{\nu}\gamma^\mu\nu) \\
& + \frac{\sqrt{g^2+g'^2}}{2}\Big[(\cos^2\theta_W - \sin^2\theta_W) Z_\mu + 2\sin\theta_W\cos\theta_W A_\mu\Big](\overline{e}_L\gamma^\mu e_L) \\
& + \frac{\sqrt{g^2+g'^2}}{2}\Big[(-2\sin^2\theta_W) Z_\mu + 2\sin\theta_W\cos\theta_W A_\mu\Big](\overline{e}_R\gamma^\mu e_R)
\end{aligned}$$

Electromagnetic Interaction [Example: Electrons]

$$
\begin{aligned}
\mathcal{L}_{\text{em}} &= \frac{\sqrt{g^2+g'^2}}{2} 2 \sin\theta_W \cos\theta_W \cdot A_\mu \cdot \left[(\overline{e}_L \gamma^\mu e_L) + (\overline{e}_R \gamma^\mu e_R) \right] \\
&= \frac{gg'}{\sqrt{g^2+g'^2}} \cdot A_\mu \cdot (\overline{e} \gamma^\mu e) \\
&= q_e \cdot A_\mu \cdot j^\mu_{\text{em}}
\end{aligned}
$$

$\rightarrow$ QED **vector current** j^μ_{em} recovered

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$$
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\mathcal{L}_{\text{em}} &= \frac{\sqrt{g^2+g'^2}}{2} 2 \sin\theta_W \cos\theta_W \cdot A_\mu \cdot \left[(\bar{e}_L \gamma^\mu e_L) + (\bar{e}_R \gamma^\mu e_R) \right] \\
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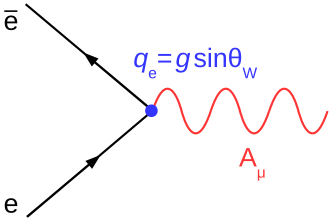
\end{aligned}
$$

$\rightarrow$ QED **vector current** j^μ_{em} recovered

- Electron charge related to electroweak **coupling constants** g and g'

$$
q_e = \frac{gg'}{\sqrt{g^2+g'^2}} = g \sin\theta_W = g' \cos\theta_W
$$



Electromagnetic Interaction [Example: Electrons]

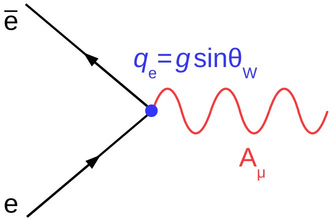
$$\begin{aligned}\mathcal{L}_{em} &= \frac{\sqrt{g^2+g'^2}}{2} 2\sin\theta_W\cos\theta_W \cdot A_\mu \cdot \left[(\overline{e}_L\gamma^\mu e_L) + (\overline{e}_R\gamma^\mu e_R) \right] \\ &= \frac{gg'}{\sqrt{g^2+g'^2}} \cdot A_\mu \cdot (\overline{e}\gamma^\mu e) \\ &= q_e \cdot A_\mu \cdot j^\mu_{em}\end{aligned}$$

$\rightarrow$ QED **vector current** j^μ_{em} recovered

- Electron charge related to electroweak **coupling constants** g and g'

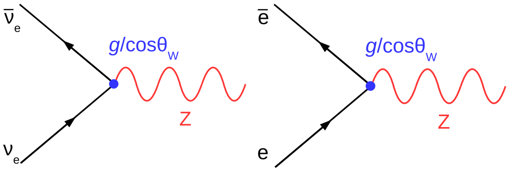
$$q_e = \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\theta_W = g'\cos\theta_W$$

- Photon field A_μ couples "as desired"
  - Photon couples to **all charged particles**
  - **Symmetric** coupling for left-handed and right-handed components

