

Seismic Modelling

The Reflectivity Method

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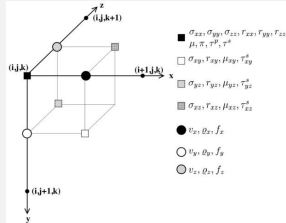
Wave equations

$$p_{ij} = \lambda \theta \delta_{ij} + 2\mu \epsilon_{ij}$$

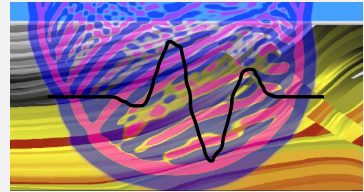
$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial p_{ij}}{\partial x_j} + f_i$$

Discretization



Solutions



Agenda

1. Introduction
2. Response of a stack of horizontal layers
3. Reflectivity method
4. Examples
5. References
6. Appendix
 - 6.1 Bessel functions
 - 6.2 Sommerfeld integral

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The Reflectivity Method

- The Reflectivity method (RM) is a powerful analytic method that allows to calculate *complete* synthetic seismograms for layered acoustic, elastic, visco-elastic, anisotropic media.
- The RM was the first method that allowed for the *interpretation of complete seismograms (traveltimes and amplitudes)* in the 1960-70s.
- It was introduced by Karl Fuchs and Gerhard Müller (Fuchs 1968, Fuchs & Müller 1971, Mueller 1985).
- It is still widely used today as an efficient algorithm to compute full seismograms on small computers (→ exercise).
- Typical applications are
 - surface waves in layered media
 - crustal and upper mantle deep seismic sounding
 - core/mantle inner core/outer core boundaries for long period body waves
 - concepts are also used in seismic migration

The Reflectivity Method

The Reflectivity Method is based on two fundamental concepts:

- 1 The calculation of the response of a stack of horizontal layers to an incident harmonic wave with certain frequency and angle of incidence: $R_{pp}(\omega, \varphi)$. This response can be computed by a matrix formalism (Haskell 1953). It takes most of the computing time in the RM.
- 2 The decomposition of a spherical wave radiating from a point source above the stack of layers into plane waves and vice versa using the Sommerfeld integral. This requires the numerical integration of $R_{pp}(\omega, \varphi)$ over φ or wavenumber k .

The Reflectivity Method

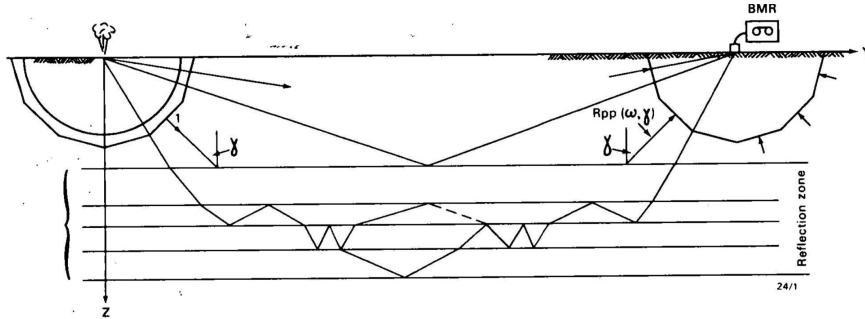


Figure 1: Calculation of the Point Source Response of a layered medium (Fuchs 1980). All wave phenomena in the layered medium are considered: surface waves, multiple reflections and refractions, mode conversions P-S, S-P etc. Attenuation and Anisotropy can be considered as well.

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Reflection and transmission of a layered acoustic medium

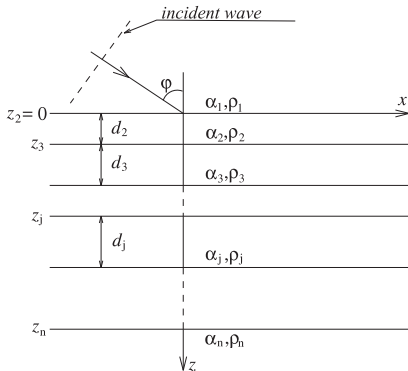


Figure 2: Calculation of the reflection and transmission response of a stack of liquids (acoustic approximation).

