Karlsruhe Institute of Technology

Institut für Experimentelle Teilchenphysik

Teilchenphysik II - W, Z, Higgs am Collider

Lecture 06: Higgs Mechanism

PD Dr. K. Rabbertz, **Dr. Nils Faltermann** | 26. Mai 2023

Recap: Electroweak Theory

- **Interactions as consequence of local gauge invariance**
  - Invariance requires introduction of gauge fields
  - Geometrical interpretation: gauge bosons transport phase information between space-time points
- Extension to non-Abelian gauge theories $\rightarrow$ **Electroweak gauge group SU**$(2)_L \times$ **U**$(1)_Y$
  - Physical gauge boson ($W^{\pm}$, Z, γ) superposition of underlying gauge fields W^a (from $SU(2)_L$) and B (from $U(1)_Y$)
  - Chiral theory: interaction different for left- and right-handed states

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- So far very successful in describing observed phenomena:
  - Unification of weak and electromagnetic force at higher energy scales
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- So far very successful in describing observed phenomena:
  - Unification of weak and electromagnetic force at higher energy scales
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- **BUT, there are also problems:**
  1) Gauge-boson mass terms violate gauge invariance
     $\rightarrow$ a problem of gauge theories in general
  2) Fermion mass terms violate invariance under electroweak ($SU(2)_L \times U(1)_Y$) symmetry
     $\rightarrow$ follows from the chiral structure
  3) Longitudinal WW boson scattering violates unitarity
     $\rightarrow$ cross section diverges with higher energy

Problem of Massive Gauge Bosons

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  - Vector field (photon) transforms as $\mathrm{A}_\mu \rightarrow \mathrm{A}'_\mu = \mathrm{A}_\mu - \frac{1}{q}\partial_\mu\alpha$
  - Transformation of mass terms

$$\frac{1}{2}m_\mathrm{A}^2\mathrm{A}_\mu\mathrm{A}^\mu \rightarrow \frac{1}{2}m_\mathrm{A}^2\mathrm{A}'_\mu\mathrm{A}'^\mu$$

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$$
\begin{aligned}
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= & \frac{1}{2} m_A^2 A_\mu A^\mu - \frac{1}{q} m_A^2 \left(A_\mu \partial^\mu \alpha + A^\mu \partial_\mu \alpha\right) + \frac{1}{2q^2} m_A^2 \partial_\mu \alpha \partial^\mu \alpha \\
\neq & \frac{1}{2} m_A^2 A_\mu A^\mu
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✗ Gauge-boson **mass terms break local gauge invariance**

- Property of all gauge-field theories

✗ **Fundamental problem**: W and Z bosons have masses!

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✓ No problem with fermion masses for $\mathrm{U}(1)$ transformations

✓ Similarly, no problem in $\mathrm{SU}(3)$ (non-Abelian gauge group)

Problem of Massive Fermions!

- $SU(2)_L \times U(1)_Y$ transformations act differently on chiral components
- Decomposition of mass term

$$m_f \overline{\psi}\psi = m_f \left(\overline{\psi}_R \psi_L + \overline{\psi}_L \psi_R\right)$$

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- **Left- and right-handed components transform differently!**

$$\psi_L \to \psi'_L = \mathrm{e}^{i\alpha^a \tau^a + i\alpha Y} \psi_L \quad \text{(component of isospin doublet, } I = \tfrac{1}{2}\text{)}$$
$$\psi_R \to \psi'_R = \mathrm{e}^{i\alpha Y} \psi_R \quad \text{(isospin singlet, } I = 0\text{)}$$

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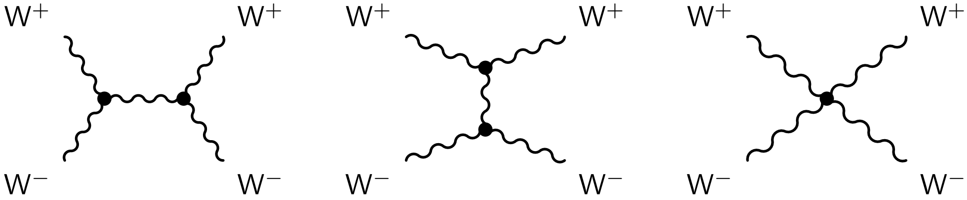
✗ Left- and right-handed fermions transform differently under $SU(2)_L \times U(1)_Y$
✗ **Fermion mass terms in chiral theory are not gauge invariant**

Unitarity Violation

- Several Standard Model scattering cross-sections violate unitarity, i. e. become divergent at large $\sqrt{s}$
  - $e^+e^- \rightarrow$ WW (for $m_e \neq 0$)
  - WW $\rightarrow$ WW scattering



$\rightarrow$ **theory becomes non-renormalizable**

How to Solve the Problems?

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      "Broken symmetries, massless particles and gauge fields"
      Physics Letters. 12 (2): 132–201.
    - Englert, F.; Brout, R. (1964)
      "Broken symmetry and the mass of gauge vector mesons"
      Physical Review Letters. 13 (9) 321–23.
    - Higgs, P. W. (1964)
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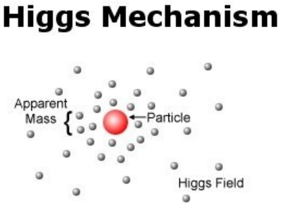
    - Guralnik, G.S.; Hagen, C.R.; Kibble, T.W.B. (1964)
      "Global conservation laws and massless particles"
      Physical Review Letters. 13 (20): 585–87.

How to Solve the Problems?

- New **background field that has non-zero amplitude** v in ground state everywhere
  - **Particles interact with the field and get 'slowed down'**: movement as if they have mass
  - Mass explained as restoring force

$$m \propto v \qquad (v = \text{field amplitude})$$



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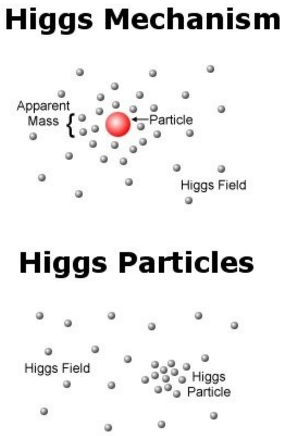

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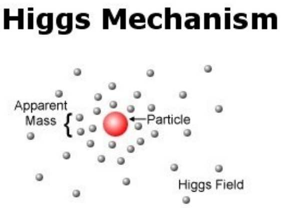
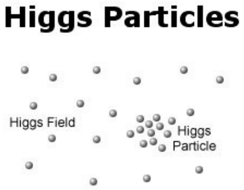
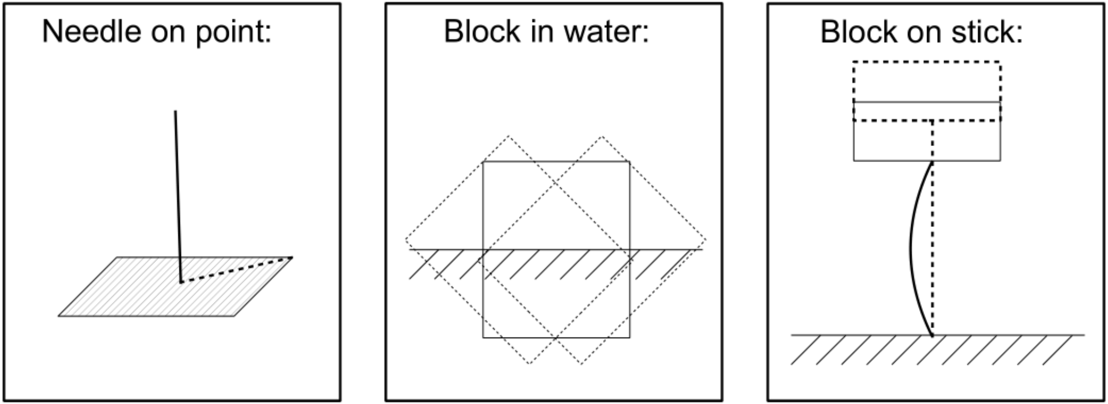
- In the Standard Model
  - Weak interactions themselves have infinite range and are described by gauge-invariant theory
  - **Interactions are screened by background field**: effective masses for the gauge bosons
  - SSB: field spontaneously takes ground-state which does not have symmetry





