Karlsruher Institut für Technologie

Vorlesung:

Teilchenphyisk I (Particle Physics I)

Units and Conventions

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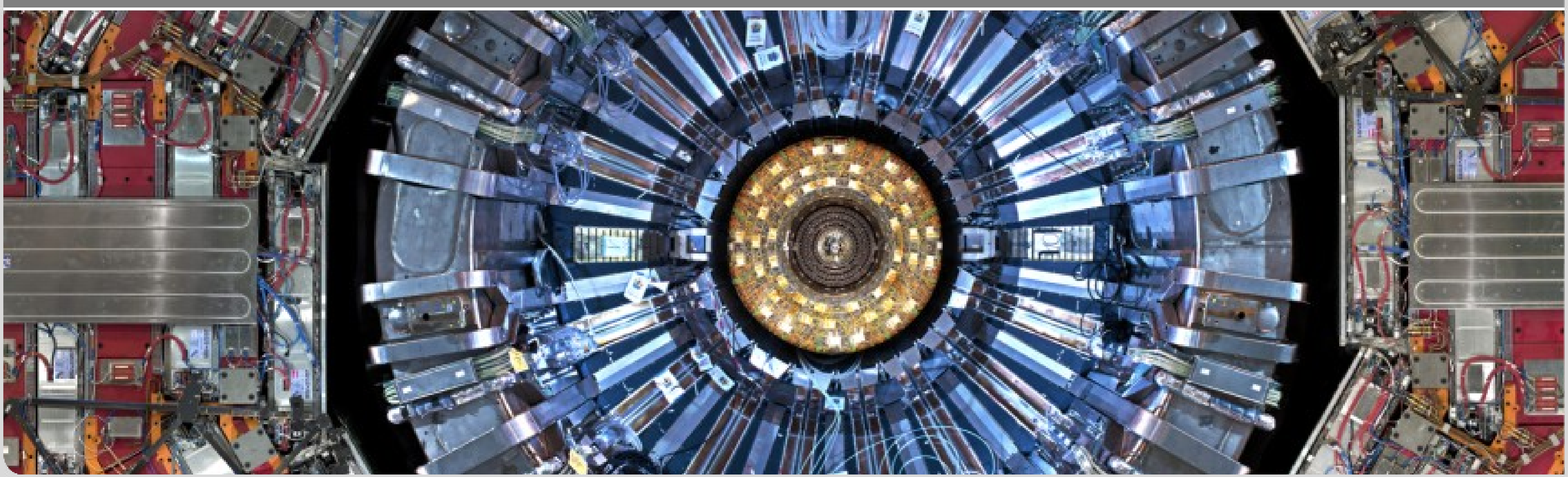


Natural Units

SI units, ptb.de

- Common choice in particle Physics

  „natural units“ with $\hbar = c = 1$

  - very pratical in particle physics,

but different from SI unit system

- Consequences:

- $c = 1 \rightarrow [L] = [T]$ (length and time have same unit)
- $\hbar = 1 \rightarrow [E][T] = 1$, from Heisenberg's uncertainty principle

$\rightarrow$ Length and time have units of 1/energy: $[L] = [E]^{-1}$, $[T] = [E]^{-1}$

- $E^2 = (pc)^2 + (mc^2)^2$, $c = 1$

$\rightarrow$ Momentum and mass have unit of energy: $[p] = [E]$, $[m] = [E]$

Relativistic Kinematics

- ***4-vectors*** in 4-dimensional space-time (Minkowski space):

$$x^\mu = (t, x, y, z) \quad \text{with} \quad g_{\mu\nu} x^\mu x^\nu = x_\nu x^\nu = t^2 - x^2 - y^2 - z^2$$

- ***Lorentz transformation*** from inertial frame of reference S to S' (moving with velocity v relative to S)
    - Relativistic ***boost***: $\beta = \frac{v}{c}$ (in 3D: $\vec{\beta} = \frac{\vec{v}}{c}$)
    - Relativistic γ ***factor***: $\gamma = (1 - \beta^2)^{-1/2}$
    - Lorentz transformation

      in 1 spatial dimension:

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

Units of Energy

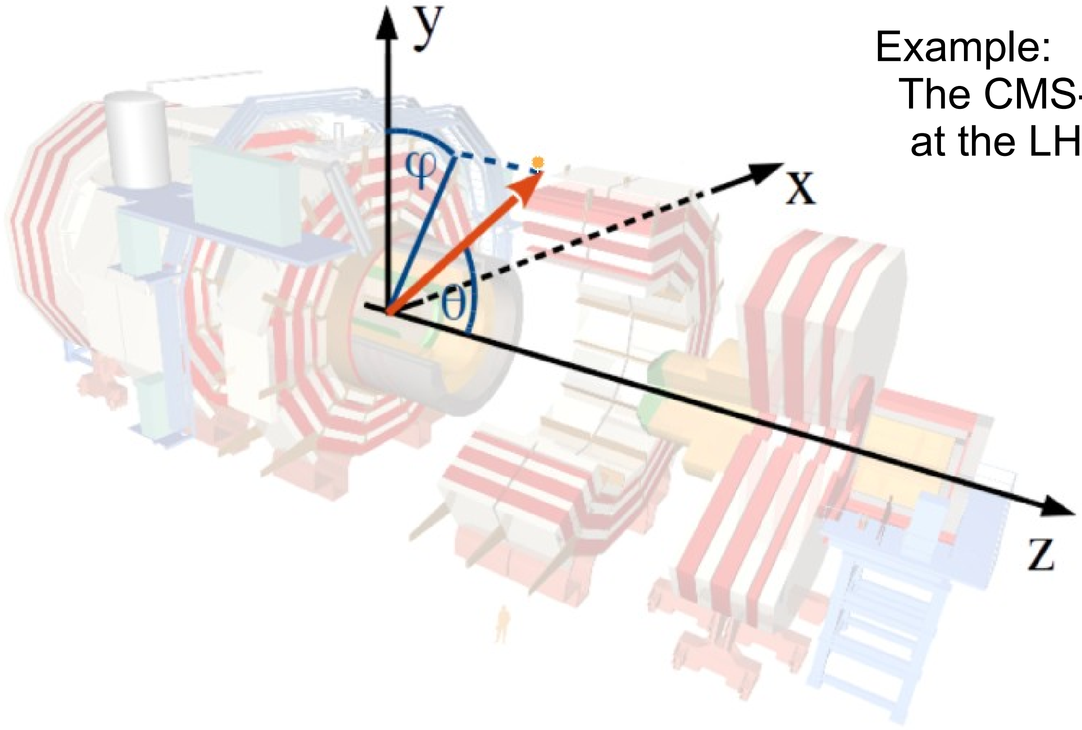
- Unit of energy in particle physics
  - Electronvolt (eV)
  - 1 eV = amount of energy gained by particle with charge $e = 1.609 \cdot 10^{-19}$ C of a single electron (elementary charge) moving across an electric potential difference of 1 V
- ***Typical energy scales*** in particle and astroparticle physics
  - Neutrino masses: meV
  - Flavour physics: GeV
  - Physics at the LHC: TeV
  - Cosmic rays: up to EeV
- Useful ***identities***

$$\hbar = 6.6 \cdot 10^{-25}\,\text{GeV}\,\text{s} \quad \rightarrow \quad 1\,\text{GeV}^{-1} \approx 6.6 \cdot 10^{-25}\,\text{s}$$

$$\hbar c = 197\,\text{MeV}\,\text{fm} \quad \rightarrow \quad 1\,\text{fm} \approx 5\,\text{GeV}^{-1}$$

Kinematics at Collider Detectors

- Conventions for kinematic variables at collider detectors
  (motivated by cylindrical symmetry of detectors)
  - ***Right-handed cylindrical coordinates*** system
  - Azimuthal angle ϕ: angle to x axis in xy plane
  - Polar angle θ: angle to z axis (beam axis)



Example:
The CMS-Detector
at the LHCm http://cms.cern

Rapidity

- **Rapidity** y very conveniently used to describe a Lorentz boost (here along the *z* axis)

$$\begin{pmatrix} ct' \\ z' \end{pmatrix} = \begin{pmatrix} \cosh y & -\sinh y \\ -\sinh y & \cosh y \end{pmatrix} \begin{pmatrix} ct \\ z \end{pmatrix}$$

with $$y = \tanh^{-1} \beta_z = \frac{1}{2} \ln \left(\frac{1+\beta_z}{1-\beta_z} \right) = \frac{1}{2} \ln \left(\frac{E+p_z}{E-p_z} \right)$$

$$\beta_z = \frac{v_z}{c}$$ is the component parallel to z axis

- relation y ↔ β is similar to
  α (the angle between a straight line and the x axis) ↔ s (the slope, with $\alpha = \tan^{-1}(s)$):
- angles or rapidities are additive, slopes or (relativistic) velocities are not
  (two subsequent rotations or Lorentz boosts)
- upon a global rotation or Lorentz boost of the coordinate system,
  difference in angles or rapidities remain constant

Rapidity and Pseudo-Rapidity

At hadron colliders,

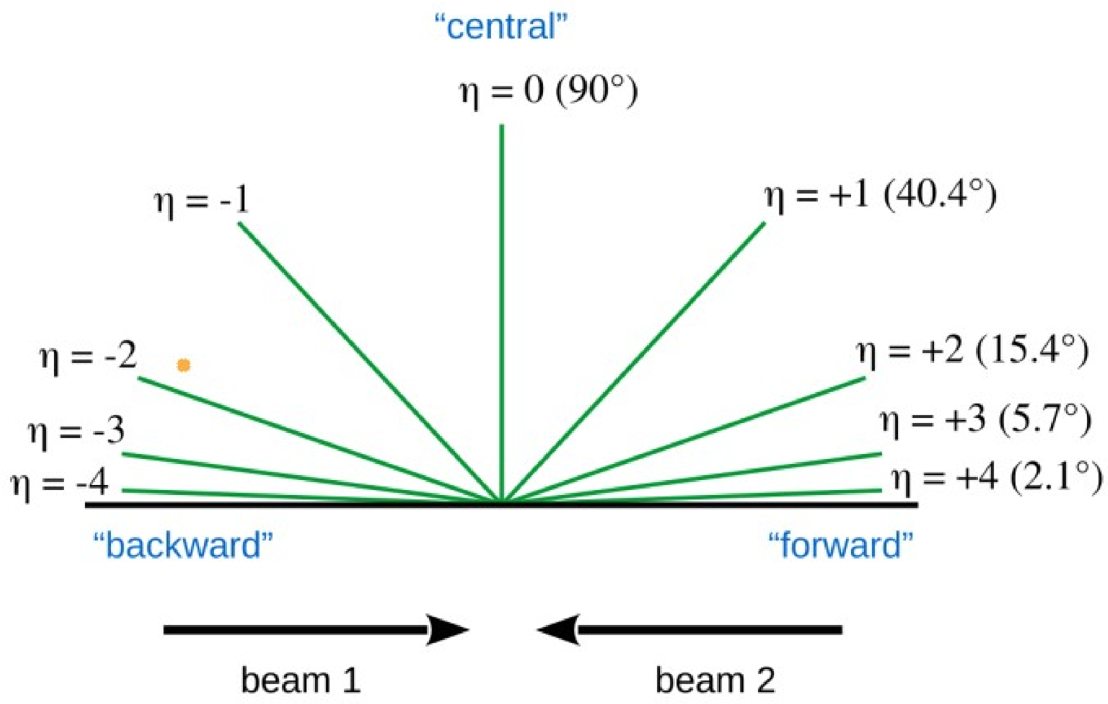
where the centre-of-mass system of a collision is not at rest,

y is the proper variable,

or the pseudo-rapidity $\eta = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right]$

for massless particles (or $p_z >> m$) $\eta = y$

note: η is easy to measure, y is not !



