Vorlesung 2

Tuesday, 30. April 2019 09:40

Lecture 2 30.04.2019

Quantumbit = Qubit

- quantum two-level system (TLS)
- pseudo spin 1/2
- independent of the qubit realization

E_e E_1 —— $|e\rangle\ |1\rangle\ |\downarrow\rangle$ $\qquad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$\alpha, \beta \in \mathbb{C}$

E_g E_0 —— $|g\rangle\ |0\rangle\ |\uparrow\rangle$ $\qquad \langle\psi|\psi\rangle = |\alpha|^2 + |\beta|^2 = 1$

vector rep. $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $\quad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ $\quad |\psi\rangle = \begin{pmatrix}\alpha\\\beta\end{pmatrix}$

Measurement

probabilities: $|\langle 0|\psi\rangle|^2 = |\alpha|^2$

$|\langle 1|\psi\rangle|^2 = |\beta^2|$

using methods of NMR

important protocols:

- Rabi → population oscillation
- Relaxation measurement ⇒ T_1
- Ramsey fringes ⇒ T_2^*
- Hahn spin echo ⇒ T_2

Hamiltonian of a TLS

$$\mathcal{H} = \frac{1}{2}\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22}\end{pmatrix} \quad \text{Hermitian matrix}$$

$H_{11}, H_{22} \in \mathbb{R}$

$H_{21} = H_{12}^*$

- we choose often $H_{11} > H_{22}$
- substruct global energy offset

$$\mathcal{H} = \frac{1}{2}\begin{pmatrix} \varepsilon & \Delta - i\tilde{\Delta} \\ \Delta + i\tilde{\Delta} & -\varepsilon\end{pmatrix} = \frac{\varepsilon}{2}\sigma_z + \frac{\tilde{\Delta}}{2}\sigma_y + \frac{\Delta}{2}\sigma_x$$

σ_i = Pauli matrix

$$\varepsilon = \frac{H_{11} - H_{22}}{2} > 0 \qquad \Delta, \tilde{\Delta} \in \mathbb{R}$$

Natural or physical basis $\{|\varphi_+\rangle, |\varphi_-\rangle\}$

$$\mathcal{H}_0 = \frac{1}{2}\begin{pmatrix}\varepsilon & 0 \\ 0 & -\varepsilon\end{pmatrix} = \frac{\varepsilon}{2}\sigma_z$$

$$\mathcal{H}_0|\varphi_\pm\rangle = \pm\varepsilon|\varphi_\pm\rangle$$

Diagonalization of $\mathcal{H}$

Eigenvalues

$$\det\begin{vmatrix}\varepsilon - 2E_\pm & \Delta - i\tilde{\Delta} \\ \Delta + i\tilde{\Delta} & -\varepsilon - 2E_\pm\end{vmatrix} = 0$$

$$\Rightarrow E_\pm = \pm\frac{1}{2}\sqrt{\varepsilon^2 + \Delta^2 + \tilde{\Delta}^2} = \pm\hbar\omega_q$$