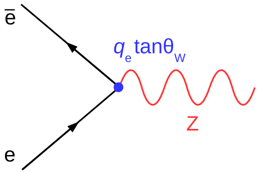


Z Boson Exchange [Example: 1. Generation Leptons]

$$\begin{aligned}
\mathcal{L}_Z &= -\frac{\sqrt{g^2+g'^2}}{2} J^\mu_{NC} Z_\mu \\
&= -\frac{g}{2\cos\theta_W}\left[\overline{\nu}_e\gamma^\mu\nu_e - \left(\cos^2\theta_W - \sin^2\theta_W\right)\overline{e}_L\gamma^\mu e_L + 2\sin^2\theta_W\overline{e}_R\gamma^\mu e_R\right]Z_\mu \\
&= -\frac{g}{2\cos\theta_W}\left[\overline{\nu}_e\gamma^\mu\tfrac{1}{2}(1-\gamma_5)\nu_e - \overline{e}\gamma^\mu\tfrac{1}{2}(1-\gamma_5)e + 2\sin^2\theta_W\left(\overline{e}\gamma^\mu e\right)\right]Z_\mu \\
&= -\frac{g}{\cos\theta_W}\left[I_3^\nu\,\overline{\nu}_e\tfrac{1}{2}\gamma^\mu(1-\gamma_5)\nu_e + I_3^e\,\overline{e}\gamma^\mu\tfrac{1}{2}(1-\gamma_5)e\right]Z_\mu - q_e\tan\theta_W\overline{e}\gamma^\mu e\,Z_\mu
\end{aligned}$$



Left-handed (V-A) couplings



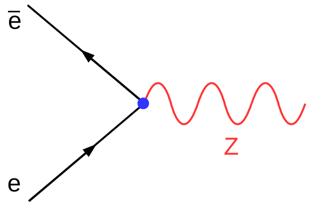
Vector couplings

Example: Z Boson Coupling to Electrons

$$\mathcal{L}_{Ze} = -\frac{g}{2\cos\theta_W}\Big[-\bar{e}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)e + 2\sin^2\theta_W\,(\bar{e}\gamma^\mu e)\Big]Z_\mu$$

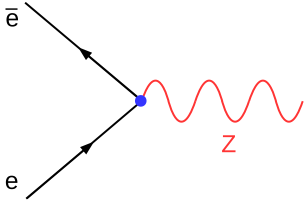
$$= -\frac{g}{4\cos\theta_W}\Big[\bar{e}\gamma^\mu(4\sin^2\theta_W - 1 + \gamma_5)\Big]e\,Z_\mu$$

$$= -\frac{g}{4\cos\theta_W}\,\bar{e}\gamma^\mu\big(C_V + C_A\gamma_5\big)e\,Z_\mu$$



Example: Z Boson Coupling to Electrons

$$\mathcal{L}_{Ze} = -\frac{g}{2\cos\theta_W}\left[-\bar{e}\gamma^\mu \tfrac{1}{2}(1-\gamma_5)e + 2\sin^2\theta_W\,(\bar{e}\gamma^\mu e)\right]Z_\mu$$
$$= -\frac{g}{4\cos\theta_W}\left[\bar{e}\gamma^\mu(4\sin^2\theta_W - 1 + \gamma_5)\right]e\,Z_\mu$$
$$= -\frac{g}{4\cos\theta_W}\,\bar{e}\gamma^\mu\left(C_V + C_A\gamma_5\right)e\,Z_\mu$$



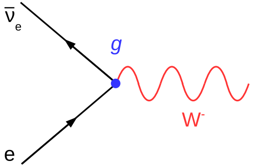
$$C_V = 4\sin^2\theta_W - 1 \text{ (vector coupling)}$$
$$C_A = 1 \text{ (axial-vector coupling)}$$

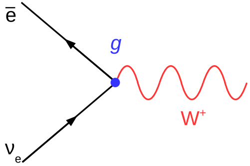
With $\sin^2\theta_W = 0.22$: C_V is small

$\rightarrow$ **electron couples mostly via axial-vector couplings to the Z boson**

Electroweak Interactions [Example: 1. Generation Leptons]

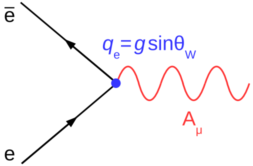
Charged-current interactions (W boson exchange)