SSB in Classical Mechanics

- Symmetry is present in the system (i. e. the Lagrangian)
- But it is broken in the energy ground-state



φ symmetry — axis-symmetry — φ symmetry

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→ **Higgs mechanism**

  - Solves all the discussed problems
  - Introduces a **fundamental scalar particle: the Higgs boson**

Simple Example of SSB

- **Illustrate idea** of Higgs field and **spontaneous symmetry breaking**
- Real scalar field $\phi(x)$ in specific potential $V(\phi)$

$$\mathcal{L} = \underbrace{\frac{1}{2}\left(\partial_\mu \phi(x)\right)\left(\partial^\mu \phi(x)\right)}_{T(\phi)} - \underbrace{\left[\frac{1}{2}\mu^2\phi^2(x) + \frac{1}{4}\lambda\phi^4(x)\right]}_{V(\phi)}$$

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  - Two possibilities for sign of μ^2
- Investigate particle spectrum: **investigate $\mathcal{L}$ around energy ground-state** (*vacuum expectation value* or short *vacuum*)

Energy ground-state at minimum of Hamiltonian density

$$\mathcal{H} = \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)}(\partial_0 \phi) - \mathcal{L} = \frac{1}{2}[(\partial_0\phi)^2 + (\nabla\phi)^2] + V(\phi)$$

Lowest energy if $\phi(x) = \phi_0 =$ const and $V(\phi_0)$ minimal

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    - Ground state retains symmetry in $\phi \to -\phi$

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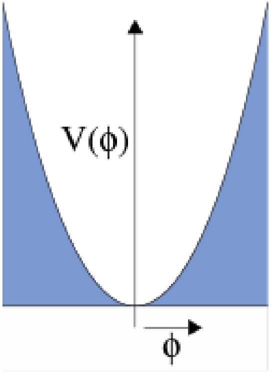

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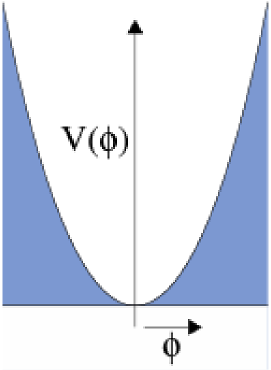
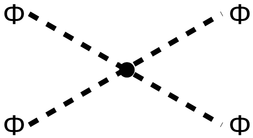
$\rightarrow$ **free scalar particle with mass μ and four-point self-interaction**

- Mass = excitation against "restoring force"





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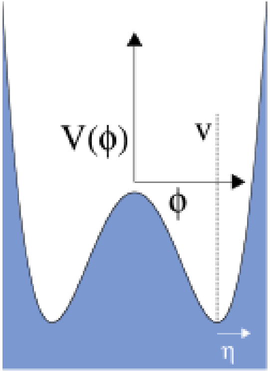
  - No stable minimum of $V(\phi)$ at $\phi(x) = 0$ (perturbation theory will not converge)
  - Ground state(s) located at $\boxed{\phi_0 = \sqrt{-\frac{\mu^2}{\lambda}} \equiv v}$
  - Study **states close to minimum**:

    $\boxed{\phi(x) \equiv v + \eta(x)}$ (perturbations $\eta(x)$ around v)



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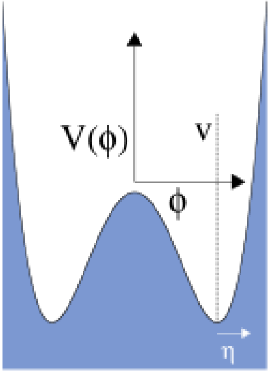
Kinetic term: $T = \frac{1}{2}\left[\partial_\mu(v+\eta)\partial^\mu(v+\eta)\right]$

$= \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta)$, since $\partial_\mu v = 0$

Potential term: $V = \frac{1}{2}\mu^2(v+\eta)^2 + \frac{1}{4}\lambda(v+\eta)^4$

$= \lambda v^2\eta^2 + \lambda v\eta^3 + \frac{1}{4}\lambda\eta^4 - \underbrace{\frac{1}{4}\lambda v^4}_{\text{const}}$, since $\mu^2 = -\lambda v^2$



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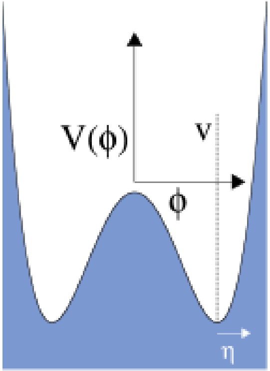

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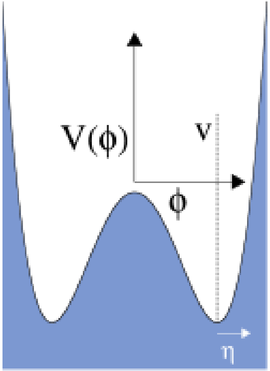
  - Scalar particle η with mass $\boxed{\frac{1}{2}m_\eta^2 \equiv \lambda v^2 = -\mu^2 \Rightarrow m_\eta = \sqrt{2\lambda v^2}}$
  - Additional 3- and 4-point self-interactions



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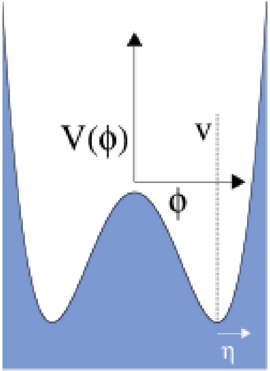

Symmetry in ϕ retained but ground state not symmetric in η: $\mathcal{L}(\eta) \neq \mathcal{L}(-\eta)$

$\rightarrow$ **spontaneous symmetry breaking (SSB)**

Intermediate Summary - SSB

- Lagrangian for scalar field ϕ without mass terms + potential $V(\phi)$ with minimum (= ground-state of system) at $\phi \equiv v \neq 0$
- Particle spectrum obtained by investigating $\mathcal{L}$ close to the minimum: expansion of ϕ around the minimum v
- **Adding V leads to massive scalar particle (consequence of ‘restoring force’ in potential) with self-interaction**
  - Keeps the full Lagrangian invariant under the original symmetry (here: global phase transformation)
  - But the energy ground-state is *not* invariant under this symmetry

  $\rightarrow$ “spontaneous symmetry breaking”

$\rightarrow$ tools needed for the Higgs mechanism

Breaking Global Gauge Symmetry

- Example: complex scalar field $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$

- **Higgs potential** $\boxed{V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4}$

- Lagrangian $\mathcal{L} = (\partial_\mu \phi^*)(\partial^\mu \phi) - V(\phi)$
- $V = V(|\phi|^2) \rightarrow$ **invariant under global U(1) transformations**

$$\begin{aligned} \phi &\rightarrow e^{i\alpha}\phi \\ \phi^* &\rightarrow e^{-i\alpha}\phi^* \end{aligned} \qquad \alpha = \text{const}$$