Rapidity and invariant cross section

As a consequence of the above (dy being Lorentz-invariant w.r.t. boosts in z),

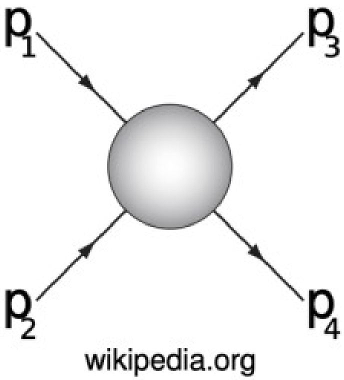
$$\frac{d\sigma}{dy}, \quad \frac{d\sigma}{d\phi}$$ and hence the double-differential cross section $$\frac{d^2\sigma}{dy\,d\phi}$$

are Lorentz-invariant w.r.t. boosts in z-direction

$$\frac{d\sigma}{d\eta}$$ is invariant for high-energetic particles with negligible rest mass

Mandelstam Variables

- ***2 → 2 scattering process*** with 4-momenta p_i $(i = 1, \ldots 4)$



- 4-momentum conservation: $p_1 + p_2 = p_3 + p_4$

- ***Mandelstam variables*** (4-momentum of intermediate particle)

$$
\boxed{
\begin{aligned}
s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \\
t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 \\
u &= (p_1 - p_4)^2 = (p_2 - p_3)^2
\end{aligned}
}
$$

  - s, t, u are Lorentz invariant, have unit $[\text{energy}]^2$

Cross section measurement

Master formula:

signal candidates

background events

$$\sigma = \frac{N_{cand} - N_{bkg}}{\epsilon \cdot \int L}$$

acceptance and efficiency

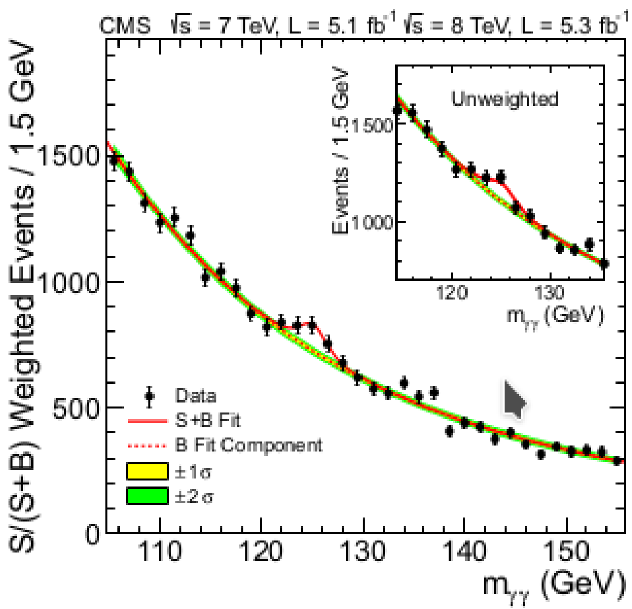
integrated Luminosity

product of many factors:
– detector acceptance
– trigger efficiency
– reconstruction efficiency
– selection efficiency
– background rejection efficiency

usually determined using well-known reference reaction:

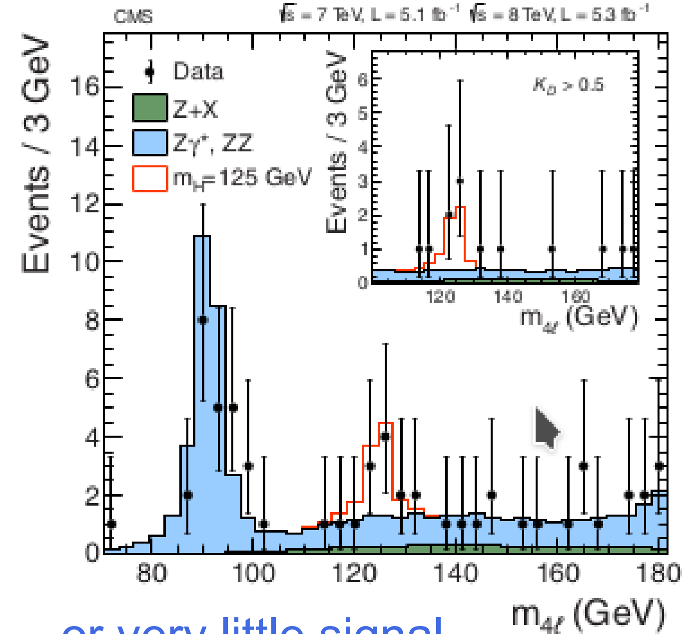
$$\int L = N_{ref} \,/\, \sigma_{ref}$$

Signal & Background



CMS 2012, „Observation of a new boson ...“

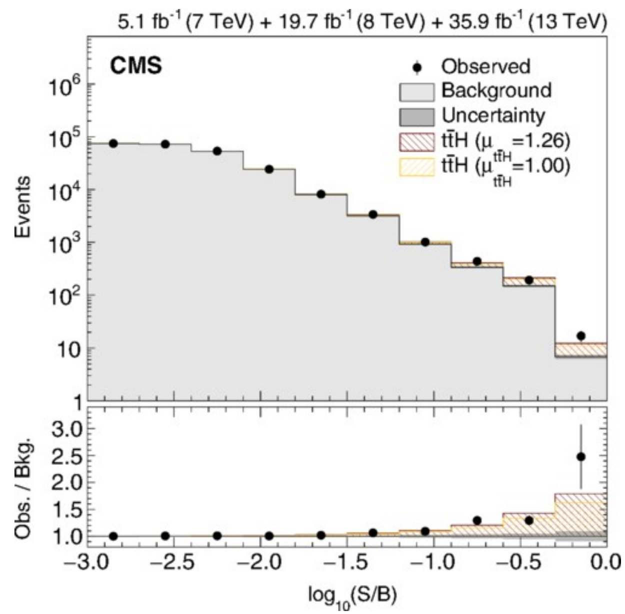
very high background ...



… or very little signal

CMS 2012, „Observation of a new boson ...“

very sophisticated „discriminating variable“ (artificial neural network)



CMS 2012, „Ovservation of ttH production“

Cross Section measurement: uncertainties

by error propagation →

$$\frac{\delta\sigma}{\sigma} = \sqrt{\frac{\delta N_{cand}^2 + \delta N_{bkg}^2}{(N_{cand} - N_{bkg})^2} + \left(\frac{\delta\epsilon}{\epsilon}\right)^2 + \left(\frac{\delta \int L}{\int L}\right)^2}$$

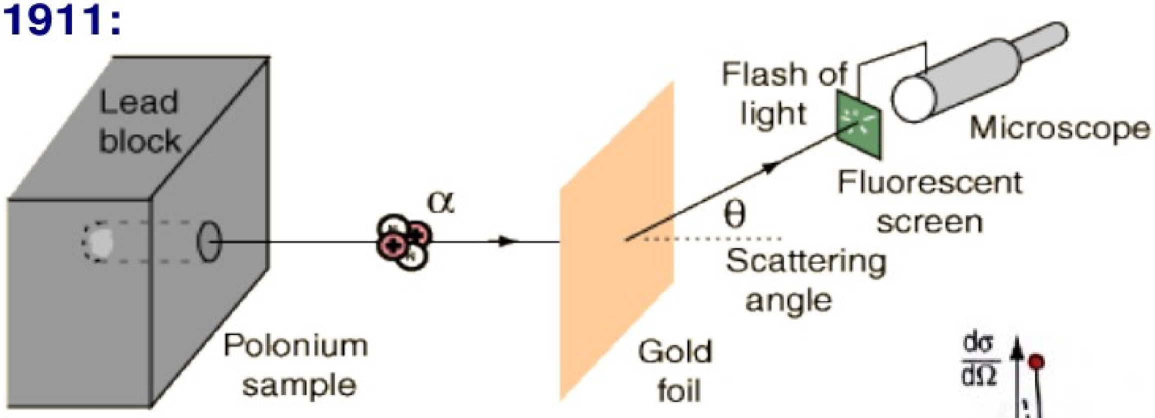
This is the error you want to <u>minimize</u>

- with signal as large as possible
- background as small as possible
- nonetheless, want large efficiency
- luminosity error small (typically beyond your control, also has a "theoretical" component)

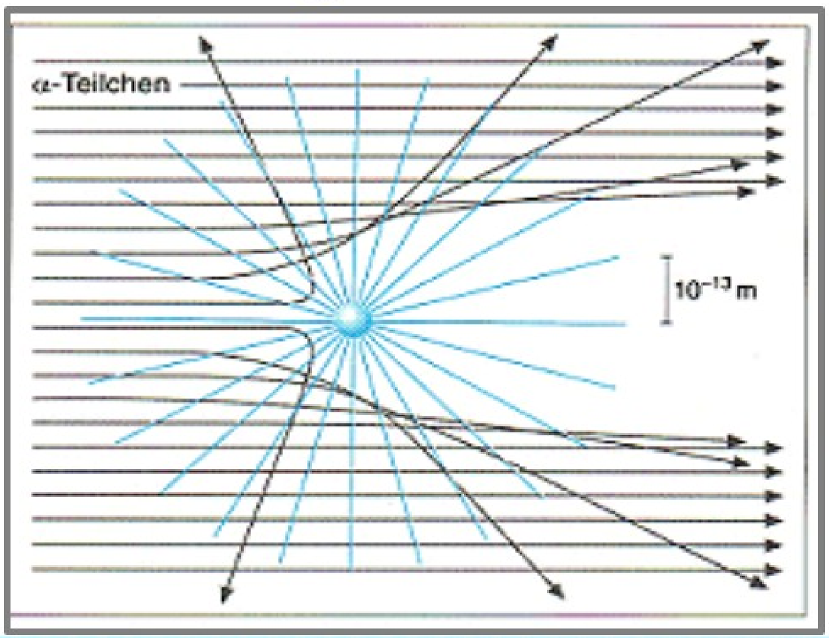
Reprise: the mother of all particle physics experiments