The displacement potential Φ_j in the j -th layer ($j = 1, 2, \dots, n$) satisfies

$$\frac{\partial^2 \Phi_j}{\partial x^2} + \frac{\partial^2 \Phi_j}{\partial z^2} = \frac{1}{\alpha_j^2} \frac{\partial^2 \Phi_j}{\partial t^2}$$

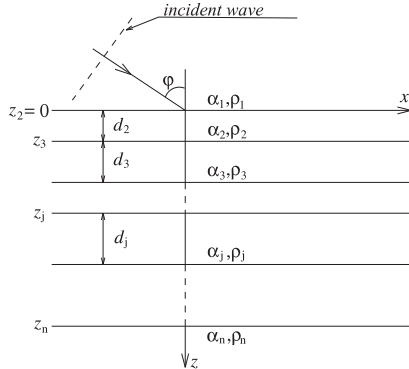
Potential ansatz

$$\begin{aligned} \Phi_j = & A_j \exp [i (\omega t - k_j x - l_j (z - z_j))] \quad (1) \\ & + B_j \exp [i (\omega t - k'_j x + l'_j (z - z_j))] \end{aligned}$$

$$\frac{\omega^2}{\alpha_j^2} = k_j^2 + l_j^2 = k_j'^2 + l_j'^2 \quad (2)$$

$$B_n = 0. \quad (3)$$

Incident P-wave

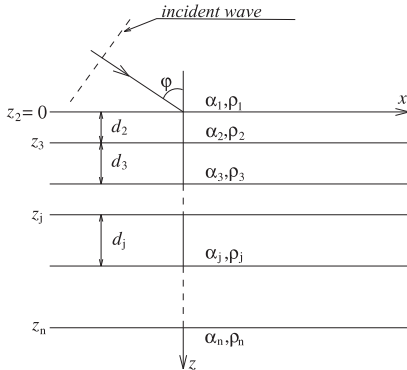


$A_1 \exp [i (\omega t - k_1 x - l_1 z)]$ of Φ_1 is interpreted as incident *P*-wave. Therefore

$$\begin{aligned} k_1 &= \frac{\omega}{\alpha_1} \sin \varphi \\ l_1 &= \frac{\omega}{\alpha_1} \cos \varphi \end{aligned} \quad (4)$$

The incident P-wave is described by amplitude A_1 , angular frequency ω and incident angle φ .

Reflectivity R_{pp} and transmittivity B_{pp}



$B_1 \exp [i (\omega t - k'_1 x + l'_1 z)]$ of Φ_1 is the reflected wave. We want to compute the displacement reflection coefficient R_{pp} (also called Reflectivity) and the displacement refraction coefficient B_{pp} (also called Transmittivity) of the layered medium

$$\begin{aligned}
 R_{pp}(\omega, \varphi) &= \frac{B_1}{A_1} \\
 B_{pp}(\omega, \varphi) &= \frac{\alpha_1}{\alpha_n} \cdot \frac{A_n}{A_1} \quad (5)
 \end{aligned}$$

Displacement boundary conditions

The boundary conditions require continuity of the vertical displacement at each interface

$z = z_2, z_3, \dots, z_n$

$$\frac{\partial \Phi_j}{\partial z} = \frac{\partial \Phi_{j-1}}{\partial z} \quad (6)$$