- $\mu^2 > 0$: ground state at $|\phi_0| = 0$

  $\rightarrow$ 2 massive scalar particles with additional self-interaction

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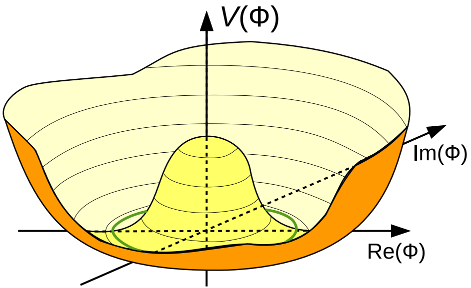
- $\mu^2 < 0$: infinitely many ground states on circle with

$$\boxed{|\phi| = \sqrt{\frac{1}{2}(\phi_1^2 + \phi_2^2)} = \sqrt{\frac{-\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}}$$



Breaking Global Gauge Symmetry

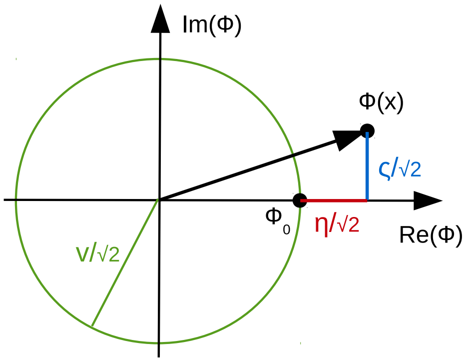
- Choose real ground state (U(1) symmetry!)

$$\boxed{\phi_0 = \frac{v}{\sqrt{2}} = \sqrt{\frac{-\mu^2}{2\lambda}}}$$

- Study perturbation around ϕ_0:

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + \eta(x) + i\zeta(x)\right)$$

$\eta(x)$, $\zeta(x)$: infinitesimal field amplitudes



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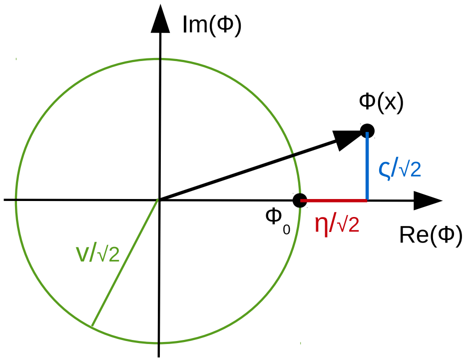
$$= \tfrac{1}{2}\left(\partial_\mu\eta\right)\left(\partial^\mu\eta\right) + \tfrac{1}{2}\left(\partial_\mu\zeta\right)\left(\partial^\mu\zeta\right), \quad \partial_\mu v = 0$$

$$V = \mu^2|\phi|^2 + \lambda|\phi|^4$$
$$= -\tfrac{1}{2}\lambda v^2\left[(v+\eta)^2+\zeta^2\right] + \tfrac{1}{4}\lambda\left[(v+\eta)^2+\zeta^2\right]^2, \quad \mu^2 = -\lambda v^2$$
$$= +\lambda v^2\eta^2 + \mathcal{O}(\eta^3, \eta^4, \zeta^4, \eta\zeta^2, \eta^2\zeta^2, \ldots)$$



Breaking Global Gauge Symmetry

- Full Lagrangian after symmetry breaking

$$\mathcal{L} = \underbrace{\tfrac{1}{2}\left(\partial_\mu \eta\right)\left(\partial^\mu \eta\right) - \lambda v^2 \eta^2}_{\text{massive scalar particle}} + \underbrace{\tfrac{1}{2}\left(\partial_\mu \zeta\right)\left(\partial^\mu \zeta\right)}_{\text{massless scalar particle}} + \underbrace{\text{higher-order terms}}_{\text{self interaction}}$$

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- ζ: massless scalar particle “**Goldstone Boson**”
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Breaking Global Gauge Symmetry

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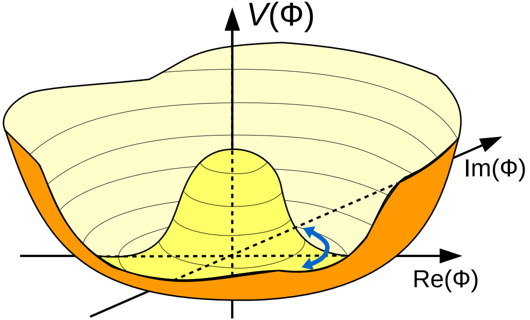
Goldstone Theorem *For each generator of a spontaneously broken[1] continuous symmetry[2], a massless spin-zero particle will appear*

[1] a symmetry of $\mathcal{L}$ that is not present in the ground state
[2] that ‘connects’ the ground states

Intermediate Summary

Spontaneously breaking a continuous global symmetry leads to the appearance of a massless Goldstone boson



Higgs Mechanism: Breaking Local Symmetry

Example QED: local U(1) symmetry

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- Invariance under local U(1) gauge transformations

$$\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$$

achieved by introduction of covariant derivative

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + iqA_\mu \quad \text{with} \quad A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{q}\partial_\mu\alpha(x)$$

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  (also transforms under U(1)!)
- Local-U(1) **gauge-invariant Lagrangian** for Higgs and photon field
  (omitting fermion terms)

$$\boxed{\mathcal{L} = (D_\mu\phi)^\dagger (D^\mu\phi) - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

  with Higgs potential $V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$ with $\mu^2 < 0$

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$$\begin{aligned} T &= (D_\mu \phi)^\dagger (D^\mu \phi) \\ &= \frac{1}{2} \left[(\partial_\mu - iqA_\mu)(v + \eta - i\zeta) \right] \left[(\partial^\mu + iqA^\mu)(v + \eta + i\zeta) \right] \end{aligned}$$

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&= \frac{1}{2}\Big[\left(\partial_\mu \eta\right)\left(\partial^\mu \eta\right) + \left(\partial_\mu \zeta\right)\left(\partial^\mu \zeta\right) \\
&\quad + q^2\left(\left(v + \eta\right)^2 + \zeta^2\right) A_\mu A^\mu + 2qvA_\mu\left(\partial^\mu \zeta\right)\Big] + \text{higher orders}
\end{aligned}
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$$
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Higgs Mechanism: Breaking Local Symmetry

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$$\rightarrow \mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu\eta)^2 - \lambda v^2\eta^2}_{\text{massive scalar boson}} + \underbrace{\tfrac{1}{2}(\partial_\mu\zeta)^2}_{\text{Goldstone boson}} \underbrace{- \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}q^2v^2A_\mu A^\mu}_{\text{photon with mass term}}$$

$$+ \underbrace{qvA_\mu(\partial^\mu\zeta)}_{?} + \text{interaction } \eta/\zeta A_\mu + \text{ self-interaction } \eta/\zeta$$

Rewriting Lagrangian in Unitary Gauge

- Terms involving ζ and A_μ:

$$\frac{1}{2}\left(\partial_\mu\zeta\right)^2+\frac{1}{2}q^2v^2A_\mu A^\mu+qvA_\mu\left(\partial^\mu\zeta\right)=\frac{1}{2}q^2v^2\left[A_\mu+\frac{1}{qv}\left(\partial^\mu\zeta\right)\right]^2$$