Rutherford scattering experiment: α -particles on gold foil

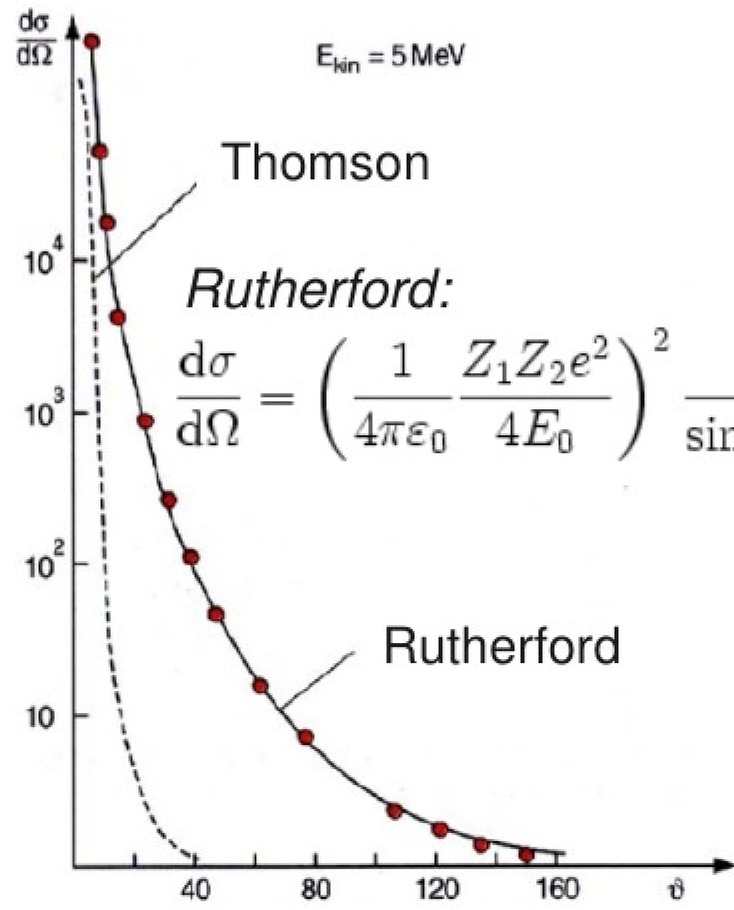
1911:



Observation:



Rutherford **cross section**



Rutherford:

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{4E_0} \right)^2 \frac{1}{\sin^4\left(\frac{\vartheta}{2}\right)}$$

Cross section: relation to theoretical calculations

cross section:

transition rate initial → final state

in theory

Fermi's golden rule

$$\lambda_{i\to f} = 2\pi |M_{fi}|^2 \rho$$

amplitude or "matrix element" of underlying process (M_{fi})

phase space (ρ)

Cross Section

$$\sigma_{i\to f} = \frac{|\mathcal{M}|^2 \cdot [\text{Phase space}]}{[\text{Colliding particle flux}]}$$

experimentally

determined from

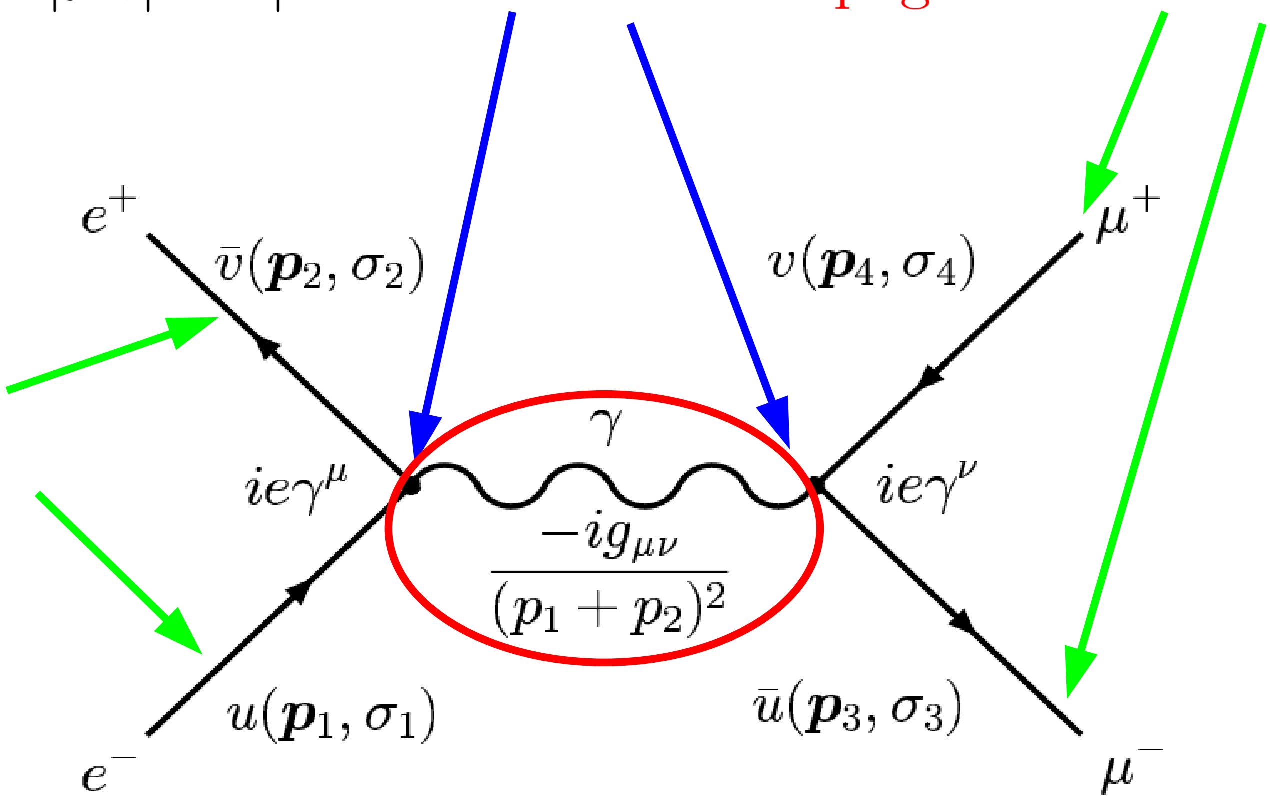
$\mathrm{N}_{\mathrm{cand}}$: number of observed events
N_{bkd} : number of expected background events
ϵ : acceptance $\cdot$ efficiency
f : flux
T : measurement time

