Seismic Modelling — Exercise 1 Analytical Solutions

In this exercise we calculate and compare analytical solutions of the Helmholtz equation given by:

$$\left[\Delta - \frac{1}{c^2}\frac{1}{\partial t^2}\right] p(x, x_s = 0, t) = -4\pi f(x_s = 0, t) \tag{1}$$

at distance $x = R$ via the convolution

$$\hat{p}^{1D,2D,3D}(R, 0, \omega) = \hat{G}^{1D,2D,3D}(R, 0, \omega)\hat{f}(\omega) \tag{2}$$

with $\hat{G}^{1D,2D,3D}(R, 0, \omega)$ defined by by the following Greens functions

$$\text{1D} : \Delta = \left[\frac{\partial^2}{\partial x^2}\right], \qquad \hat{G}^{1D}(x, x_s, \omega) = \frac{2\pi i}{k} e^{-ikr} \tag{3}$$

$$\text{2D} : \Delta = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right], \qquad \hat{G}^{2D}(x, x_s, \omega) = \sqrt{\frac{2\pi}{kr}} e^{-ikr} e^{-i\pi/4} \tag{4}$$

$$\text{3D} : \Delta = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right], \qquad \hat{G}^{3D}(x, x_s, \omega) = \frac{e^{-ikr}}{r} \tag{5}$$

with $r = ||x - x_s||$ and $k = \omega/c$.

We assume a homogeneous acoustic medium with $c = 500$ m/s and a shifted Ricker signal as source wavelet with a center frequency of $f_c = 50$ Hz located at $x = 0$

$$f(x_s = 0, t) = (1 - 2\tau^2)e^{-2\tau^2}, \quad \tau = \pi(t - t_d)f_c, \quad t_d = 1/f_c \tag{6}$$

Tasks:

1. Plot the Ricker signal given in equation 6. (5 points)
2. Calculate the analytical solutions for the 1D, 2D, and 3D case via equation 2 at distances $R = 20m$ and $R = 100m$. (15 points)
3. Compare the analytical solutions and discuss the main differences. (5 points).