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  - A_μ fixed up to a term $\frac{1}{q}\partial_\mu \alpha(x)$ (because $A_\mu \to A'_\mu = A_\mu - \frac{1}{q}\partial_\mu \alpha(x)$)

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$$\begin{aligned} A'_\mu &= A_\mu + \frac{1}{qv}(\partial_\mu\zeta) \\ \phi' &= e^{i\alpha}\phi = e^{-i\frac{1}{v}\zeta}\phi \\ &= e^{-i\frac{1}{v}\zeta}\frac{1}{\sqrt{2}}(v+\eta+i\zeta) \end{aligned}$$

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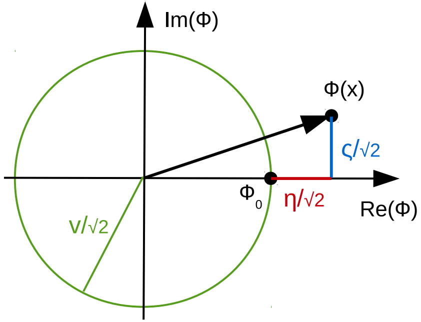

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&= \tfrac{1}{2} (\partial_\mu \eta)^2 + \tfrac{1}{2} q^2 (v + \eta)^2 A_\mu^2 - \underbrace{\left(\lambda v^2 \eta^2 + \lambda v \eta^3 + \tfrac{1}{4} \lambda \eta^4 - \tfrac{1}{4} \lambda v^4 \right)}_{= V(\phi)} \\
&= \underbrace{\tfrac{1}{2} (\partial_\mu \eta)^2 - \lambda v^2 \eta^2}_{\text{massive Higgs boson}} + \underbrace{\tfrac{1}{2} q^2 v^2 A_\mu^2}_{\text{photon mass}} + \underbrace{q^2 v A_\mu^2 \eta + \tfrac{1}{2} q^2 A_\mu^2 \eta^2}_{\text{Higgs-photon interaction}} - \underbrace{\lambda v \eta^3 - \tfrac{1}{4} \lambda \eta^4}_{\text{Higgs self-interaction}}
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- From point-of-view of the gauge field, two interpretations
  1. Photon field interacts with external background (Higgs) field: *dynamic* mass term
  2. Background field unknown: interpretation as massive photon field

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"The gauge boson has eaten up the Goldstone boson and has become fat on it."

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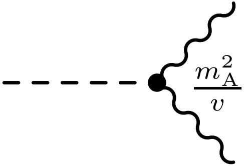
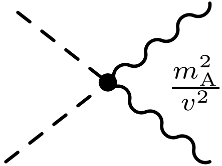
  - Photon-Higgs four-point interaction





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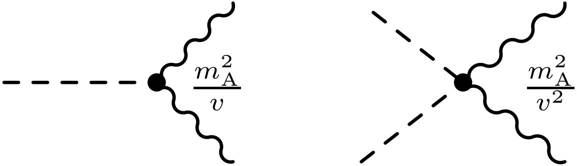
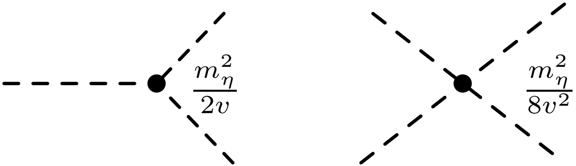


- Higgs self-interaction
  - Three-point self-coupling
  - Four-point self-coupling



This is not the Complete Story

- Previous discussion was just an example to illustrate the Higgs mechanism: Apparently, **there is no charged Higgs field** with $v > 0$ because the **photon is massless**!
- But principle can be applied to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry of the Standard Model

The Standard-Model Higgs Field ϕ

- $\mathcal{L}_{\text{SM}}$ should retain all gauge symmetries: add Higgs field ϕ as **left-chiral weak-isospin doublet of two complex fields**

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

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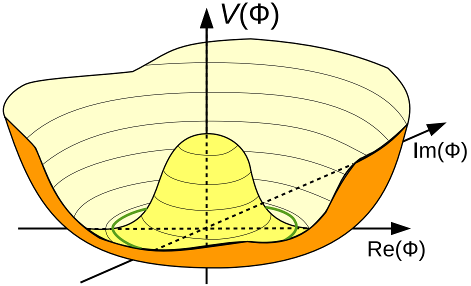
- Lagrangian for the Higgs field

$$\mathcal{L}_{\text{Higgs}} = (\partial_\mu \phi^\dagger)(\partial^\mu \phi) - V(\phi)$$

$$V(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4$$

with $\mu^2 < 0$ (SSB!)



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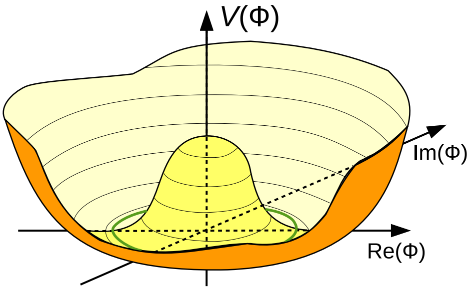
- The Standard-Model Lagrangian becomes

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{CC}} + \mathcal{L}_{\text{NC}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}}$$



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enforced by **covariant derivative**

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$\text{SU}(2) \times \text{U}(1)$ hypercharges of ϕ

field	Y_ϕ	I_3	Q
ϕ^+	+1	$+1/2$	$+1$
ϕ^0		$-1/2$	0

$Q = I_3 + \frac{Y}{2}$ (Gell-Mann–Nishijima)

Choice of Vacuum

- Ground state ϕ_0 with non-zero amplitude $\phi_0 \equiv v/\sqrt{2}$ ($\rightarrow$ SSB)
- Choose **ground state** with $I_3 = -\frac{1}{2}$, $Q = 0$ (i. e. $\phi^+ = 0$):

$$\boxed{\phi_0 = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \ , \ v = \sqrt{-\frac{\mu^2}{\lambda}}}$$

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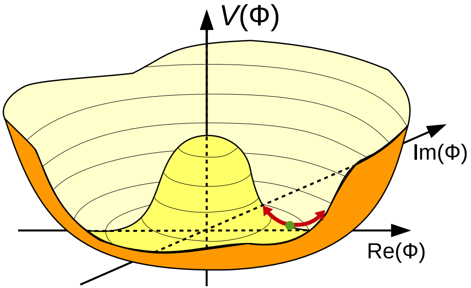
- Expansion of ϕ around ground state in **unitarity gauge**

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

Vacuum expectation value $v \neq 0$: gauge-boson masses

Radial excitation: the Higgs boson



Goldstone boson (term $i\zeta$) eliminated by gauge transformation

Dynamic Term in $\mathcal{L}_{\text{Higgs}}$

- **Covariant derivative** will give rise to
  - Masses for gauge bosons ($\propto v$)
  - Interactions between gauge bosons and Higgs boson ($\propto v\text{H}$, $\propto \text{H}^2$)

$$(D_\mu \phi) \;= \frac{\partial_\mu}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \text{H} \end{pmatrix} + \frac{i}{\sqrt{2}} \left[\frac{g}{2} \tau_a W_\mu^a + \frac{g'}{2} Y_\phi B_\mu \right] \begin{pmatrix} 0 \\ v + \text{H} \end{pmatrix}$$

Dynamic Term in $\mathcal{L}_{\text{Higgs}}$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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$\rightarrow$ Pauli matrices