$$\sigma_{i\to f} = \frac{N_{cand} - N_{bkg}}{\epsilon \cdot f} \frac{1}{T}$$

Matrix elements and Feynman Diagrams

Matrix elements are represented by **“Feynman diagrams”**

$$|\mathcal{M}|^2 = |\text{Vertex factors} \cdot \text{Propagator} \cdot \text{External lines}|^2$$



- elements (external lines, vertices, internal lines) represent terms in Lagrange density of the underlying theory

Calculations of the phase space density in reactions involving many particles (and hence a high-dimensional phase space) require **Monte Carlo** techniques

Reprise: electron-nucleon scattering

Including the electron **spin** and **recoil of the nucleus**,
the Rutherford cross section becomes the **Mott cross section**:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{(Z\alpha)^2 E^2}{4k^4 \sin^4 \frac{\theta}{2}} \left(1 - \frac{k^2}{E^2} \sin^2 \frac{\theta}{2}\right)$$

with the electron momentum k = | $\mathbf{k_i}$| = | $\mathbf{k_f}$ | and electron energy E

Taking into account also the finite size of the nucleus,
the cross section is multiplied by the form factor F($\mathbf{q}^2$)

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left|F(\mathbf{q}^2)\right|$$

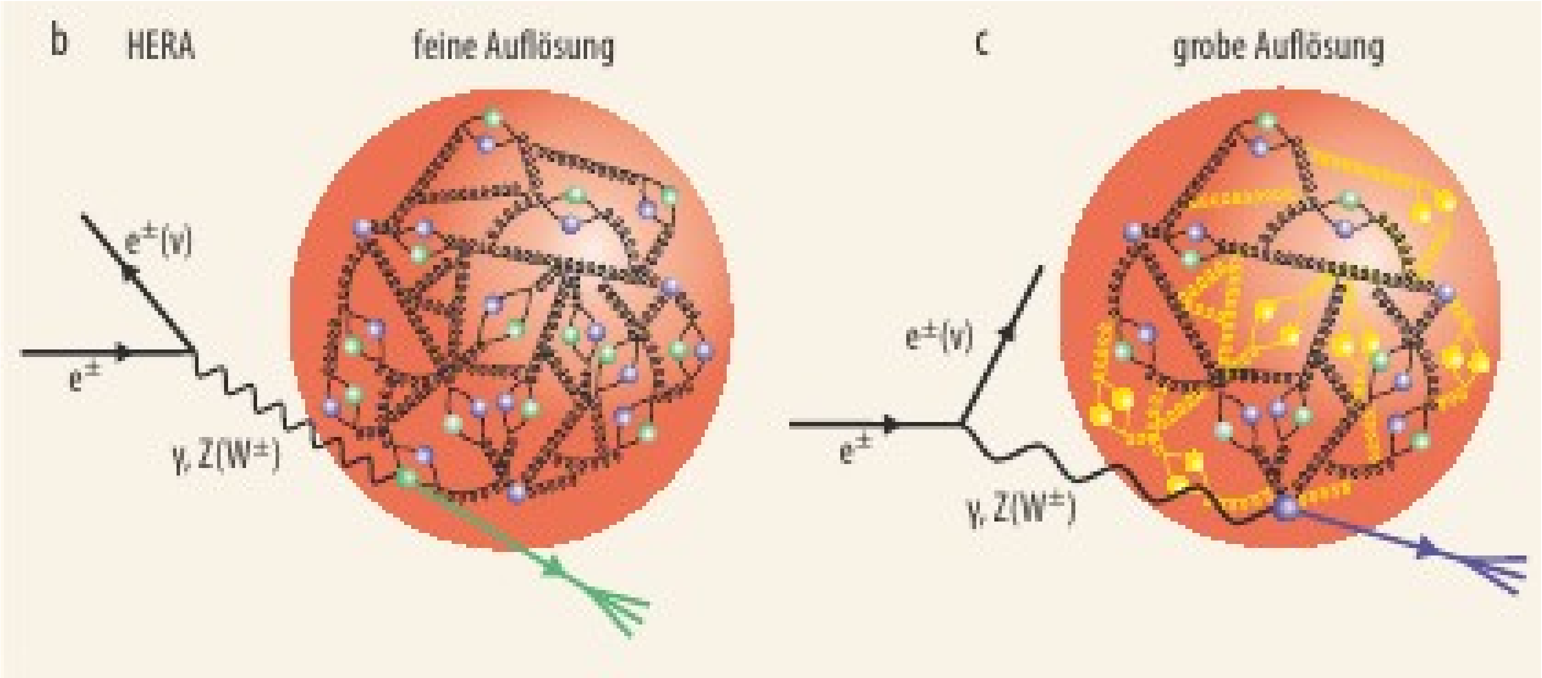
F($\mathbf{q}^2$) is the Fourier transform of the charge density ρ(**r**), Q $\mathbf{q} = \mathbf{k}_i - \mathbf{k}_f$ ist the momentum transfer

Reprise: the Proton

in fact, the proton is much more complicated:

composed of
- **valence quarks**
- **sea quarks**
- **gluons** (carry 50% of momentum)

