Vorlesung 7

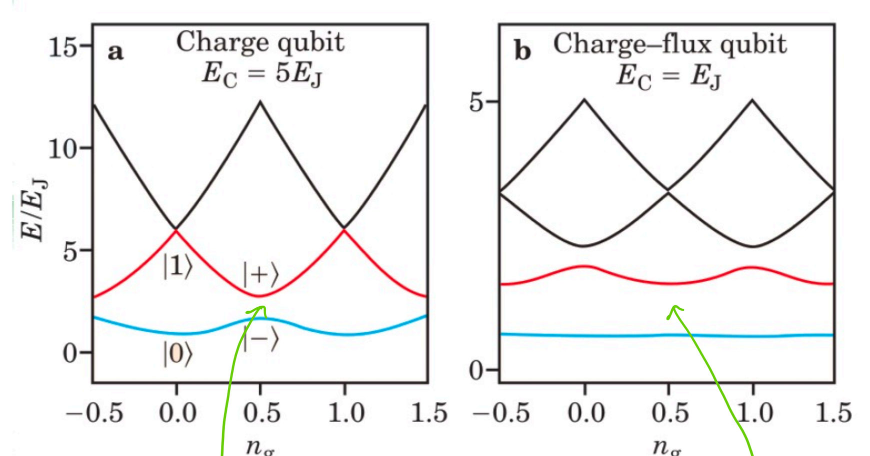
Wednesday, 15. July 2020 09:27

2.6 Capacitively-shunted CPB

The ratio E_J/E_C

So far, $E_C \gg E_J$ (charge qubit)

E_J/E_C affects anharmonicity and sensitivity to charge noise



- for E_J/E_C very large, the Hamiltonian begins to approach a harmonic oscillator (see below) ~ small anharmonicity
- need to find some middle ground

Anharmonicity

- As E_J/E_C is raised, the $\cos\delta$ term dominates in the picture of a particle in a periodic potential, the atomic wells become deeper ⇒ tight binding model
  ⇒ Energie will thus depend on the shape of the potential near the bottom of the well at $\delta = 0$

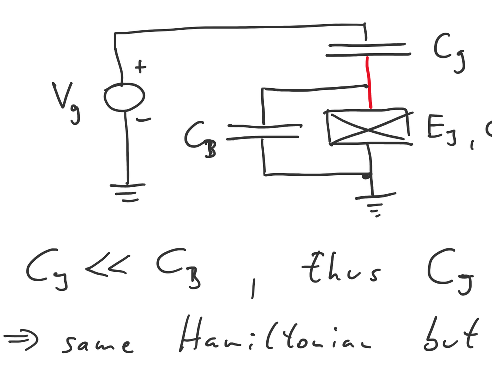
$$\cos\delta \approx 1 - \delta^2/2 + O(\delta^4)$$

⇒ only perturpation of harmonic oscillator

$$\frac{E_{n2} - E_{01}}{E_{01}} \approx -\left(8E_J/E_C\right)^{-1/2}$$

Shunting the CPB

- standard way to increase E_J/E_C
  ⇒ add a large shunt capacitance



$C_J \ll C_B$, thus C_J can be absorbed into C_B

⇒ same Hamiltonian but $C_J \to C_B$

⇒ $E_C \approx$ hundreds of MHz $\quad E_C = \frac{1}{2C}Q^2$

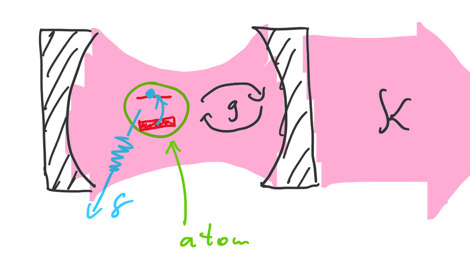
$E_J \approx$ tens of GHz

- how to couple to such a qubit, which is no longer dominated by charging energy?

2.7 Circuit QED ← Quantum Electrodynamics

- analogous to Cavity QED (confined atom-light interaction)
- provide a controllably isolated environment for qubit
- inhibiting spontaneous decay (similar to Purcell effect)
- allows non-destructive measurements
- allows coupling of qubits

Vocabulary of Cavity QED



g: coupling strength between atom and electromag. field in cavity

γ: interaction with other modes

κ: loss from imperfect cavity

ω_r: cavity mode frequency

Ω: atomic transition freq.