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\end{aligned}
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- Full dynamic term $(D^\mu\phi)^\dagger(D_\mu\phi)$ in $\mathcal{L}_{\text{Higgs}}$

$$\frac{1}{2}(\partial_\mu\text{H})^2 + \frac{g^2}{8}(v+\text{H})^2\left(|\text{W}^1|^2 + |\text{W}^2|^2\right) + \frac{1}{8}(v+\text{H})^2\left|-g\text{W}^3_\mu + g'Y_\phi B_\mu\right|^2$$

Dynamic Term in $\mathcal{L}_{\text{Higgs}}$

- Re-writing $(D^\mu \phi)^\dagger (D_\mu \phi)$ in terms of **physical bosons**:

$$\frac{1}{2}\partial^\mu \mathrm{H}\partial_\mu \mathrm{H} + \frac{g^2}{8}(v + \mathrm{H})^2 \left(|\mathrm{W}^1|^2 + |\mathrm{W}^2|^2\right) \quad + \frac{1}{8}(v + \mathrm{H})^2 \left|-g\mathrm{W}^3_\mu + g' Y_\phi B_\mu\right|^2$$

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- $\mathrm{W}^\pm$ bosons

$$\mathrm{W}^\pm_\mu = \frac{1}{\sqrt{2}}\left(\mathrm{W}^1_\mu \mp i\mathrm{W}^2_\mu\right) \quad \Rightarrow \quad |\mathrm{W}^1|^2 + |\mathrm{W}^2|^2 = |\mathrm{W}^+|^2 + |\mathrm{W}^-|^2$$

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- Weinberg rotation: photon and Z boson

$$\begin{pmatrix} \text{Z}_{\mu} \\ A_{\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} \text{W}_{\mu}^{3} \\ B_{\mu} \end{pmatrix}$$

$$\underbrace{\sin\theta_W \equiv \frac{g'}{\sqrt{g^2+g'^2}}\,, \qquad \cos\theta_W \equiv \frac{g}{\sqrt{g^2+g'^2}}}$$

$$\left(-g\text{W}_{\mu}^{3} + g'B_{\mu}\right) = -\sqrt{g^2+g'^2}\,\text{Z}_{\mu} + 0\cdot A_{\mu}$$

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Dynamic Term in $\mathcal{L}_{\text{Higgs}}$

- Higgs doublet and **choice of specific ground state** leads to

$$
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(D^\mu \phi)^\dagger (D_\mu \phi) = &\tfrac{1}{2}(\partial_\mu \mathrm{H})(\partial_\mu \mathrm{H}) \\
&+ \tfrac{1}{2} \underbrace{\tfrac{g^2}{4}}_{m_\mathrm{W} \equiv \frac{1}{2} g v} (v + \mathrm{H})^2 \left(|\mathrm{W}^+|^2 + |\mathrm{W}^-|^2 \right) + \tfrac{1}{2} \underbrace{\tfrac{g^2 + g'^2}{4}}_{m_\mathrm{Z} \equiv \frac{1}{2}\sqrt{g^2 + g'^2}\, v} (v + \mathrm{H})^2 |\mathrm{Z}|^2
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Mass terms for the $\mathrm{W}^\pm$ and Z bosons
No mass term for the photon

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Mass terms for the $\mathrm{W}^\pm$ and Z bosons
No mass term for the photon

- **Results depend on choice of Higgs-sector structure** ($v = 0$ for ϕ^+)
- Absolute masses of gauge bosons not predicted but their relation

$$\rho = \frac{m_W}{m_Z \cos\theta_W} = 1 \quad \Rightarrow \quad m_Z > m_W$$

Vacuum Expectation Value *v*

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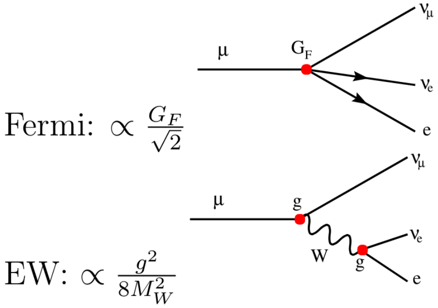
$$m_W^2 = \left(\tfrac{1}{2}gv\right)^2 \quad \text{(from Higgs mechanism)}$$

$$m_W^2 = \frac{\sqrt{2}g^2}{8G_F} \quad \text{(from Fermi theory)}$$

- $G_F = (1.16639 \pm 0.00002) \cdot 10^{-5}\,\text{GeV}^{-2}$ from muon-lifetime measurement



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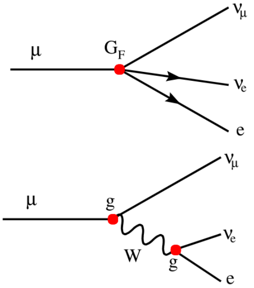
Fermi: $\propto \frac{G_F}{\sqrt{2}}$

EW: $\propto \frac{g^2}{8 M_W^2}$



$v = 246.22\,\text{GeV}$ sets the scale of electroweak symmetry breaking

The Standard Model Higgs Mechanism

- Adding ϕ as $\mathrm{SU}(2)_L$ doublet with specific non-zero ground-state

$$\begin{aligned}\mathcal{L}_{\mathrm{Higgs}} = & \frac{1}{2}\left(\partial_\mu \mathrm{H}\right)\left(\partial^\mu \mathrm{H}\right) - \lambda v^2 \mathrm{H}^2 + \lambda v \mathrm{H}^3 - \frac{1}{4}\lambda \mathrm{H}^4 \\ & + \frac{1}{2} m_\mathrm{Z}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{m_\mathrm{Z}^2}{v}\mathrm{H}\mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{1}{2}\frac{m_\mathrm{Z}^2}{v^2}\mathrm{H}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu \\ & + m_\mathrm{W}^2 \mathrm{W}^+_\mu \mathrm{W}^{-,\mu} + 2\frac{m_\mathrm{W}^2}{v}\mathrm{H}\mathrm{W}^+_\mu \mathrm{W}^{-,\mu} + \frac{m_\mathrm{W}^2}{v^2}\mathrm{H}^2 \mathrm{W}^+_\mu \mathrm{W}^{-,\mu}\end{aligned}$$

(Here, the equality $|\mathrm{W}^+|^2 + |\mathrm{W}^-|^2 = 2\mathrm{W}^+\mathrm{W}^-$ was used)

The Standard Model Higgs Mechanism

- Adding ϕ as $SU(2)_L$ doublet with specific non-zero ground-state

$$
\begin{aligned}
\mathcal{L}_{\text{Higgs}} = {} & \frac{1}{2}(\partial_\mu H)(\partial^\mu H) - \lambda v^2 H^2 + \lambda v H^3 - \frac{1}{4}\lambda H^4 \\
& + \frac{1}{2} m_Z^2 Z_\mu Z^\mu + \frac{m_Z^2}{v} H Z_\mu Z^\mu + \frac{1}{2}\frac{m_Z^2}{v^2} H^2 Z_\mu Z^\mu \\
& + m_W^2 W_\mu^+ W^{-,\mu} + 2\frac{m_W^2}{v} H W_\mu^+ W^{-,\mu} + \frac{m_W^2}{v^2} H^2 W_\mu^+ W^{-,\mu}
\end{aligned}
$$

- Masses (mass terms) for the gauge bosons $W^\pm$ and Z

w/o ϕ-W/Z interaction: d.o.f.		with ϕ-W/Z interaction: d.o.f.	
4 massless vector fields W^a, B:	8	3 massive vector fields $W^\pm$, Z:	9
2 complex Higgs fields:	4	1 massless vector field A:	2
		1 massive scalar:	1
total number d.o.f.:	12	total number d.o.f.:	12

