Exercise 1 of the Lecture "Seismic wave modelling"

Modelling of seismic waves by using Green's function

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Goal

In this exercise we solve the forward problem of acoustic wave propagation in a homogeneous model by using the Green's function (analytical solution). We will compare the numerical solutions to this analytical solution in the future to evaluate the accuracy of the numerical methods.

Introduction

In this exercise we use a simple mathematical model of wave propagation phenomena, i.e. acoustic wave equation with a constant-density model:

$$\left[\frac{1}{v^2(\mathbf{x})}\frac{\partial^2}{\partial t^2} - \bigtriangledown^2\right] p(\mathbf{x},t) \ = s(\mathbf{x},t), \tag{1}$$

where pressure p is a time- and space-dependent field and s is the source of disturbances. $\bigtriangledown^2$ is the Laplacian operator (e.g., $\bigtriangledown^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in 3D). $\mathbf{x}$ denotes spatial coordinates, which can be 1D, 2D, or 3D.

If the domain is unbounded, the sound velocity is spatially-independent (i.e., $v(\mathbf{x}) = v$) and the source $s(\mathbf{x},t) = \delta(\mathbf{x} - \mathbf{x}_s)\delta(t)$, then the equation (1) has a simple analytical solution:

1D solution:

$$G^{1D}(x,t) = \frac{v}{2}H(t - \frac{r}{v}), \tag{2}$$

2D solution:

$$G^{2D}(x,y,t) = \frac{1}{2\pi}\frac{H\left(t - \frac{r}{v}\right)}{\sqrt{t^2 - \frac{r^2}{v^2}}}, \tag{3}$$

3D solution:

$$G^{3D}(x, y, z, t) = \frac{1}{4\pi r}\delta(t - \frac{r}{v}), \tag{4}$$

where δ denotes Dirac delta function, H is the Heaviside (sometimes called step) function, and $r = ||\mathbf{x} - \mathbf{x}_s||_2$ in which $\mathbf{x}_s$ is the source coordinates (e.g., $r = \sqrt{(x - x_s)^2 + (y - y_s)^2}$ in 2D case).
After calculating the Green's function of the model, the solution of pressure for any source $s(\mathbf{x}, t)$ is simply the acoustic Green's function convolved with the source signature:

$$p(\mathbf{x}, t) = s(t) * G(\mathbf{x}, t; \mathbf{x}_s) \tag{5}$$

where * represents convolution. So $G(\mathbf{x}, t; \mathbf{x}_s)$ is the pressure when $s(t) = \delta(t)$.

Exercise

1. Prepare a matlab script to simulate the analytical seismic wavefield in 1D, 2D, and 3D models. (5 points)
2. Plot the snapshots of 2D Green's function (or the wavefields) at t= 0.250 s, 0.500 s, 0.750 s, and 1.000 s by using function *imagesc*. (1 points)
3. Plot the analytical solution at all receivers' positions as a shot gather. Pick and use the arrival times of the waveforms to calculate the seismic-wave velocity of the model. (2 points)
4. Plot and compare the 1D, 2D and 3D analytical solutions at a trace that is 1050 m away from the source. Explain the reasons for the differences in their waveforms. (2 points)

Model information:
Size of the model : (1D) 2500 m, (2D) 2500 * 2500 m, (3D) 2500 * 2500 * 2500 m
Velocity of the model: 1500 m/s
Source location: (1D) 1250 m, (2D) (1250, 1250) m, (3D) (1250, 1250, 1250) m
Receivers' location:
1D: (100),(200),(300), ..., (2500) m
2D: (100, 1250), (200, 1250), (300, 1250), ..., (2500, 1250) m
3D: (100, 1250, 1250), (200, 1250, 1250), (300, 1250, 1250), ..., (2500, 1250, 1250) m
Source: Ricker wavelet (you can define your own peak frequency, e.g., 50 Hz)
Sampling rate of the data: 1000 Hz (0.001 s/sample)
Total recording time: 1.200 s

Reports and scripts

Name your repeort (in pdf format) and matlab script as "exercise1_YN.pdf" and "exercise1_YN.m", respectively, where YN is your name. Send them to tingting.liu@kit.edu, yudi.pan@kit.edu and thomas.bohlen@kit.edu before **9.May.2019**.