$\{|g\rangle, |e\rangle\}$: two-level atomic system

Hamiltonians

$$\mathcal{H}_{atom} = \frac{\hbar\Omega}{2}\sigma_z \quad \leftarrow \text{pauli matrix}$$

$$\mathcal{H}_{cavity} = \hbar\omega\left(a^+a + \tfrac{1}{2}\right) \quad \text{(harmonic oszillator)}$$

- atomic dipole moment couples with the electric fields of the cavity mode
- electric field is analogous to the position operator of the harmonic oscillator ⇒ $\mathcal{E}_{rms}(a + a^+)$
  ↑ root-mean-square electric field of a single cavity mode
- dipole moment D of the atom is off-diagonal in the Pauli basis since atomic energy eigenstates themselves have no dipole moments
  ⇒ $D = d\sigma_x = d(\sigma^+ + \sigma^-)$ where $d = |\langle g|D|e\rangle|$

  Interaction $\mathcal{H}_{int} = \mathcal{E}_{rms}\, d\,(a + a^+)(\sigma^+ + \sigma^-)$
- Using the rotating wave approximation to eliminate the quickly oscillating terms

$$\Rightarrow \mathcal{H}_{int} = \hbar g\left(a\sigma^+ + a^+\sigma^-\right) \qquad g = \mathcal{E}_{rms}d/\hbar$$

$$\Rightarrow \boxed{\mathcal{H}_{JC} = \frac{\hbar\Omega}{2}\sigma_z + \hbar\omega\left(a^+a + \tfrac{1}{2}\right) + \hbar g\left(a\sigma^+ + a^+\sigma^-\right)}$$

Jaynes-Cummings Hamiltonian

- works well in the "strong coupling" limit: $g \gg \kappa, \gamma$
  that is, other cavity modes are far detuned and leakage is small
- scales: optical transitions (350 THz for Cesium)
  microwave transitions (51 GHz for Rydberg atoms)
  dipole moments (ea_0 or $10^3\, ea_0$, respectively)
  ⇒ coupling $g = 110 \times 2\pi$ MHz or $24 \times 2\pi$ kHz
  ⇒ time scale $2\pi/g$ for coupling effects is longer than the transition time scale by a factor of $> 10^6$
  ⇒ extremely high quality factors are needed $Q \sim 10^7 - 10^8$
- diagonalizing the Jaynes-Cummings Hamiltonian
  ⇒ "dressed states" which mix the atomic eigenstates with the cavity states

excited states: (atom, cavity photon number)

$$|\overline{+,n}\rangle = +\cos\theta_n|e,n\rangle + \sin\theta_n|g,n+1\rangle$$
$$|\overline{-,n}\rangle = -\sin\theta_n|e,n\rangle + \cos\theta_n|g,n+1\rangle$$

mixing angle θ_n: $\quad \theta_n = \frac{1}{2}\tan^{-1}\left(\frac{2g\sqrt{n+1}}{\Delta}\right)$

