Introduction to theoretical foundations III
(from experimenter's viewpoint)

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Summary

- Symmetries are a basic principle of physics
- Principle of local gauge invariance
  - Postulate of local gauge invariance of the Lagrange density
    → leads to interaction terms with gauge bosons as mediators
- QED: Symmetry under U(1) gauge transformation → photon exchange
- Cross section from Lagrange density
  - Fermi's golden rule:
    matrix element squared X phase space → cross section
  - Feynman rules:
    - set of rules how to calculate matrix elements
    - can be read off Lagrange density (at leading order …)
    - represented by Feynman graphs

The standard model so far

- **Postulation of local U(1) gauge symmetry → Lagrangian of QED**

$$\mathcal{L}_{\mathrm{QED}} = \overline{\psi}(i\gamma^\mu D_\mu - m)\psi - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

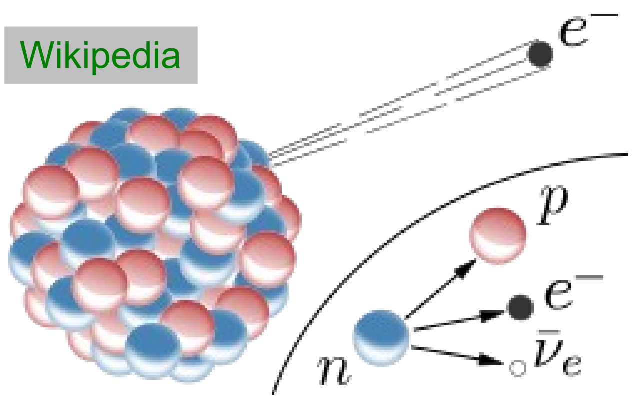
$$= \underbrace{\overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi}_{\text{free fermion}} - \underbrace{q(\overline{\psi}\gamma^\mu\psi)A_\mu}_{\text{interaction}} - \underbrace{\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{\text{photon gauge field}}$$

- **We have fermions and (massless) vector bosons**
- **We have a conserved quantum number q identifiable with the electric charge**
- **We have electromagnetic interactions**

⟶ Quantum version of Maxwell's unified theory of electricity and magnetism

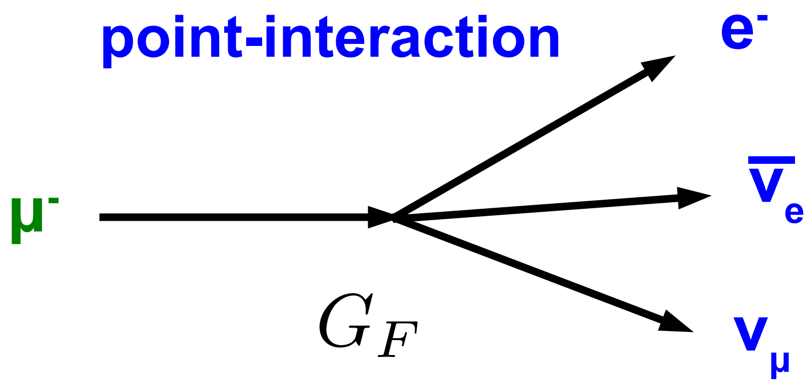
β-decay

Wikipedia



μ-decay

point-interaction



Versuch einer Theorie der β-Strahlen. I[1]).

Von **E. Fermi** in Rom.

Mit 3 Abbildungen. (Eingegangen am 16. Januar 1934.)

Eine quantitative Theorie des β-Zerfalls wird vorgeschlagen, in welcher man die Existenz des Neutrinos annimmt, und die Emission der Elektronen und Neutrinos aus einem Kern beim β-Zerfall mit einer ähnlichen Methode behandelt, wie die Emission eines Lichtquants aus einem angeregten Atom in der Strahlungstheorie. Formeln für die Lebensdauer und für die Form des emittierten kontinuierlichen β-Strahlenspektrums werden abgeleitet und mit der Erfahrung verglichen.