We insert our ansatz

$$\Phi_j = A_j \exp [i (\omega t - k_j x - l_j (z - z_j))] + B_j \exp [i (\omega t - k'_j x + l'_j (z - z_j))]$$

and obtain

$$\begin{aligned} -l_j A_j \exp [-ik_j x] + l'_j B_j \exp [-ik'_j x] &= -l_{j-1} A_{j-1} \exp [i (-k_{j-1} x - l_{j-1} d_{j-1})] \\ &\quad + l'_{j-1} B_{j-1} \exp [i (-k'_{j-1} x + l'_{j-1} d_{j-1})] \end{aligned} \quad (7)$$

with layer thickness $d_{j-1} := z_j - z_{j-1}$, ($d_1 = 0$)

Stress boundary conditions

The boundary conditions require continuity of the normal stress

$$\rho_{zz} = \lambda \nabla^2 \Phi = \rho \partial^2 \Phi / \partial t^2$$

at each interface $z = z_2, z_3, \dots, z_n$.

$$\rho_j \frac{\partial^2 \Phi_j}{\partial t^2} = \rho_{j-1} \frac{\partial^2 \Phi_{j-1}}{\partial t^2} \quad (8)$$

We insert our ansatz

$$\Phi_j = A_j \exp [i (\omega t - k_j x - l_j (z - z_j))] + B_j \exp [i (\omega t - k'_j x + l'_j (z - z_j))]$$

and obtain

$$\begin{aligned} \rho_j A_j \exp [-ik_j x] + \rho_j B_j \exp [-ik'_j x] &= \rho_{j-1} A_{j-1} \exp [i (-k_{j-1} x - l_{j-1} d_{j-1})] \\ &\quad + \rho_{j-1} B_{j-1} \exp [i (-k'_{j-1} x + l'_{j-1} d_{j-1})] \end{aligned} \quad (9)$$

Boundary Conditions

In order to fulfill the boundary conditions the exponential terms depending on x must cancel. With eq. 4 this gives

$$k'_n = k_n = k'_{n-1} = k_{n-1} = \dots = k'_1 = k_1 = \frac{\omega}{\alpha_1} \sin \varphi \quad (10)$$

which is an alternative form of Snell's law. With

$$\frac{\omega^2}{\alpha_j^2} = k_j^2 + l_j^2 = k_j'^2 + l_j'^2$$

this leads to

$$l'_j = l_j = \left(\frac{\omega^2}{\alpha_j^2} - k_1^2 \right)^{\frac{1}{2}} = \frac{\omega}{\alpha_j} \left(1 - \frac{\alpha_j^2}{\alpha_1^2} \sin^2 \varphi \right)^{\frac{1}{2}}. \quad (11)$$

If $\sin \varphi > \alpha_1 / \alpha_j$, l_j must be negative imaginary so that Φ_n remains bounded for $z \rightarrow \infty$.

Matrix formalism

Equations 7 and 9 lead to the following system of equations, that relate A_j and B_j with A_{j-1} and B_{j-1} , respectively

$$\begin{aligned}
 A_j - B_j &= \frac{l_{j-1}}{l_j} [A_{j-1} e^{-il_{j-1}d_{j-1}} - B_{j-1} e^{il_{j-1}d_{j-1}}] \\
 A_j + B_j &= \frac{\rho_{j-1}}{\rho_j} [A_{j-1} e^{-il_{j-1}d_{j-1}} + B_{j-1} e^{il_{j-1}d_{j-1}}]
 \end{aligned}$$

In matrix form

$$\begin{pmatrix} A_j \\ B_j \end{pmatrix} = \frac{e^{-il_{j-1}d_{j-1}}}{2l_j\rho_j} \begin{pmatrix} l_{j-1}\rho_j + l_j\rho_{j-1} & (-l_{j-1}\rho_j + l_j\rho_{j-1})e^{2il_{j-1}d_{j-1}} \\ -l_{j-1}\rho_j + l_j\rho_{j-1} & (l_{j-1}\rho_j + l_j\rho_{j-1})e^{2il_{j-1}d_{j-1}} \end{pmatrix} \cdot \begin{pmatrix} A_{j-1} \\ B_{j-1} \end{pmatrix} = m_j \cdot \begin{pmatrix} A_{j-1} \\ B_{j-1} \end{pmatrix} \quad (12)$$

where m_j is the layer matrix.

