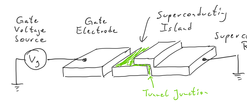
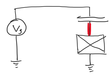
# Vorlesung 5-6

Monday, 29. June 2020 21:07

## 2.5 Cooper Pair Box (CPB) (late 90's)

- charge qubit: charge degree of freedom is used for coupling and interaction
- CPB consists of a superconducting island into which Cooper pairs may tunnel via a Josephson junction



Hamiltonian

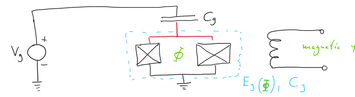
$$\mathcal{H} = \underbrace{\frac{(Q_J - C_J V_g)^2}{2\,C_\Sigma}}_{\text{capacitive / charging energy}} - \underbrace{E_J \cos\delta}_{\text{Josephson-inductive energy}} \qquad C_\Sigma = C_g + C_J$$

$\delta$: phase across the junction
$Q_J$: charge of the island

- with charging energy scale $E_c = \frac{e^2}{2\,C_\Sigma}$
  number of Cooper pairs $\boxed{n = Q_J/2e}$ $\quad n_g = \frac{C_g V_g}{2e}$

$$\mathcal{H} = 4\,E_c\,(n - n_g)^2 - E_J \cos\delta$$

### Split Junction - split Cooper Pair Box




- two parallel junctions replace the single junction
- this pair merely creates an effective single junction
- Josephson energy can be tuned in situ by applying a magnetic field

### Quantization of Hamiltonian at the sweet spot

- quantize CPB circuit using the commutation relation $[n, \delta] = i$
- for dominating charging energy of the island $E_c \gg E_J$
  $\Rightarrow$ choice of basis states $\{|n\rangle\}$ number of Cooper pairs

a) charge basis

Eigenvalue problem:

$$\left[4\,E_c\,(\hat{n} - n_g)^2 - E_J \cos\left(2\pi\frac{\Phi}{\Phi_0}\right)\right]\psi_m = E_m\,\psi_m$$

i) $\hat{n}|n\rangle = n|n\rangle \qquad n \in \mathbb{Z}$

ii) $\hat{n} = \sum_{n=-\infty}^{\infty} n\,|n\rangle\langle n| \qquad \sum |n\rangle\langle n| = \mathbb{1}$ ← negative in respect to the effect

(iii) $\cos\left(2\pi\frac{\Phi}{\Phi_0}\right) = \frac{1}{2}\left(e^{i2\pi\Phi/\Phi_0} + e^{-i2\pi\Phi/\Phi_0}\right)$

$[\hat{n}, \hat{\varphi}] = i \qquad [\hat{n}, e^{i\varphi}] = e^{i\varphi}$

$e^{-i2\pi\Phi/\Phi_0}|n\rangle = |n \pm 1\rangle$

$$\Rightarrow H = \sum_n 4\tilde{E}_c\,(n - n_g)^2 |n\rangle\langle n| - \sum_n \frac{E_J}{2}\Big[\underbrace{|n+1\rangle\langle n|}_{e^{-i2\pi\Phi/\Phi_0}} + \underbrace{|n\rangle\langle n+1|}_{e^{-i2\pi\Phi/\Phi_0}}\Big]$$

$$\begin{array}{c} \langle -N| \\ \langle -N+1| \\ \vdots \\ \\ \langle N+1| \end{array} \begin{pmatrix} 4\tilde{E}_c(-N-n_g)^2 & -\frac{E_J}{2} & 0 & \cdots & \\ -\frac{E_J}{2} & 4E_c(-N+1-n_g)^2 & -\frac{E_J}{2} & \cdots & \\ 0 & -\frac{E_J}{2} & \ddots & \ddots & \\ \vdots & & \ddots & \ddots & -\frac{E_J}{2} \\ & & & -\frac{E_J}{2} & 4\tilde{E}_c(+N-n_g)^2 \end{pmatrix}$$