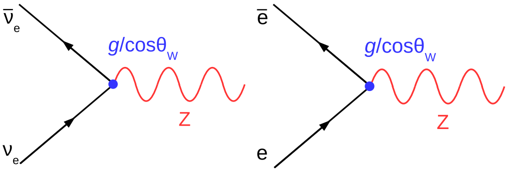


Left-handed (V-A) couplings

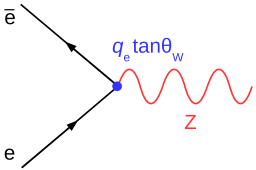
Neutral-current interactions (photon and Z boson exchange)



Vector couplings



Left-handed (V-A) couplings



Vector couplings

Electroweak Lagrangian (without gauge boson mass terms)

$$\mathcal{L}_{\mathrm{EWK}} = \boxed{i\overline{\psi}_L \gamma^\mu D_\mu \psi_L + i\overline{\psi}_R \gamma^\mu D_\mu \psi_R}$$

1. Covariant derivatives

$$\begin{aligned} D_\mu \psi_L &= \left(\partial_\mu - i\frac{g}{2}\tau^a \mathrm{W}^a_\mu - i\frac{g'}{2} Y_L \mathbb{1}_2 \mathrm{B}_\mu\right)\psi_L \\ D_\mu \psi_R &= \left(\partial_\mu - i\frac{g'}{2} Y_R \mathrm{B}_\mu\right)\psi_R \end{aligned}$$

Electroweak Lagrangian (without gauge boson mass terms)

$$\mathcal{L}_{\text{EWK}} = \boxed{i\overline{\psi}_L\gamma^\mu D_\mu\psi_L + i\overline{\psi}_R\gamma^\mu D_\mu\psi_R} \boxed{-\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu}}$$

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$$\begin{aligned} D_\mu\psi_L &= \left(\partial_\mu - i\frac{g}{2}\tau^a W^a_\mu - i\frac{g'}{2}Y_L\mathbb{1}_2 B_\mu\right)\psi_L \\ D_\mu\psi_R &= \left(\partial_\mu - i\frac{g'}{2}Y_R B_\mu\right)\psi_R \end{aligned}$$

Electroweak Lagrangian (without gauge boson mass terms)

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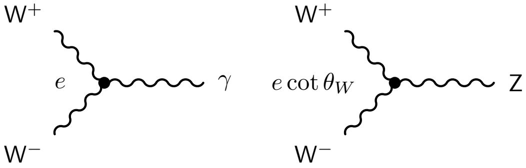
2. Field strength tensors

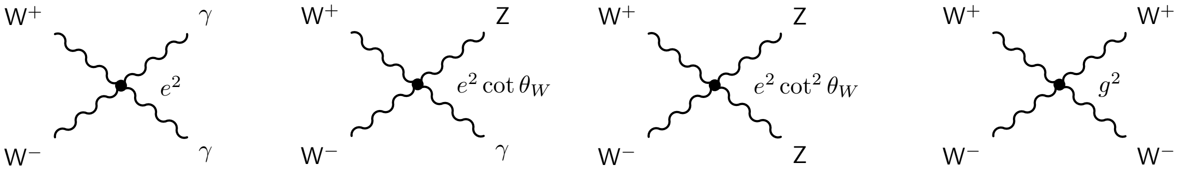
$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu \underbrace{- g\epsilon^{ijk} W^j_\mu W^k_\nu}_{\text{gauge boson self-interaction}} \end{aligned}$$

Gauge Boson Self-Interaction

Triple gauge couplings



Quartic gauge couplings



Summary

- Gauge groups of the Standard Model: $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- Electroweak gauge group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ has peculiar structure
  - Physical gauge boson ($W^\pm$, Z, γ) superposition of underlying gauge fields W^a (from $\mathrm{SU}(2)_L$) and B (from $\mathrm{U}(1)_Y$)
  - Interaction different for left- and right-handed states, leading to e. g. parity violation
- So far, gauge bosons remain massless to not break gauge invariance
  - Other mass generating mechanism is needed...