Matrix formalism

Repeated application of eq. 12 gives

$$\begin{pmatrix} A_n \\ B_n \end{pmatrix} = \underline{m}_n \cdot \underline{m}_{n-1} \cdot \dots \cdot \underline{m}_3 \cdot \underline{m}_2 \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \underline{M} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \quad (13)$$

The total layer matrix $\underline{M}(\omega, \varphi)$ can be calculated efficiently for a given frequency ω and angle of incidence φ of the incident P-wave. The l_j 's are calculated by eq. 15

$$l_j = \frac{\omega}{\alpha_j} \left(1 - \frac{\alpha_j^2}{\alpha_1^2} \sin^2 \varphi \right)^{\frac{1}{2}}$$

Reflectivity R_{pp} and transmittivity B_{pp}

1 Reflectivity

$$B_n = M_{21}A_1 + M_{22}B_1 = 0$$
$$R_{pp}(\omega, \varphi) := \frac{B_1}{A_1} = -\frac{M_{21}}{M_{22}} \quad (14)$$

2 Transmittivity

$$A_n = M_{11}A_1 + M_{12}B_1$$
$$B_{pp}(\omega, \varphi) := \frac{A_n}{A_1} = M_{11} - \frac{M_{12}M_{21}}{M_{22}} = \frac{\alpha_1}{\alpha_n} \left(M_{11} - \frac{M_{12}M_{21}}{M_{22}} \right) \quad (15)$$

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Reflectivity method

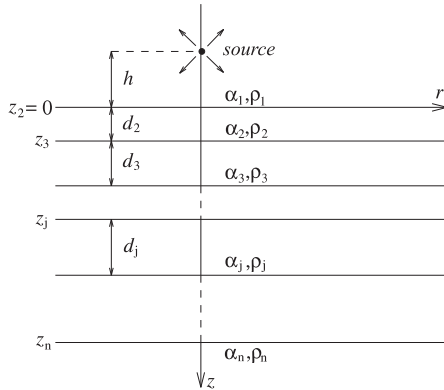


Figure 3: Reflectivity method: Calculation of point source response of a stack of homogeneous layers.

- Now we calculate the **point source response** of the stack of homogeneous layers.
- This is called **Reflectivity Method** (Fuchs 1968, Fuchs & Müller 1971).
- We assume that we **know** $R_{pp}(\omega, \varphi)$ from the product of layer matrices.

Potential Ansatz

The displacement potential of a point source located at height h and frequency ω is (see lecture TSW):

$$\Phi_{1e} = \frac{1}{R} e^{i\omega \left(t - \frac{R}{\alpha_1} \right)}, \quad R^2 = r^2 + (z + h)^2 \quad (16)$$

Wave equations in cylindrical coordinates (r, z)

$$\frac{\partial^2 \Phi_j}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi_j}{\partial r} + \frac{\partial^2 \Phi_j}{\partial z^2} = \frac{1}{\alpha_j^2} \frac{\partial^2 \Phi_j}{\partial t^2} \quad (17)$$

Potential Ansatz

Elementary solutions of this equation are

$$J_0(kr) \exp [i (\omega t \pm l_j (z - z_j))] , \quad l_j = \left(\frac{\omega^2}{\alpha_j^2} - k^2 \right)^{\frac{1}{2}} \quad (18)$$

$J_0(kr)$ is the Bessel function (see Appendix) of first kind and zeroth order.
Equation (18) is an analogue to the solutions

$$e^{-ikx} \cdot e^{i(\omega t \pm l_j(z-z_j))}$$

of the 2D wave equation in cartesian coordinates

$$\frac{\partial^2 \Phi_j}{\partial x^2} + \frac{\partial^2 \Phi_j}{\partial z^2} = \frac{1}{\alpha_j^2} \frac{\partial^2 \Phi_j}{\partial t^2}$$

