Seismic Modelling — Exercise 3: 2D Finite Difference Modelling

In this exercise we solve the 2-D acoustic wave equation with the Finite Difference (FD) method.

Theory

We consider the 2-D second order acoustic wave equation

$$\frac{\partial^2 p(x,z,t)}{\partial t^2} = c^2(x)\left(\frac{\partial^2 p(x,z,t)}{\partial x^2} + \frac{\partial^2 p(x,z,t)}{\partial z^2}\right) \tag{1}$$

The pressure $p(x,z,t)$ at the location (x,z) at time t is discretized with $p(x,z,t) = p(ih, jh, n\triangle t) = p^n_{i,j}$.
In the third lecture we derived the following explicit FD update scheme of accuracy order O(2,2M):

$$p^{n+1}_{i,j} = 2p^n_{i,j} - p^{n-1}_{i,j} + r^2\left(-2a_0 p^n_{i,j} + \sum_{m=1}^{M} a_m\left(p^n_{i,j+m} + p^n_{i,j-m}\right) + \sum_{m=1}^{M} a_m\left(p^n_{i+m,j} + p^n_{i-m,j}\right)\right) \tag{2}$$

where $r_{i,j} = \frac{c_{i,j}\triangle t}{h}$ denotes the Courant number.
The FD coefficients a_m can be determined by solving

$$\sum_{m=1}^{M} a_m m^2 = 1 \quad \text{and} \quad \sum_{m=1}^{M} a_m m^{2k} = 0 \quad \text{with} \quad k = 2,...,M \quad \text{and} \quad a_0 = 2\sum_{m=1}^{M} a_m \tag{3}$$

The update scheme 2 is stable if

$$r_{i,j} = \frac{c_{i,j}\triangle t}{h} \leq \frac{2}{\sqrt{2}\sqrt{\sum\limits_{m=1}^{M} 2|a_m| + |a_0|}} \tag{4}$$

Source

As seismic source wavelet we assume a shifted Ricker signal with a center frequency f_c

$$f(t) = (1 - 2\tau^2)e^{-\tau^2}, \quad \tau = \pi(t - t_d)f_c, \quad t_d = 1/f_c \tag{5}$$

The source is implemented by adding the samples to the pressure field at the source location:

$$p(x_s, z_s, t) = p(x_s, z_s, t) + (2\pi r_j^2)\cdot f(t) \tag{6}$$

Reference numerical model

We define the reference numerical setup as follows. We assume a grid size of $nx = 250, nz = 250$ grid points. The grid spacing is $h = 2$m. The time step interval is $dt = 2.0\cdot 10^{-3}$s. The source is located at $x_s = 150, z_s = 250$m and has a center frequency of $f_c = 25$ Hz. The receiver is positioned at $x_r = 350, z_s = 250$m. The seismic velocity of the homogenous model is $c = 500$ m/s. The propagation time of waves shall be $T = 0.8$s.

Tasks

1. Write a computer program that iterates equation 2 for a given discrete (heterogenous) velocity model $c_{i,j}$. Explain the details of the numerical algorithm. (10 points)
2. Produce snapshots of the wave field at times $t = 0.1, 0.2, ...T$ for accuracy orders $M = 2$ and $M = 4$. Describe the propagation of waves. (10 points)
3. Produce synthetic seismograms recorded at (x_r, z_r) for M=2,4,8,12 and $dt = 2.0, 1.0, 0.5\cdot 10^{-3}$ and compare them with the analytical solution (exercise 1). Which accuracy order M and dt is required to obtain sufficient accuracy ? (30 points)