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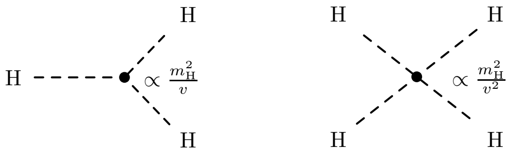
$$\mathcal{L}_{\mathrm{Higgs}} = \frac{1}{2}\left(\partial_\mu \mathrm{H}\right)\left(\partial^\mu \mathrm{H}\right) - \frac{1}{2} m_{\mathrm{H}} \mathrm{H}^2 + \frac{m_{\mathrm{H}}^2}{2v}\mathrm{H}^3 - \frac{m_{\mathrm{H}}^2}{8v^2}\mathrm{H}^4$$
$$+\frac{1}{2} m_{\mathrm{Z}}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{m_{\mathrm{Z}}^2}{v}\mathrm{H}\mathrm{Z}_\mu \mathrm{Z}^\mu + \frac{1}{2}\frac{m_{\mathrm{Z}}^2}{v^2}\mathrm{H}^2 \mathrm{Z}_\mu \mathrm{Z}^\mu$$
$$+m_{\mathrm{W}}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + 2\frac{m_{\mathrm{W}}^2}{v}\mathrm{H}\mathrm{W}_\mu^+ \mathrm{W}^{-,\mu} + \frac{m_{\mathrm{W}}^2}{v^2}\mathrm{H}^2 \mathrm{W}_\mu^+ \mathrm{W}^{-,\mu}$$

- Masses (mass terms) for the gauge bosons $\mathrm{W}^\pm$ and Z
- A massive scalar particle H (Higgs boson) with self-interaction
  - Higgs-boson mass $m_{\mathrm{H}} = \sqrt{2\lambda v^2}$
  - Three-point Higgs-boson self-coupling
  - Four-point Higgs-boson self-coupling



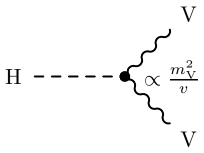
The Standard Model Higgs Mechanism

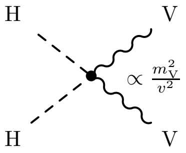
Karlsruhe Institute of Technology

- Adding ϕ as $SU(2)_L$ doublet with specific non-zero ground-state

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2}\left(\partial_\mu H\right)\left(\partial^\mu H\right) - \frac{1}{2} m_H H^2 + \frac{m_H^2}{2v} H^3 - \frac{m_H^2}{8v^2} H^4$$
$$+ \frac{1}{2} m_Z^2 Z_\mu Z^\mu + \frac{m_Z^2}{v} H Z_\mu Z^\mu + \frac{1}{2}\frac{m_Z^2}{v^2} H^2 Z_\mu Z^\mu$$
$$+ m_W^2 W_\mu^+ W^{-,\mu} + 2\frac{m_W^2}{v} H W_\mu^+ W^{-,\mu} + \frac{m_W^2}{v^2} H^2 W_\mu^+ W^{-,\mu}$$

- Masses (mass terms) for the gauge bosons $W^\pm$ and Z
- A massive scalar particle H (Higgs boson) with self-interaction
- Interactions of the Higgs boson with the $W^\pm$ and Z bosons
  - V-Higgs three-point interaction
  - V-Higgs four-point interaction





Reminder: Problem of Massive Fermions

- $SU(2)_L \times U(1)_Y$ transformations act differently on chiral components
- Decomposition of mass term

$$m_f \overline{\psi}\psi = m_f \left(\overline{\psi}_R \psi_L + \overline{\psi}_L \psi_R\right)$$

- **Left- and right-handed components transform differently!**

$$\psi_L \to \psi'_L = e^{i\alpha^a \tau^a + i\alpha Y}\psi_L \quad \text{(component of isospin doublet, } I = \tfrac{1}{2}\text{)}$$
$$\psi_R \to \psi'_R = e^{i\alpha Y}\psi_R \quad \text{(isospin singlet, } I = 0\text{)}$$

✗ Left- and right-handed fermions transform differently under $SU(2)_L \times U(1)_Y$

✗ **Fermion mass terms in chiral theory are not gauge invariant**

$\mathbf{SU(2)_L \times U(1)_Y}$ Invariant Fermion-Mass Term

- Higgs field can also be used to generate mass terms for fermions!
- Terms as the following are gauge invariant under $SU(2)_L \times U(1)_Y$

$$\boxed{\mathcal{L}_{\text{Yukawa}} = -y_f \overline{\psi}_L \phi \psi_R - y_f \overline{\psi}_R \phi^\dagger \psi_L}$$

y_f: “**Yukawa coupling**”

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y_f: "**Yukawa coupling**"

$$\begin{aligned}
\overline{\psi}_L \phi \psi_R \to & \left(\overline{\psi}_L A^\dagger_{Y_L} B^\dagger\right)\left(A_{Y_\phi} B \phi\right)\left(A_{Y_R}\psi_R\right) \\
&= A^\dagger_{Y_L} A_{Y_\phi} A_{Y_R} \overline{\psi}_L B^\dagger B \phi \psi_R \\
&= e^{i\frac{g'}{s}(-Y_L + Y_\phi + Y_R)\alpha(x)} \overline{\psi}_L \underbrace{B^\dagger B}_{=1} \phi \psi_R \\
&= e^{i\frac{g'}{s}(-(-1)+(+1)+(-2))\alpha(x)} \overline{\psi}_L \phi \psi_R \\
&= \overline{\psi}_L \phi \psi_R
\end{aligned}$$

...and analogously for $\overline{\psi}_R \phi^\dagger \psi_L$

Transformations:

$$\mathrm{U}(1)_Y \;:\; A_Y \equiv e^{i\frac{g'}{2} Y\alpha(x)}$$

$$\mathrm{SU}(2)_L \;:\; B \equiv e^{i\frac{g}{2}\tau^a \alpha^a(x)}$$

Hypercharges Y, e. g. for e:

e_L	-1
e_R	-2
ϕ	+1

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- Summary: under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ transformations

Dirac mass terms	$m_f(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L)$	break invariance
Yukawa mass terms	$y_f(\overline{\psi}_L \phi \psi_R + \overline{\psi}_R \phi^\dagger \psi_L)$	are invariant

Coupling to Higgs field restores gauge invariance!

. . . and how does this help?

Example: Electron Mass

- Expand ϕ around vacuum $|\phi_0| = \frac{v}{\sqrt{2}}$: $\phi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \mathrm{H} \end{pmatrix}$

$$\mathcal{L}^{\mathrm{e}}_{\mathrm{Yukawa}} = -y_e \overline{\psi}_L \phi e_R - y_e \overline{e}_R \phi^\dagger \psi_L$$

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$$
\begin{aligned}
\mathcal{L}^{\mathrm{e}}_{\mathrm{Yukawa}} &= -y_e \overline{\psi}_L \phi e_R - y_e \overline{e}_R \phi^\dagger \psi_L \\
&= -y_e \frac{1}{\sqrt{2}} \left[(\overline{\nu} \quad \overline{e})_L \begin{pmatrix} 0 \\ v + \mathrm{H} \end{pmatrix} e_R + \overline{e}_R (0 \quad v + \mathrm{H}) \begin{pmatrix} \nu \\ e \end{pmatrix}_L \right]
\end{aligned}
$$

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&= -\frac{y_e}{\sqrt{2}} (v + H) [\underbrace{\overline{e}_L e_R + \overline{e}_R e_L}_{=\overline{e}e}]
\end{aligned}$$