Precision study of proton composition in electron-proton scattering
HERA at DESY in Hamburg

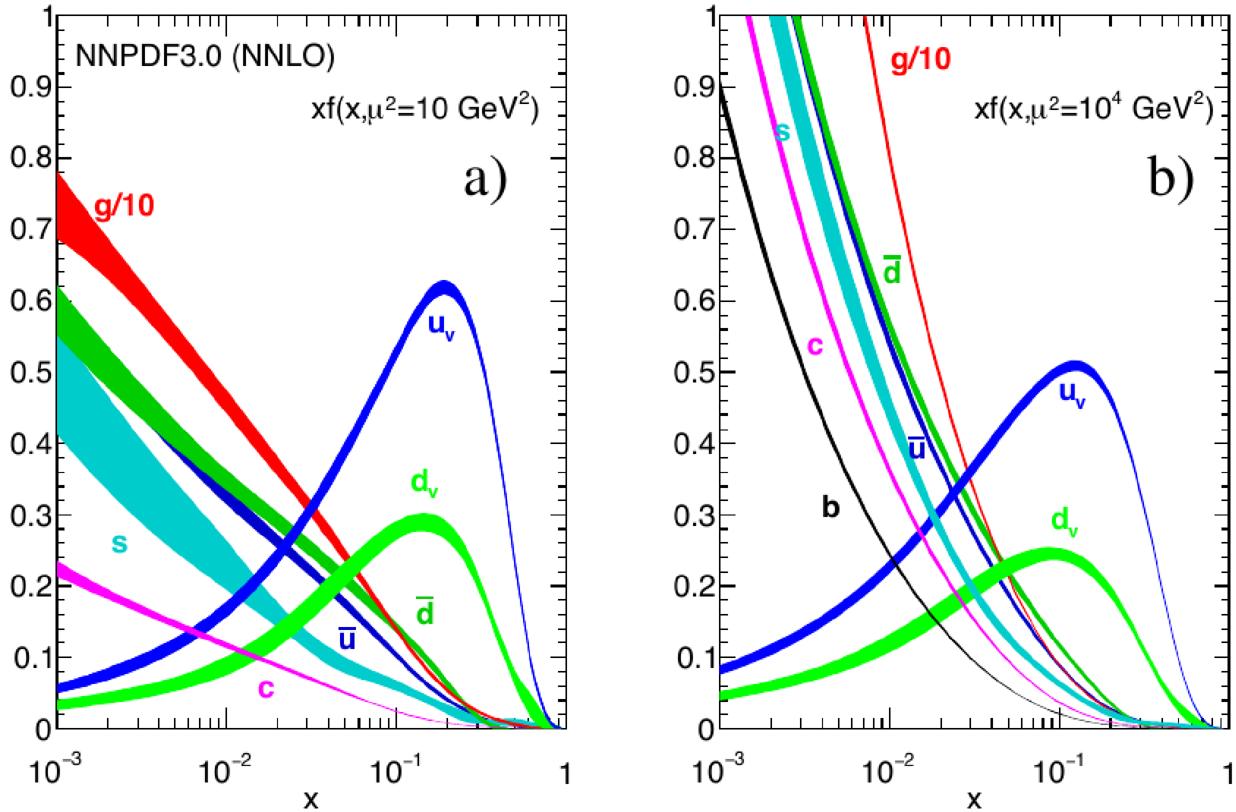


Parton Density Distributions (PDFs) ...

… describe the contributions of qurks, anti-quarks and gluons to the proton momentum:

PDFs are determined from measurements of suitable processes

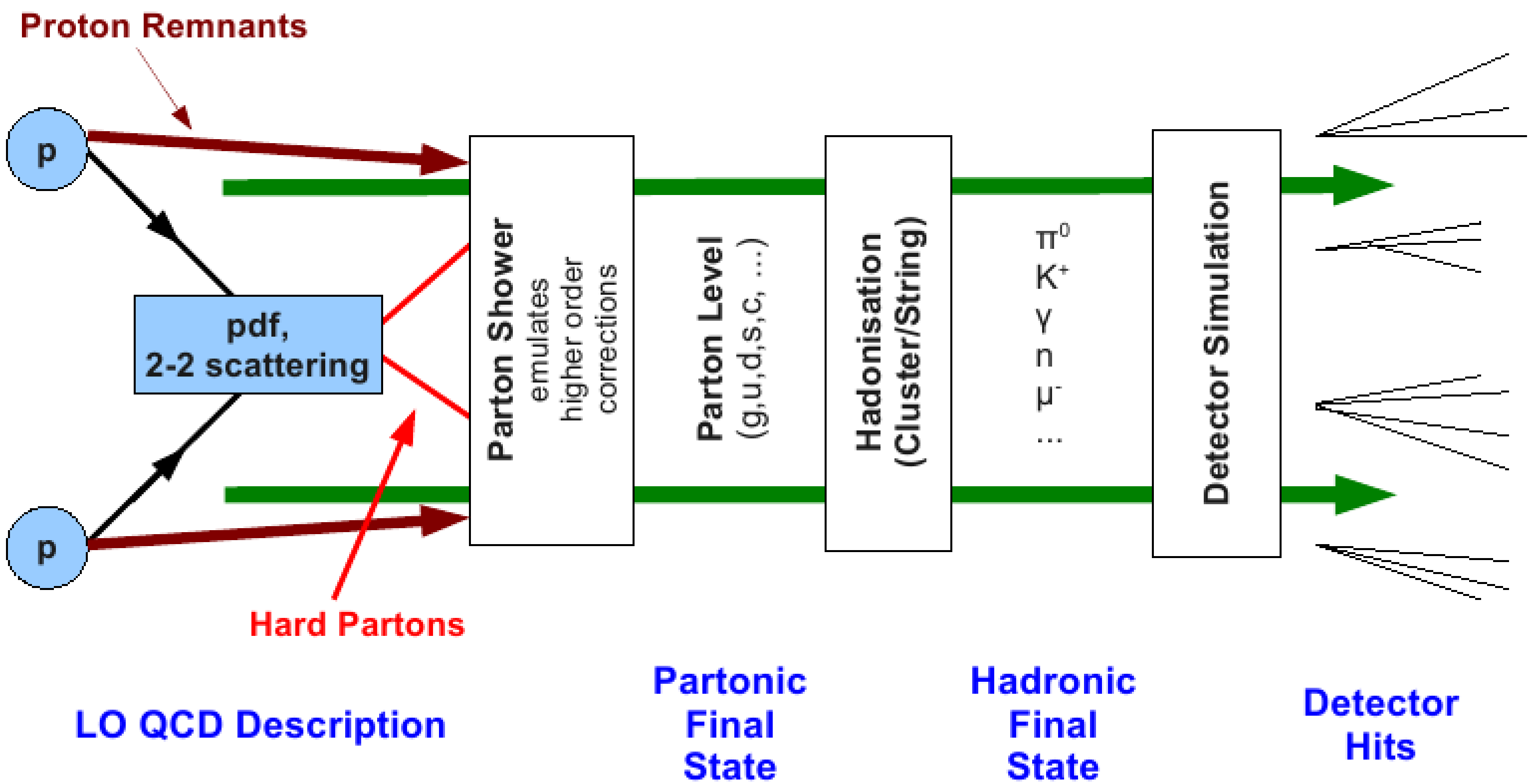
example:
PDF set by the NNPDF group →



PDG 2018

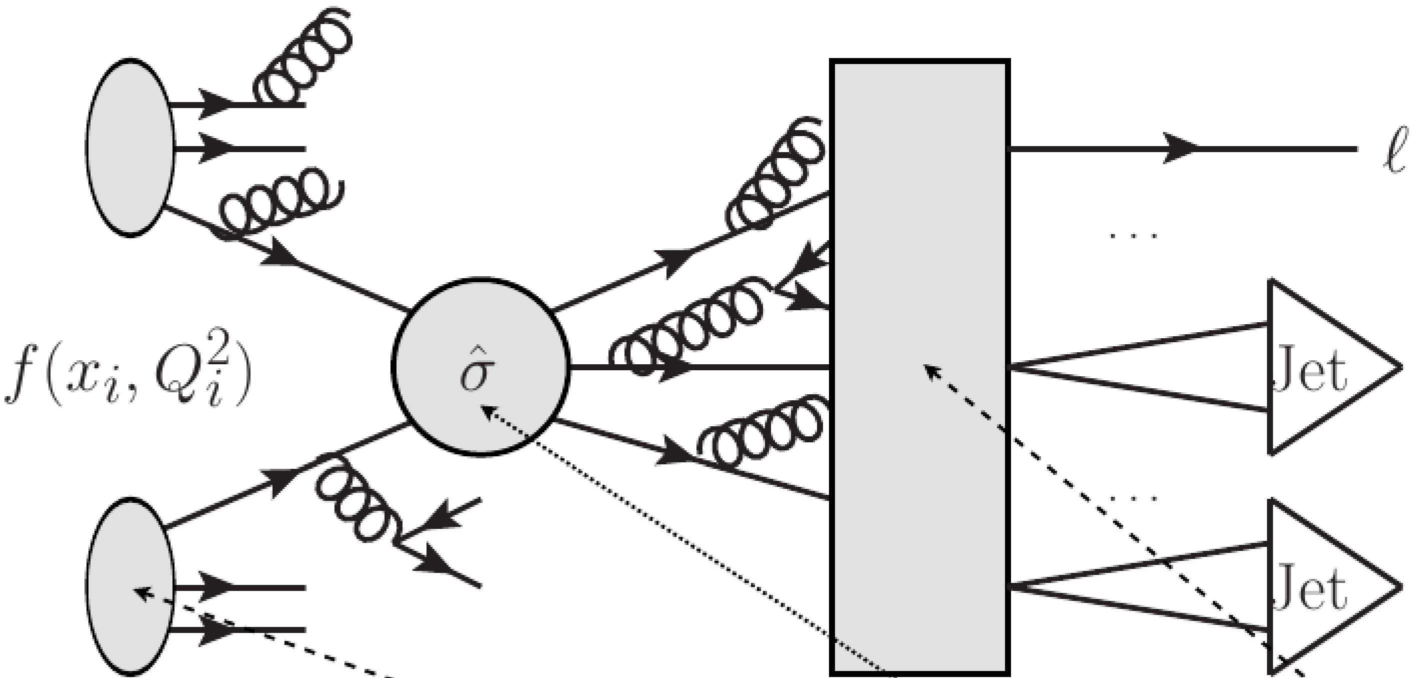
Parton Densitiy Functions (PDFs) have to be taken into account when calculating cross sections at hadron colliders.

pp → final state is a multi-step process



Calculation of Cross sections

σ = PDFs ⊗ 2→n process ⊗ hadroniszation



$$\sigma_{QCD} = \sum_{jk} \int dx_j \, dx_k \; f_j(x_j, \mu_F^2)\, f_k(x_k, \mu_F^2) \cdot \hat{\sigma}(x_j x_k s, \mu_F^2, \mu_R^2) \otimes \text{hadronization}$$

Complicated process – use MC techniques to calculate cross sections, phenomenological modes to describe hadronization process (quarks → jets)