Potential Ansatz

Together with (18), the functions

$$\int_0^\infty f(k) J_0(kr) e^{i(\omega t \pm l_j(z-z_j))} dk \quad \text{with} \quad l_j(k) = \left(\frac{\omega^2}{\alpha_j^2} - k^2 \right)^{\frac{1}{2}} \quad (19)$$

are also solutions of (17) if the integral converges. Therefore, we come to the ansatz

$$\Phi_j = \int_0^\infty J_0(kr) \left\{ A_j(k) e^{i(\omega t - l_j(z-z_j))} + B_j(k) e^{i(\omega t + l_j(z-z_j))} \right\} dk. \quad (20)$$

Incident wave

A spherical wave can be written as a superposition of plane harmonic waves (Sommerfeld integral, see Appendix):

$$\frac{1}{R} e^{i\omega \left(t - \frac{R}{\alpha_1}\right)} = \int_0^\infty J_0(kr) \frac{k}{i l_1} e^{i(\omega t - l_1 |z+h|)} dk, \quad l_1(k) = \left(\frac{\omega^2}{\alpha_1^2} - k^2 \right)^{\frac{1}{2}}, \quad k = \frac{\omega}{\alpha_1} \sin \varphi \quad (21)$$

We can therefore interpret the first part of Φ_1 with $z_1 = 0$ as the incident wave

$$\Phi_{1e} = \int_0^\infty J_0(kr) A_1 e^{i(\omega t - l_1(z-h))} dk, \quad A_1 = \frac{k}{i l_1} \quad (22)$$

Displacements

The second part is the reflected wave

$$\Phi_{1r} = \int_0^\infty J_0(kr) B_1 e^{i(\omega t + l_1(z-h))} dk \quad (23)$$

With

$$B_1(k) = A_1(k) R_{pp}(\omega, k) = \frac{k}{il_1} R_{pp}(\omega, k)$$

we get

$$\Phi_{1r} = \int_0^\infty \frac{k}{il_1} J_0(kr) R_{pp}(\omega, k) e^{i(\omega t + l_1(z-h))} dk$$

Displacements

The vertical displacement is

$$w_{1r}(r, z, \omega, t) = \frac{\partial \Phi_{1r}}{\partial z} = e^{i\omega t} \int_0^\infty k J_0(kr) R_{pp}(\omega, k) e^{il_1(z-h)} dk \quad (24)$$

The horizontal displacement is (with $J'_0(x) = -J_1(x)$)

$$u_{1r}(r, z, \omega, t) = \frac{\partial \Phi_{1r}}{\partial r} = e^{i\omega t} \int_0^\infty \frac{-k^2}{il_1} J_1(kr) R_{pp}(\omega, k) e^{il_1(z-h)} dk. \quad (25)$$

The integrals in (24) and (25) can be computed numerically.

Impulsive excitation

The transition to impulse excitation

$$\Phi_{1e} = \frac{1}{R} F \left(t - \frac{R}{\alpha_1} \right)$$

can be performed by Fourier transformation. Let $\bar{F}(\omega)$ denote the Fourier spectrum of $F(t)$. Then

$$\Phi_{1e} = \frac{1}{2\pi R} \int_{-\infty}^{+\infty} \bar{F}(\omega) e^{i\omega \left(t - \frac{R}{\alpha_1} \right)} d\omega$$

The corresponding displacements of the reflected wave are obtained via

$$\left. \begin{matrix} W_{1r}(r, z, t) \\ U_{1r}(r, z, t) \end{matrix} \right\} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \bar{F}(\omega) \left\{ \begin{matrix} w_{1r}(r, z, \omega, t) \\ u_{1r}(r, z, \omega, t) \end{matrix} \right\} d\omega \quad (26)$$