Example: Electron Mass

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&= -\underbrace{\frac{y_e}{\sqrt{2}} v}_{e \text{ mass}} \overline{e}e - \underbrace{\frac{y_e}{\sqrt{2}} \mathrm{H}\overline{e}e}_{\mathrm{H}\overline{e}e \text{ interaction}} \equiv -m_e \overline{e}e - \frac{m_e}{v} \mathrm{H}\overline{e}e
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&= -y_e \frac{1}{\sqrt{2}} \left[(\overline{\nu} \quad \overline{e})_L \begin{pmatrix} 0 \\ v + \mathrm{H} \end{pmatrix} e_R + \overline{e}_R (0 \quad v + \mathrm{H}) \begin{pmatrix} \nu \\ e \end{pmatrix}_L \right] \\
&= -\frac{y_e}{\sqrt{2}} (v + \mathrm{H}) \underbrace{[\overline{e}_L e_R + \overline{e}_R e_L]}_{=\overline{e}e} \\
&= -\underbrace{\frac{y_e}{\sqrt{2}} v}_{e \text{ mass}} \overline{e}e - \underbrace{\frac{y_e}{\sqrt{2}} \mathrm{H}\overline{e}e}_{\mathrm{H}\overline{e}e \text{ interaction}} \equiv -m_e \overline{e}e - \frac{m_e}{v} \mathrm{H}\overline{e}e
\end{aligned}$$

- **Yukawa coupling of electron with Higgs field**
  - Electron-mass term (cf. Dirac equation) in gauge-invariant way! Electron mass: $m_e = \frac{y_e}{\sqrt{2}} v$
- In addition: **interaction of electron with Higgs boson** $\propto m_e$
- **No prediction of electron mass**: free parameter y_e

Fermion Masses

- $\overline{\psi}_L \phi \psi_R + \overline{\psi}_R \phi^\dagger \psi_L$: masses only for 'down'-type fermions
- Additional **term for 'up'-type fermions**:

$$\boxed{\overline{\psi}_L \phi^c \psi_R \,, \qquad \phi^c \equiv i\tau_2 \phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^{-*} \end{pmatrix}}$$

ϕ^c: charge conjugate of ϕ $Y_{\phi^c} = -1$

- Conjugate ϕ^c transforms in same way as ϕ under $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$: above terms are gauge invariant

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- **Fermion-mass terms** (without *h.c.* terms):

$$\text{d-type:} \quad -y_d(\overline{u}_L \quad \overline{d}_L)\phi d_R = -\frac{y_d}{\sqrt{2}}(\overline{u}_L \quad \overline{d}_L)\begin{pmatrix} 0 \\ v \end{pmatrix} d_R = -\frac{y_d}{\sqrt{2}} v \overline{d}_L d_R$$

$$\text{u-type:} \quad -y_u(\overline{u}_L \quad \overline{d}_L)\phi^c u_R = -\frac{y_u}{\sqrt{2}}(\overline{u}_L \quad \overline{d}_L)\begin{pmatrix} v \\ 0 \end{pmatrix} u_R = -\frac{y_u}{\sqrt{2}} v \overline{u}_L u_R$$

- $\mathcal{L}_{\text{Yukawa}}$ for generation i (massless neutrinos)

$$\mathcal{L}_{\text{Yukawa}} = -y_i^d \overline{Q}_{Li} \phi d_{Ri} - y_i^u \overline{Q}_{Li} \phi^c u_{Ri} - y_i^l \overline{L}_{Li} \phi l_{Ri} - h.c.$$

Quark Masses

- Most general case: $y_i \longrightarrow G_{ij}$ **complex matrices**

$$\mathcal{L}_{\text{Yukawa}}^{\text{quarks}} = G_{ij}\overline{\psi}_{Li}\phi\psi_{Rj} = -G^{d}_{ij}\overline{Q}'_{Li}\phi d'_{Rj} - G^{u}_{ij}\overline{Q}'_{Li}\phi^{c}u'_{Rj} - h.c.$$

  - $d', \ldots$: states in **flavour (= SU(2)-interaction) base**

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  - $d', \ldots$: states in **flavour (= SU(2)-interaction) base**

- For example, first term (after electroweak symmetry breaking)

$$G^d_{ij}\overline{Q}'_{Li}\phi d'_{Rj} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} G^d_{dd}\cdot\overline{(u \quad d)}'_L & G^d_{ds}\cdot\overline{(u \quad d)}'_L & G^d_{db}\cdot\overline{(u \quad d)}'_L \\ G^d_{sd}\cdot\overline{(c \quad s)}'_L & G^d_{ss}\cdot\overline{(c \quad s)}'_L & G^d_{sb}\cdot\overline{(c \quad s)}'_L \\ G^d_{bd}\cdot\overline{(t \quad b)}'_L & G^d_{bs}\cdot\overline{(t \quad b)}'_L & G^d_{bb}\cdot\overline{(t \quad b)}'_L \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right]\cdot\begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix}$$

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- **Lagrangian** becomes

$$\begin{aligned}\mathcal{L}^{\text{quarks}}_{\text{Yukawa}} &= -\ G^d_{dd}\frac{v}{\sqrt{2}}\cdot\overline{d}'_L d'_R - G^d_{ds}\frac{v}{\sqrt{2}}\cdot\overline{d}'_L s'_R - \ldots - h.c. \\ &= -\ M^{d'}_{dd}\cdot\overline{d}'_L d'_R - M^{d'}_{ds}\cdot\overline{d}'_L s'_R - \ldots - h.c. \\ &= -\ \underbrace{M'_{dd}\cdot\overline{d}'d'}_{d\text{-quark mass}} - \underbrace{M'_{ds}\cdot\overline{d}'s'}_{?} - \ldots\end{aligned}$$

Quark Masses

- States with proper mass terms by **diagonalizing the mass matrices**

$$M^d = V_L^d M^{d'} V_R^{d\dagger} = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} , \quad M^u = V_L^u M^{u'} V_R^{u\dagger} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

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with unitary matrices V (i. e. $V^\dagger V = \mathbb{1}$)

$$\begin{aligned} \mathcal{L}^{\text{quarks}}_{\text{Yukawa}} &= - \overline{d}'_{Li} M^{d'}_{ij} d'_{Rj} - \overline{u}'_{Li} M^{u'}_{ij} u'_{Rj} \\ &= - \underbrace{\overline{d}'_{Li} V_L^{d\dagger}} \underbrace{V_L^d M^{d'}_{ij} V_R^{d\dagger}} \underbrace{V_R^d d'_{Rj}} - \underbrace{\overline{u}'_{Li} V_L^{u\dagger}} \underbrace{V_L^u M^{u'}_{ij} V_R^{u\dagger}} \underbrace{V_R^u u'_{Rj}} \\ &= - \overline{d}_{Li} M^d_{ij} d_{Rj} - \overline{u}_{Li} M^u_{ij} u_{Rj} \end{aligned}$$

