Seismic Modelling — Exercise 2: 1D Finite Difference Modelling

In this exercise we solve the 1-D wave equation with the Finite Difference (FD) method.

Theory

We consider the 1-D second order wave equation

$$\frac{\partial^2 p(x,t)}{\partial t^2} = c^2(x)\frac{\partial^2 p(x,t)}{\partial x^2} \tag{1}$$

The pressure $p(x,t)$ at the location x at time t is discretized with $p(x,t) = p(jh, n\triangle t) = p_j^n$.
In the second lecture we derived the following explicit FD update scheme of accuracy order O(2,2M):

$$p_j^{n+1} = 2p_j^n - p_j^{n-1} + r^2\left(-a_0 p_j^n + \sum_{m=1}^{M} a_m\left(p_{j+m}^n + p_{j-m}^n\right)\right), \quad \text{with} \quad r_j = \frac{c_j\triangle t}{h} \tag{2}$$

where r_j denotes the Courant number. The FD coefficients a_m can be determined by solving

$$\sum_{m=1}^{M} a_m m^2 = 1 \quad \text{and} \quad \sum_{m=1}^{M} a_m m^{2k} = 0 \quad \text{with} \quad k = 2,...,M \quad \text{and} \quad a_0 = 2\sum_{m=1}^{M} a_m \tag{3}$$

The update scheme 2 is stable if

$$r_j = \frac{c_j\triangle t}{h} \leq \frac{2}{\sqrt{\sum\limits_{m=1}^{M} 2|a_m| + |a_0|}} \tag{4}$$

Source

As seismic source wavelet we assume a Ricker signal with a center frequency f_c

$$f(t) = (1-2\tau^2)e^{-\tau^2}, \quad \tau = \pi(t-t_d)f_c, \quad t_d = 1/f_c \tag{5}$$

The source is implemented by adding the first time derivative to the pressure field at the source location:

$$p(x_s,t) = p(x_s,t) + (2r_j\triangle t)\cdot f(t) \tag{6}$$

Reference numerical model

We define the reference numerical setup as follows. We assume a grid size of $nx = 1000$ grid points. The grid spacing is $h = 2$m. The time step interval is $dt = 2.0\cdot 10^{-3}$s. The source is located at $x_s = 300$m and has a center frequency of $f_c = 25$ Hz. The seismic velocity of the homogenous model is $c = 500$ m/s. The receiver is located at $x_r = 1300$m.

Tasks

1. Write a computer program that iterates equation 2 for a given discrete (heterogenous) velocity model c_j . Explain the details of the numerical algorithm. (10 points)
2. Simulate the pressure field in the reference model using accuracy orders $2M = 2, 4, 6, 8$. Quantify the numerical dispersion and explain the observed seismograms. Compare the numerical solutions with the analytical solution (time shifted original Ricker signal). (30 points)
3. Reduce the time step interval (Courant number) for $2M = 4$ and $2M = 8$ until the dispersion error becomes sufficiently small. Which Courant number is required ? (10 points)