Karlsruher Institut für Technologie

Institut für Theorie der Kondensierten Materie

Exercises for "Superconductivity, Josephson ..." WS 2023/2024

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Exercise 4

Tutorial on 23.01.2023

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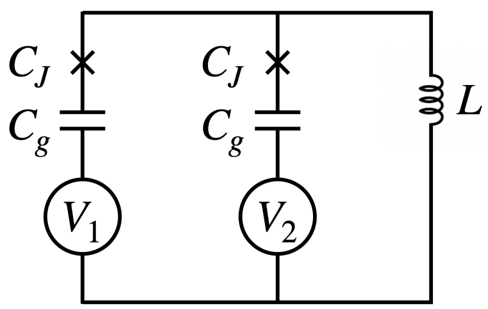
1. Inductively coupled charge qubits: (50 Punkte)



In the picture above you can see a possible realization of two charge qubits, which are coupled to one another inductively via inductance L.

As in the lecture, assign a phase to each element (in units of magnetic flux) and write down the Lagrangian. Take into account that the Josephson junctions, are characterized by Josephson energies E_J (not indicated in the picture) and by capacitances C_J respectively. Using Kirchhoff laws, express the Lagrangian as a function of the phases (and their temporal derivatives) on the Josephson junctions and that of the inductor. Carry out a Legendre transformation and determine the Hamiltonian of the system.

2. Numerical diagonalization of the charge qubit Hamiltonian : (50 Punkte)

In the lecture we have obtained the following Hamiltonian of the charge qubit

$$H = E_C(n - q_g)^2 - E_J \cos\phi \ . \tag{1}$$

Here n is the number of extra Cooper pairs on the island, q_g is the dimensionless gate charge ($q_g = Q_g/2e$). The operator $e^{i\phi}$ is defined by $e^{i\phi}|n\rangle = |n+1\rangle$. Perform numerical diagonalization of the above Hamiltonian (you can use Mathematica for example). Use the charge basis $|n\rangle$ and restrict the number of basis states (using physical arguments) in order to obtain a matrix of a finite dimension. Plot 3 lowest eigen-energies as functions of q_g. Investigate the regimes: a) $E_J \ll E_C$, b) $E_J \sim E_C$, c) $E_J \gg E_C$.