b) phase basis $\quad \hat{\varphi}|\varphi\rangle = \varphi|\varphi\rangle$

$$\hat{n} = -i\frac{\partial}{\partial\varphi}$$

$$H = 4\tilde{E}_c\left(-\frac{\partial}{\partial\varphi} - n_g\right)^2 - E_J\cos(\hat{\varphi})$$

boundary condition: $\psi_k(\varphi) = \psi_k(\varphi + 2\pi)$

transformation:

$$\tilde{\psi}_k = e^{-ing\hat{\varphi}}\,\psi_k(\varphi)$$

$$\Rightarrow H\tilde{\psi}_k = -4\tilde{E}_c\frac{\partial^2}{\partial\varphi^2}\tilde{\psi}_k(\varphi) - E_J\cos(\varphi)\,\tilde{\psi}_k(\varphi)$$

Bloch - theorem: The energy eigenstates of a particle in a periodic potential are Bloch waves

$$\varphi_{nk}(r) = e^{ikr}\,u(r)$$

↑ plane wave ← periodic fct with same periodicity as potential

$$\varphi_{nk}(\varphi) = e^{ikr}\,u(\varphi)$$

$u(\varphi) = u(\varphi + 2\pi) = \cos(\varphi)$

Ansatz: $\tilde{\psi}_k(\varphi) = e^{ik\varphi}\,u(\varphi)$

$$= e^{-ing\varphi}\,u(\varphi)$$

$\hookrightarrow u(\varphi) = \psi(\varphi)$

Fourier Ansatz:

$$\tilde{\psi}_{m,ng}(\varphi) = e^{-ing\varphi}\frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty} C_{m,l}\,e^{il\varphi} \qquad l \in \mathbb{Z}$$

$$= \frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty} C_{m,l}\,e^{i(l-ng)\varphi}$$

$l$ has an integer value (spectrum) to satisfy periodicity $(2\pi)$

$$-4\tilde{E}_c\frac{\partial^2}{\partial\varphi^2}\left(e^{-ing\varphi}\frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty} C_{m,l}\,e^{il\varphi}\right) -$$

$$\frac{E_J}{2}\left(e^{i\varphi} + e^{-i\varphi}\right)\left(\frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty} e^{i(l-ng)\varphi}\right) = E_{m,ng}\frac{1}{\sqrt{2\pi}}\sum_{l=-\infty}^{\infty} e^{i(l-ng)\varphi}\,C_{m,l}$$

$$\sum_{l=-\infty}^{\infty} 4\,E_c\,(l-ng)^2\,C_{m,l} - \frac{E_J}{2}\left(C_{m,l+1} + C_{m,l-1}\right) - E_{m,ng}\,C_{m,l} = 0$$

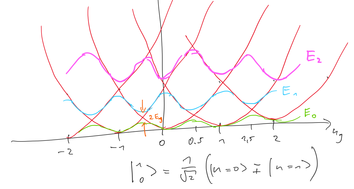
$\frac{1}{\sqrt{2\pi}}e^{-i(l-ng)\varphi}$

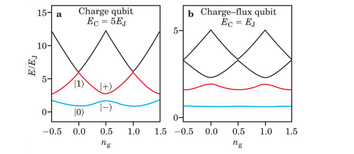
Matrix representation:

$$4\tilde{E}_c\,(l-ng)^2\,C_{m,l} - \frac{E_J}{4}\left(C_{m,l+1} + C_{m,l-1}\right) - E_{m,ng}\,C_{m,l} = 0$$

$$\tilde{H} - E\,\mathbb{1} = 0 \qquad \text{basis: Fourier coefficients} \qquad \vec{C}_m = \begin{pmatrix} C_{m,-\infty} \\ \vdots \\ C_{m,+\infty} \end{pmatrix}$$

Band structure $\quad E_m(n_g)$




$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|n=0\rangle \mp |n=1\rangle\right)$$