Eigenvectors

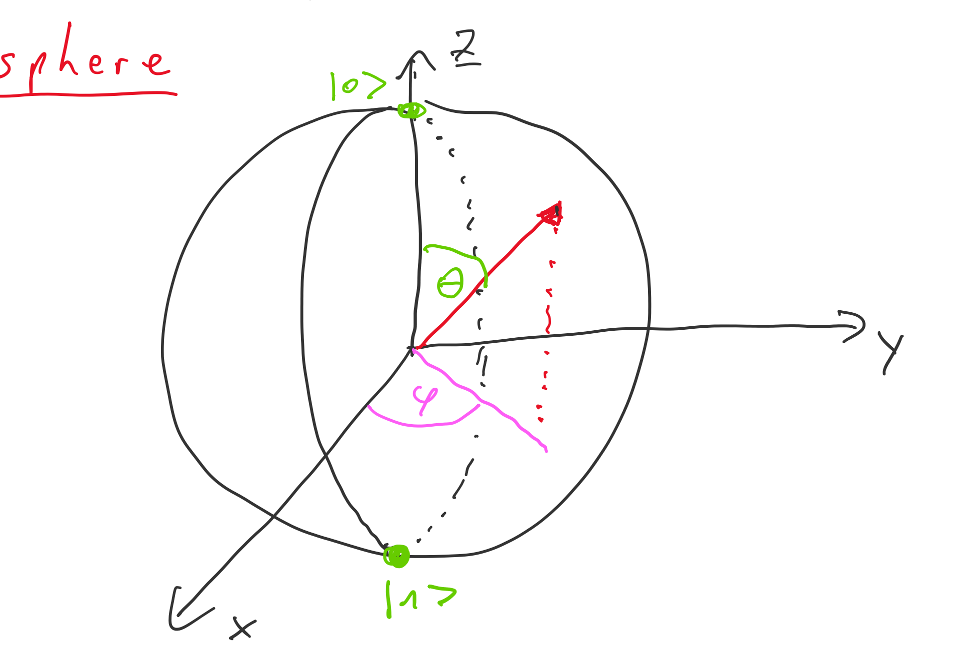
$$|\psi_+\rangle = e^{-i\varphi/2}\cos\left(\frac{\theta}{2}\right)|\varphi_+\rangle + e^{i\varphi/2}\sin\left(\frac{\theta}{2}\right)|\varphi_-\rangle$$

$$|\psi_-\rangle = -e^{-i\varphi/2}\sin\left(\frac{\theta}{2}\right)|\varphi_+\rangle + e^{i\varphi/2}\cos\left(\frac{\theta}{2}\right)|\varphi_-\rangle$$

$$\tan\theta \equiv \frac{\sqrt{\Delta^2 + \tilde{\Delta}^2}}{\varepsilon} \quad \text{with } 0 \le \theta \le \pi$$

$$\tan\varphi = \frac{\tilde{\Delta}}{\Delta} \quad \text{with } 0 \le \varphi \le 2\pi$$

Bloch sphere



- let's look to a spin 1/2 system

$$\mathcal{H}_1 = -\gamma\hbar\vec{B}\vec{S} = \frac{\gamma\hbar}{2}\begin{pmatrix} B_z & B_x - iB_y \\ B_x + iB_y & -B_z\end{pmatrix}$$

γ: gyromagnetic ratio

Quantum operation ≙ unitary transformation

⇒ 2 × 2 Matrix $A = \langle a_{ij}\rangle$

complex conjugated $A^* = \langle a_{ij}^*\rangle$

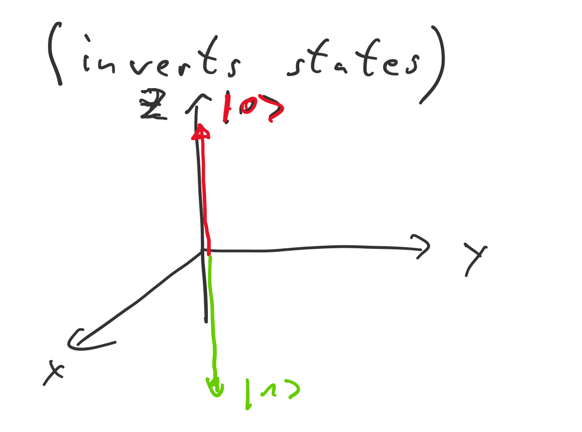
$A^\dagger = (A^*)^T$ $\quad$ T: transposed matrix

$A^\dagger$ is adjoint matrix of A

Def: A is unitary when $\boxed{A^\dagger = A^{-1}}$

1) Example: NOT gate (inverts states)

$$NOT = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$$



π-pulse (half a Rabi period)

2) Example: Hadamard gate (creates a superposition)