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$$\begin{aligned} \mathcal{L}^{\text{quarks}}_{\text{Yukawa}} &= - \overline{d}'_{Li} \, M^{d'}_{ij} \, d'_{Rj} - \overline{u}'_{Li} \, M^{u'}_{ij} \, u'_{Rj} \\ &= - \underbrace{\overline{d}'_{Li} V_L^{d\dagger}}\underbrace{V_L^d M^{d'}_{ij} V_R^{d\dagger}}\underbrace{V_R^d d'_{Rj}} - \underbrace{\overline{u}'_{Li} V_L^{u\dagger}}\underbrace{V_L^u M^{u'}_{ij} V_R^{u\dagger}}\underbrace{V_R^u u'_{Rj}} \\ &= - \overline{d}_{Li} \, M^d_{ij} \, d_{Rj} - \overline{u}_{Li} \, M^u_{ij} \, u_{Rj} \end{aligned}$$

with **quark mass-eigenstates**

$$\boxed{\begin{array}{ll} d_{Li} = (V_L^d)_{ij} d'_{Lj} & d_{Ri} = (V_R^d)_{ij} d'_{Rj} \\ u_{Li} = (V_L^u)_{ij} u'_{Lj} & u_{Ri} = (V_R^u)_{ij} u'_{Rj} \end{array}}$$

SM Interactions in Quark Mass-Eigenstates

- Electroweak interaction terms rewritten in SU(2)-interaction base

$$
\begin{aligned}
\mathcal{L}_{\text{EWK}} = &\quad i\overline{Q}'_L\gamma^\mu \left[\partial_\mu + i\tfrac{g}{2}W^a_\mu\tau^a + i\tfrac{g'}{2}Y_L B_\mu\right] Q'_L + i\overline{q'}_R\gamma^\mu \left[\partial_\mu + i\tfrac{g'}{2}Y_L B_\mu\right] q'_R \\
= &\quad i\overline{Q}'_L\gamma^\mu\partial_\mu Q'_L + i\overline{q}'_R\gamma^\mu\partial_\mu q'_R && \longrightarrow \mathcal{L}_{\text{kin}} \\
& + \overline{Q}'_L\gamma^\mu W^\pm_\mu\tau^\pm Q'_L && \longrightarrow \mathcal{L}_{\text{CC}} \\
& + \overline{Q}'_L\gamma^\mu(c^Z_L Z_\mu, c^A A_\mu)Q'_L + \overline{q}'_R\gamma^\mu(c^Z_R Z_\mu, c^A A_\mu)q'_R && \longrightarrow \mathcal{L}_{\text{NC}}
\end{aligned}
$$

SM Interactions in Quark Mass-Eigenstates

- Electroweak interaction terms rewritten in SU(2)-interaction base
- For $\mathcal{L}_{\text{NC}}$ (and similarly $\mathcal{L}_{\text{kin}}$), terms of the form

$$\overline{Q}'_L \gamma^\mu (Z_\mu, A_\mu) Q'_L = \overline{(u \;\; d)}'_{Li} \gamma^\mu (Z_\mu, A_\mu) \begin{pmatrix} u \\ d \end{pmatrix}'_{Li}$$

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$$= \overline{u}'_{Li} \gamma^\mu (Z_\mu, A_\mu) u'_{Li} + \ldots$$

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&= \overline{u}'_{Li} \gamma^\mu (Z_\mu, A_\mu) u'_{Li} + \dots \\
&= \gamma^\mu (Z_\mu, A_\mu) \overline{u}'_{Li} u'_{Li} + \dots
\end{aligned}
$$

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&= \overline{u}'_{Li} \gamma^\mu (Z_\mu, A_\mu) u'_{Li} + \ldots \\
&= \gamma^\mu (Z_\mu, A_\mu) \overline{u}'_{Li} u'_{Li} + \ldots \\
&= \gamma^\mu (Z_\mu, A_\mu) \underbrace{\overline{u}_{Li} (V_L^u)_{ij}}_{\overline{u}'_{Li}} \underbrace{(V_L^{u\dagger})_{ij} u_{Lj}}_{u'_{Li}} + \ldots
\end{aligned}$$

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&= \gamma^\mu (Z_\mu, A_\mu) \overline{u}_{Li} \underbrace{(V_L^u V_L^{u\dagger})_{ij}}_{\delta_{ij}} u_{Lj} + \dots
\end{aligned}
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\end{aligned}
$$

Kinetic and NC interaction terms act on quark mass-eigentstates

SM Interactions in Quark Mass-Eigenstates

- Electroweak interaction terms rewritten in SU(2)-interaction base
- For $\mathcal{L}_{\mathrm{CC}}$, e. g. W^+, terms of the form

$$\overline{Q}'_L \gamma^\mu \mathrm{W}^+_\mu \tau^+ Q'_L = \overline{(u \;\; d)}'_{Li} \gamma^\mu \mathrm{W}^+_\mu \tau^+ \begin{pmatrix} u \\ d \end{pmatrix}'_{Li}$$

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- Electroweak interaction terms rewritten in SU(2)-interaction base
- For $\mathcal{L}_{\text{CC}}$, e. g. W^+, terms of the form

$$\overline{Q}'_L \gamma^\mu \text{W}^+_\mu \tau^+ Q'_L = \gamma^\mu \text{W}^+_\mu \, \overline{u}'_{Li} d'_{Li} + \ldots$$

$$= \gamma^\mu \text{W}^+_\mu \, \underbrace{\overline{u}_{Li} (V^u_L)_{ij}}_{\overline{u}'_{Li}} \underbrace{(V^{d\dagger}_L)_{ij} d_{Lj}}_{d'_{Li}} + \ldots$$

SM Interactions in Quark Mass-Eigenstates

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$$
\begin{aligned}
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&= \gamma^\mu \text{W}^+_\mu \, \underbrace{\overline{u}_{Li} (V^u_L)_{ij}}_{\overline{u}'_{Li}} \underbrace{(V^{d\dagger}_L)_{ij} d_{Lj}}_{d'_{Li}} + \dots \\
&= \gamma^\mu \text{W}^+_\mu \, \overline{u}_{Li} \underbrace{(V^u_L V^{d\dagger}_L)_{ij}}_{V^{ij}_{\text{CKM}}} d_{Lj} + \dots
\end{aligned}
$$

SM Interactions in Quark Mass-Eigenstates

Karlsruhe Institute of Technology

- Electroweak interaction terms rewritten in SU(2)-interaction base
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$$\begin{aligned}
\overline{Q}'_L \gamma^\mu \text{W}^+_\mu \tau^+ Q'_L &= \gamma^\mu \text{W}^+_\mu \, \overline{u}'_{Li} d'_{Li} + \dots \\
&= \gamma^\mu \text{W}^+_\mu \, \underbrace{\overline{u}_{Li} (V^u_L)_{ij}}_{\overline{u}'_{Li}} \underbrace{(V^{d\dagger}_L)_{ij} d_{Lj}}_{d'_{Li}} + \dots \\
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\end{aligned}$$

CC act on superposition of mass-eigentstates (**quark mixing**)

$V^u_L V^{d\dagger}_L = V_{\text{CKM}}$: Cabibbo-Kobayashi-Maskawa (CKM) matrix

Convention: V_{CKM} elements such that no mixing for u-type quarks: $u'_i = u_i$

Summary: The Higgs Mechanism

- No prediction of but **allows masses of the elementary particles** without breaking local gauge invariance
  - Higgs field ϕ with spontaneously-symmetry-breaking potential
    $\rightarrow$ non-zero vacuum expectation value v of ϕ
  - Coupling of gauge bosons to ϕ (by covariant derivative) generates boson mass-terms $\propto v$ ('eat up' Goldstone bosons to gain mass)
  - In addition: Yukawa coupling of fermions to ϕ generates fermion mass-terms $\propto v$ (and introduces freedom for mixing between fermion mass- and interaction-eigenstates)
- Predicts a **massive scalar particle (the Higgs boson)**
  - Coupling to fermions and bosons depending on their masses
  - Additional self-interaction
- **Higgs sector determined by Higgs-boson mass** (free parameter)