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Refraction and Reflection at Moho

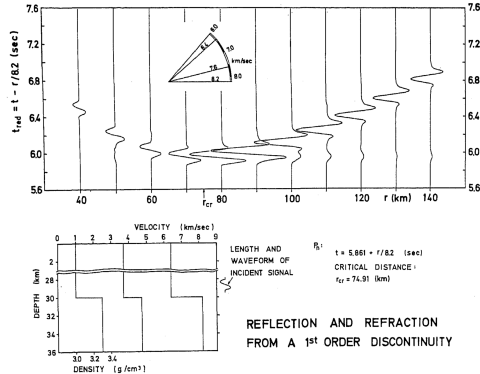


Figure 4: Moho as first order discontinuity ? (Fuchs 1968).

- Very first application: Nature of the Mohovičić-zone (Fuchs 1968)
- Beyond critical point $r^* = 74.91$ km high amplitudes and change of waveform
- NOT predictable by Zoeppritz equations (Červený effect)

Červený effect

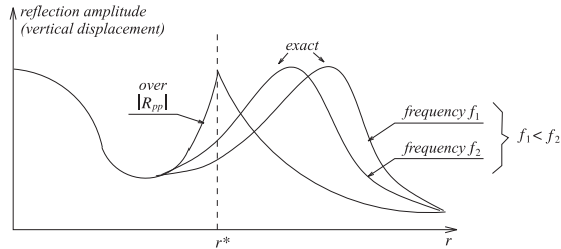


Figure 5: Reflection amplitude versus offset as a function of frequency (Červený 1961)

- According to Zoeppritz $|R_{pp}|$ has its maximum directly at the critical point. The reflectivity methods predicts a shift to larger distances.
- This is due to the interference of headwave and reflected wave, and pulse shape of the super-critical reflection.
- As an interference phenomenon, the shifting of the amplitude maximum of the reflected wave is dependent upon dominant frequency of the incident signal, depth of the reflector, and the velocity contrast at the reflector.

Complete seismograms for global earth

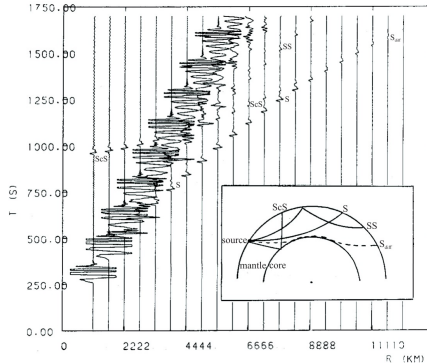


Figure 6: Global SH -seismograms at the Earth surface.

- SH -waves excited by a horizontal single force at the Earth's surface of 20 s dominant period.
- Love waves have large amplitudes
- Body wave phases are mantle wave S and SS , core reflection ScS and diffraction at the core S_{diff} .

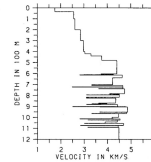
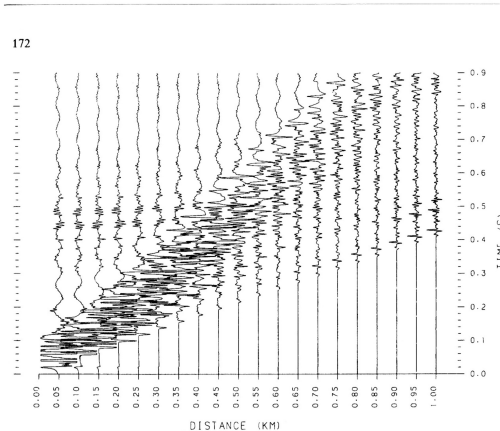


Fig. 7. *P* velocity-depth function of a model relevant to seismic prospecting for coal. The synthetic seismograms are shown in Fig. 8

Fig. 8. Vertical-component synthetic seismograms for the model of Fig. 7. The source is an explosion at depth 30 m. True amplitudes are shown, but they are partly clipped. No travel-time reduction has been applied. A weak numerical phase with (practically) infinite phase velocity is visible at short distances

Figure 7: Simulation of multiple reflections/refractions in coal (Mueller 1985).