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix} \quad \text{is unitary}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Quantum register

classical: register of bits

ex: 3bits 000, 001, 010, … 111

↝ numbers 0, 1, 2 … $2^n - 1$

quantum: analogous

ex: 2 qubits $|x_0\rangle$ and $|x_1\rangle$

register $R = |x_1\rangle|x_0\rangle = |x_1 x_0\rangle$

$|x_0\rangle = \gamma_0|0\rangle + \gamma_1|1\rangle$

$|x_1\rangle = \beta_0|0\rangle + \beta_1|1\rangle$

$R = |x_1\rangle|x_0\rangle = (\beta_0|0\rangle + \beta_1|1\rangle)(\gamma_0|0\rangle + \gamma_1|1\rangle)$

$\alpha_{ij} = \beta_i\gamma_j$

$R = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$

binary → numbers

$R = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle + \alpha_3|3\rangle$

general: $$\boxed{R = \sum_{i=0}^{2^n-1}\alpha_i|i\rangle \qquad \sum_{i=0}^{2^n-1}|\alpha_i|^2 = 1}$$

$|0\rangle = |000\ldots0\rangle$

$|1\rangle = |000\ldots1\rangle$

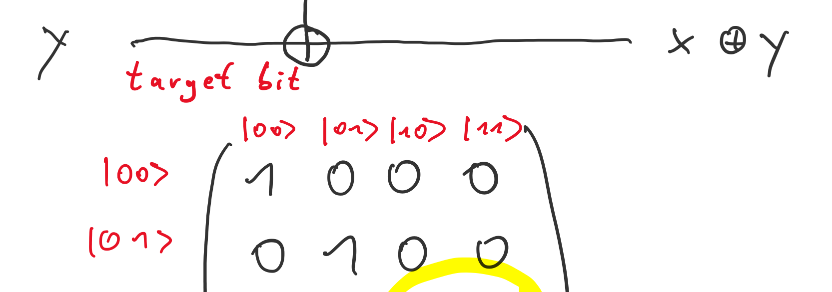
$|2^n-1\rangle = |111\ldots1\rangle$

Quantum operation

Ex: CNOT-gate: "controlled not"

the unitary version of XOR (exclusive OR)

$CNOT: |x, y\rangle \longrightarrow |x, x \oplus y\rangle$



	$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$
$\|00\rangle$	1	0	0	0
$\|01\rangle$	0	1	0	0
$\|10\rangle$	0	0	0	1
$\|11\rangle$	0	0	1	0

↝ permutation matrix

$$CNOT\begin{pmatrix}a_0\\a_1\\a_2\\a_3\end{pmatrix} = \begin{pmatrix}a_0\\a_1\\a_3\\a_2\end{pmatrix}$$

> A quantum algorithm can be decomposed to local operations involving 2 or 3 qubit

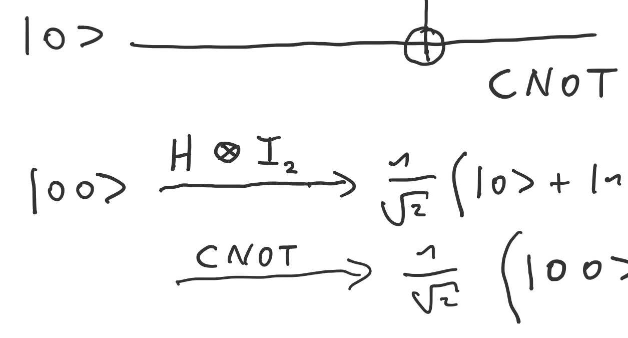
The matrix A is unitary when $A^{-1} = A^\dagger$ (↑ inverse, ↑ adjoint)

Properties

- vector length $\|\psi\| = \sqrt{\langle\psi|\psi\rangle}$ is conserved
  $\| U|\psi\rangle \| = \| |\psi\rangle \|$
- angle of scalar product is conserved
  $\langle U\phi | U\psi\rangle = \langle\phi|\psi\rangle$
- rotation about origin or reflexion along a axis going through origin
- can be reversed
- linear transformation

Entanglement

two qubit register $|b_1 b_2\rangle$



$CNOT: |x, y\rangle \to |x, y \oplus x\rangle$

$$|00\rangle \xrightarrow{H \otimes I_2} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

$$\xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$