Seismic Modelling — Exercise 4: Acoustic Reflectivity Method

In this exercise we solve the forward problem of 1D acoustic wave propagation by using the reflectivity method. We will compare the simulated waveform to the FD numerical result implemented in the second exercise.

Theory
Consider a plane P-wave that perpendicular to a layer (angle of incidence $\phi = 0°$). Since we consider a perpendicularly incident plane wave (all derivatives after x are equal to zero), the vertical **displacement** u_j in the j-th layer applies

$$\frac{\partial^2 u_j}{\partial z^2} = \frac{1}{\alpha_j^2}\frac{\partial^2 u_j}{\partial t^2}. \tag{1}$$

where α represents P-wave velocity. Assuming a plane-wave solution:

$$u_j(z,t) = \underbrace{A_j e^{i\omega\left(t-\frac{z-z_j}{\alpha_j}\right)}}_{\text{Waves in } +z\text{-direction with Amplitude } A_j} + \underbrace{B_j e^{i\omega\left(t+\frac{z-z_j}{\alpha_l}\right)}}_{\text{Wave in -}z\text{-direction with the Amplitude } B_j} \tag{2}$$

and the boundary conditions for the interfaces

$$u_j(z_j,t) = u_{j-1}(z_j,t) \tag{3}$$

$$\rho_j\alpha_j^2\frac{\partial u_j(z_j,t)}{\partial z} = \rho_{j-1}\alpha_{j-1}^2\frac{\partial u_{j-1}(z_j,t)}{\partial z} \tag{4}$$

The following system of equations can be derived for the amplitudes of the up- and down-going waves in the j-th layer in the frequency domain, where $I_j = \alpha_j \cdot \rho_j$ is the impedance:

$$\begin{pmatrix} A_j \\ B_j \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{2}\left(1+\frac{I_{j-1}}{I_j}\right)e^{-i\omega\frac{d_{j-1}}{\alpha_{j-1}}} & \frac{1}{2}\left(1-\frac{I_{j-1}}{I_j}\right)e^{i\omega\frac{d_{j-1}}{\alpha_{j-1}}} \\ \frac{1}{2}\left(1-\frac{I_{j-1}}{I_j}\right)e^{-i\omega\frac{d_{j-1}}{\alpha_{j-1}}} & \frac{1}{2}\left(1+\frac{I_{j-1}}{I_j}\right)e^{i\omega\frac{d_{j-1}}{\alpha_{j-1}}} \end{pmatrix}}_{\text{layer matrix } m_j} \begin{pmatrix} A_{j-1} \\ B_{j-1} \end{pmatrix} \tag{5}$$

where d_j is the thickness of the j-th layer and ω is angular frequency ($\omega = 2 * \pi * f$). Assuming a two-layer overlaying half-space model (e.g., Model 1), then the amplitudes A_3 and B_3 in the half-space can be written as

$$\begin{pmatrix} A_3 \\ B_3 \end{pmatrix} = m_3 \cdot m_2 \cdot m_1 \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\begin{pmatrix} A_0 \\ B_0 \end{pmatrix}. \tag{6}$$

With the conditions $B_3 = 0$ and $R = \frac{B_0}{A_0}$, one can finally obtain the reflectivity over the layers

$$R = -\frac{M_{21}}{M_{22}}. \tag{7}$$

Model 1: A layered model
(Air above layer 1: $\alpha_0 = 330$ m/s and $\rho_0 = 1$ kg/m^3)
layer 1: $\alpha_1 = 1200$ m/s, $\rho_1 = 2100$ kg/m^3 (z < 600 m),
layer 2: $\alpha_2 = 1600$ m/s, $\rho_2 = 2200$ kg/m^3 (600 m <= z < 1400 m),
halfspace: $\alpha_3 = 2000$ m/s, $\rho_3 = 2200$ kg/m^3 (1400 m <= z)
Source wavelet: 10 Hz Ricker wavelet
Total recording time: 6 s

Tasks

1. Implement the reflectivity method by conducting the following steps:
   (a) Calculate the Fourier transform of the source signal by using the function $fast\_fourier$ $\_transform$. (5 points)
   (b) Compute the layer matrices m_1, m_2 and m_3 of Model 1 by using equation 5. In matlab you have to do this for every layer matrix. Create a three-dimensional matrix of 2*2*`ns`, where `ns` is the number of samples. In these matrices, the values calculated with equation 5 must be written for each frequency. (10 points)
   (c) Calculate the reflectivity of the model by first multiplying the individual layer matrices (equation 6) and then determine the reflectivity via equation 7. (5 points)
   (d) Calculate the synthetic seismogram of the reflected waves in the frequency domain by multiplying the reflectivity of the model with the Fourier transform of the source signal. (5 points)
   (e) Determine the seismogram of the reflected waves in the time domain by using inverse Fourier transform $inverse\_fast\_fourier\_transform$. (5 points)
2. Calculate the synthetic seismograms for Model 1. Plot the synthetic seismograms **between 0.3 s and 6 s** by using the function *plot*. Explain the wavetype of each signal (e.g., reflected wave from the first interface, multiples) in the synthetic seismograms. (15 points)
3. Compare it to the 1D FD result in Exercise 1 by setting the source and receiver at the same location. Explain the differences. (15 points)