The Reflectivity Method - Summary

The Reflectivity method is an efficient analytic method to calculate *complete* synthetic seismograms for layered media.

- 1 We first calculate the response of a stack of layers for an incident harmonic plane wave $R_{pp}(\omega, \varphi)$ by a matrix formalism.
- 2 Then we apply a numerical integration of $R_{pp}(\omega, \varphi)$ over wavenumber k to obtain the point source response.
- 3 We consider many frequencies by multiplication with the source spectrum and inverse Fourier transformation.

The method can be applied to efficiently calculate complete seismograms on a broad range of spatial scales (global seismology to near surface) where the changes in elastic parameters vary along one direction only.

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Bessel functions

The **differential equation** of the Bessel function of integer order $n = 0, 1, 2, \dots$ is

$$x^2 y'' + xy' + (x^2 - n^2) y = 0. \quad (27)$$

The two linearly independent solutions of this equation are

$$\begin{aligned} y &= J_n(x) = \text{Bessel function of first kind and } n - \text{order} \\ y &= Y_n(x) = \text{Bessel function of second kind and } n - \text{th order} \\ &\quad \text{or } \textit{Neumann's function of } n - \text{th order.} \end{aligned}$$

Graphical images of Bessel functions

The graphic representation for $x \geq 0$ of Bessel and Neumann functions:

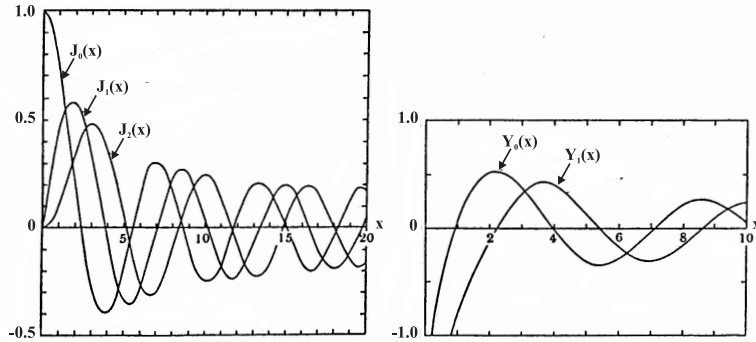


Figure 8: Graphs of Bessel and Neumann functions.

Hankel functions

Neumann's functions have a singularity at $x = 0$.

The **Hankel functions**, or Bessel functions of the third kind, are defined as

$$H_n^{(1)}(x) = J_n(x) + iY_n(x) \quad \text{Hankel function of first kind} \quad (28)$$

$$H_n^{(2)}(x) = J_n(x) - iY_n(x) \quad \text{Hankel function of second kind} . \quad (29)$$

$H_n^{(1)}(x)$ and $H_n^{(2)}(x)$ are linearly independent. The general solution of (27) is, therefore, (with the arbitrary constants A,B,C,D) either

$$y = AJ_n(x) + BY_n(x)$$

or

$$y = CH_n^{(1)}(x) + DH_n^{(2)}(x).$$

Analogies to Trigonometric Functions

There are analogies between Bessel functions and trigonometric functions. Trigonometric functions follow from the equation of oscillation:

$$y'' + n^2 y = 0$$

Fundamental solutions are $(\cos(nx), \sin(nx))$, and (e^{inx}, e^{-inx}) .