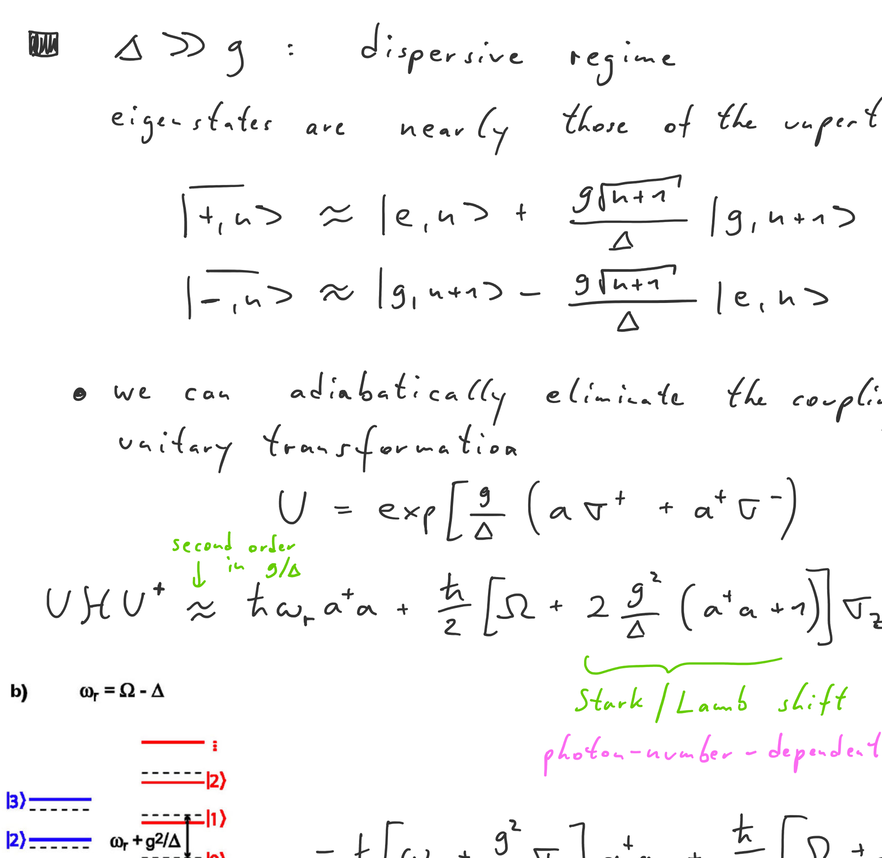
detuning $\Delta = \Omega - \omega_r$

Energies $\quad E_{\overline{\pm,n}} = (n+1)\hbar\omega_r \pm \frac{\hbar}{w}\sqrt{4g^2(n+1) + \Delta^2}$

$$E_{g,0} = -\frac{\hbar\Delta}{2}$$

- limiting cases:
  - $\Delta \ll g$: atom is resonant with cavity : $\theta_n = \frac{\pi}{4}$
    eigenstates are equal combinations of excited atoms with n photons and de-excited atoms with $n+1$ photons
    ⇒ spectrum : set of doublets

    an excited atom is not an eigenstate of the combined system
    for ex. placing an initially excited atom into an initially empty cavity ⇒ oscillation between $|e,0\rangle$ and $|g,1\rangle$ at frequence $2g$ (Rabi frequency)



  - $\Delta \gg g$: dispersive regime
    eigenstates are nearly those of the unperturbed Hamiltonian

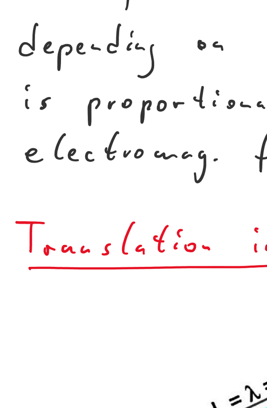
$$|\overline{+,n}\rangle \approx |e,n\rangle + \frac{g\sqrt{n+1}}{\Delta}|g,n+1\rangle$$
$$|\overline{-,n}\rangle \approx |g,n+1\rangle - \frac{g\sqrt{n+1}}{\Delta}|e,n\rangle$$

- we can adiabatically eliminate the coupling via the unitary transformation

$$U = \exp\left[\frac{g}{\Delta}\left(a\sigma^+ + a^+\sigma^-\right)\right]$$

$$U\mathcal{H}U^+ \approx \hbar\omega_r a^+a + \frac{\hbar}{2}\left[\Omega + 2\frac{g^2}{\Delta}\left(a^+a + 1\right)\right]\sigma_z$$

(second order in g/Δ; Stark / Lamb shift; photon-number-dependent shift)



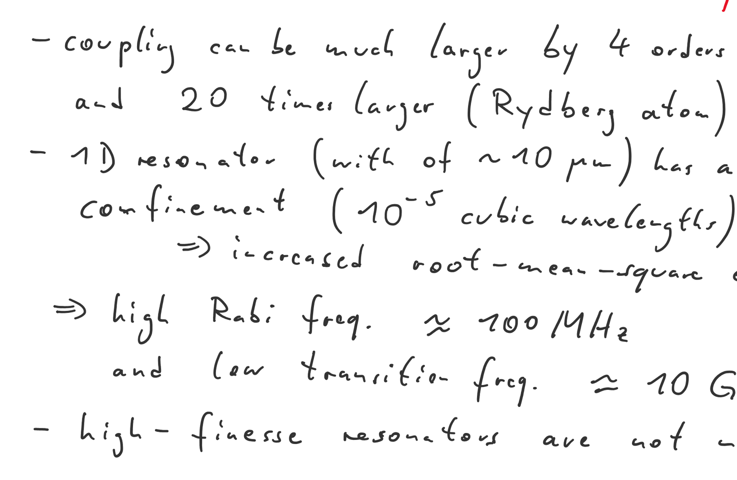
$$= \hbar\left[\omega_r + \frac{g^2}{\Delta}\sigma_z\right]a^+a + \frac{\hbar}{2}\left[\Omega + \frac{g^2}{\Delta}\right]\sigma_z$$

shift of cavity freq.
if atom is in $|e\rangle \sim \omega_r + g^2/\Delta$
$|g\rangle \sim \omega_r - g^2/\Delta$