The analogies are

Bessel functions	Trigonometric functions
$J_n(x)$	$\cos nx$
$Y_n(x)$	$\sin nx$
$H_n^{(1)}(x)$	$e^{inx} = \cos nx + i \sin nx$
$H_n^{(2)}(x)$	$e^{-inx} = \cos nx - i \sin nx.$

Asymptotic representation

For $x \gg 1$ the functions can be approximated by

$$\left. \begin{aligned} J_n(x) &\simeq \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \cos\left(x - \frac{n\pi}{2} - \frac{\pi}{4}\right) \\ Y_n(x) &\simeq \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \sin\left(x - \frac{n\pi}{2} - \frac{\pi}{4}\right) \\ H_n^{(1)}(x) &\simeq \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \exp\left[i\left(x - \frac{n\pi}{2} - \frac{\pi}{4}\right)\right] \\ H_n^{(2)}(x) &\simeq \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \exp\left[-i\left(x - \frac{n\pi}{2} - \frac{\pi}{4}\right)\right] \end{aligned} \right\} \quad (30)$$

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Sommerfeld Integral

The Sommerfeld integral describes the rule for superposition of homogeneous (real l_1) and inhomogeneous plane waves (complex l_1) to build a spherical wave.

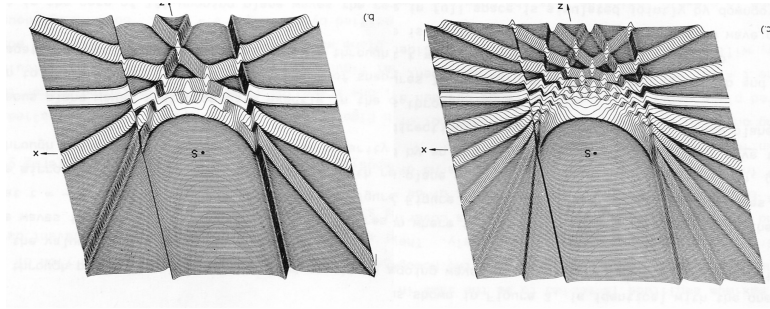


Figure 9: Superposition of homogeneous plane waves. 7 plane waves (left), 13 plane waves (right) (Tygel & Hubral 1987).

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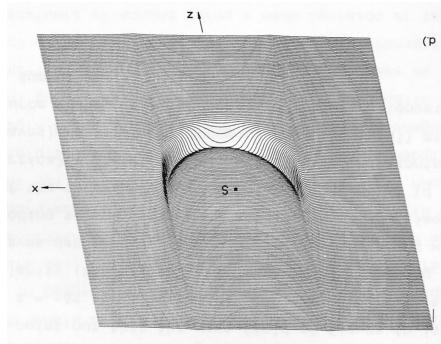


Figure 10: Superposition of 181 homogeneous plane waves. (Tygel & Hubral 1987).

Questions

- 1 Here we describe the RM for acoustic media. Can the RM method be extended to elastic media (P-SV) as well ? Are Rayleigh waves included in the RM for elastic media ?
- 2 Do we need a separate RM for SH/Love waves ?
- 3 Which parts of the RM are computationally most expensive ?
- 4 Can we easily model specific reflections only and ignore all other reflections and multiples ?
- 5 Can the RM also be used to calculate the transmittivity of a stack of layers ? Can the RM be applied to calculate VSP seismograms ?
- 6 Does the calculation time of the RM depend upon (a) number of layers ? (b) frequency bandwidth of the source, (c) distance between source and receiver ? (d) number of sources and receivers ?

Questions

- 7 Let us compare the RM with the 2D FD method for a layered model consisting of many layers such as the example on slide 30. Which method may be faster especially for large distances between source and receiver ? Should the RM and 2D FD method generally produce the same results (seismograms) ? Which of the two methods is more realistic ?
- 8 What is the so-called Cervený effect (slide 28)? Does this mean that the reflection amplitudes extracted from observed seismograms do not completely mimic the trend of reflection coefficients ?
- 9 Let us consider the Sommerfeld integral on slide 22 and the appendix. Can you think of other applications of this superposition ? (e.g. scattering)