Purcell effect:

cavity can enhance or reduce the spontaneous emission depending on detuning, because the atomic level decay is proportional to the density of states of the local electromag. field at the atomic freq. (Fermi's golden rule)

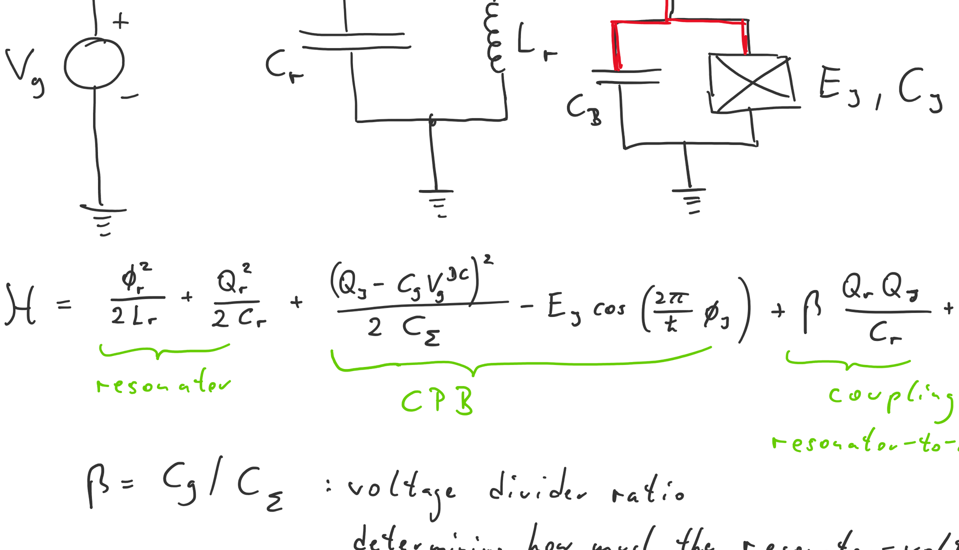
Translation into Circuit QED (cQED) [Cavity QED, CQED]



Why circuit QED is easier than Cavity QED?

- coupling can be much larger by 4 orders of magnitude (atom) and 20 times larger (Rydberg atom)
- 1D resonator (with of ~10 µm) has a smaller volume of confinement (10^{-5} cubic wavelengths)
  ⇒ increased root-mean-square electric field
  ⇒ high Rabi freq. ≈ 100 MHz and low transition freq. ≈ 10 GHz
- high-finesse resonators are not needed

Circuit QED Hamiltonian



$$\mathcal{H} = \underbrace{\frac{\phi_r^2}{2L_r} + \frac{Q_r^2}{2C_r}}_{\text{resonator}} + \underbrace{\frac{\left(Q_J - C_gV_g^{DC}\right)^2}{2C_\Sigma} - E_J\cos\left(\frac{2\pi}{\hbar}\phi_J\right)}_{\text{CPB}} + \underbrace{\beta\frac{Q_rQ_J}{C_r}}_{\text{resonator-to-qubit}} + \underbrace{\frac{C_{in}Q_rV_g}{C_r}}_{\text{resonator-to-gate}}$$

(coupling: resonator-to-qubit, resonator-to-gate)

$\beta = C_g/C_\Sigma$: voltage divider ratio determining how much the resonator-voltage is seen by the qubit

V_{DC} : DC baising

quantization ⇒ Jaynes-Cumming

$$\mathcal{H} = \hbar\omega_r\left(a^+a + \tfrac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma_z - \hbar g\left(a^+\sigma^- + a\sigma^+\right)$$

$$g = \left(2e\beta V^0_{rms}/\hbar\right)\langle e|n|g\rangle \quad \leftarrow n:\ \text{number operator}$$

for the charge-qubit, $\langle e|n|g\rangle = 1/2$ at the sweet-spot