Fermi, Z. Phys., 1934, 88, 16; Nuovo Cim., 1934, 11, 1

β-decay

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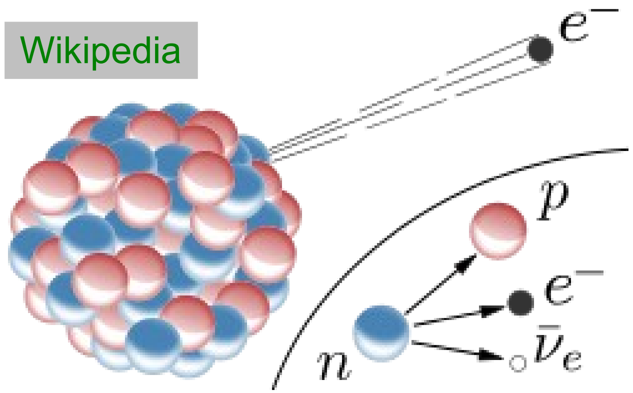

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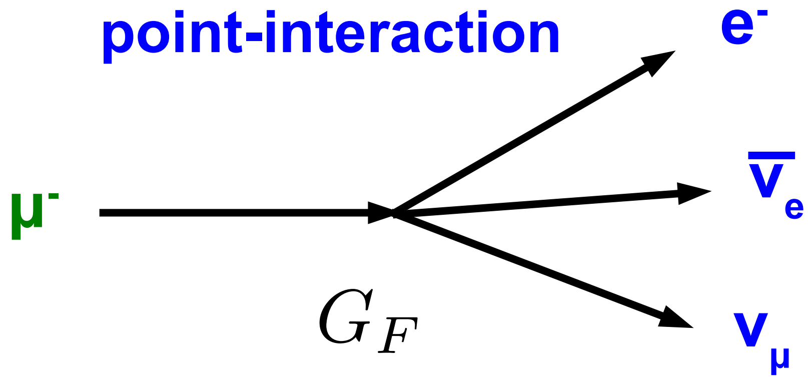

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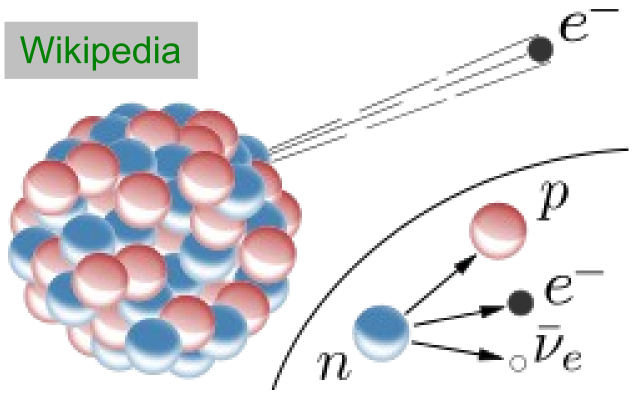
Fermi's four-fermion coupling

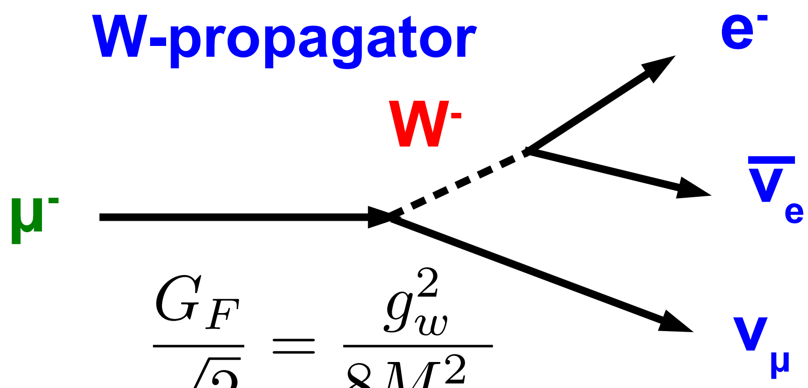
β-decay

Wikipedia



μ-decay

W-propagator



$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$$

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Problems with weak reactions

- Fermi theory corresponds to contact interaction
  - Coupling constant G_F has dimensions $[G_F] = [E]^{-2}$

$$G_F \approx 1.166 \cdot 10^{-5} \mathrm{GeV}^{-2}$$

  - Cross sections grow beyond all bounds

$$\sigma \sim G_F^2 E_{\mathrm{cms}}^2 = G_F^2 \cdot s$$

- Interaction becomes very weak at large distances (low energies)
- Parity conservation is maximally violated
  - Weak reactions differentiate between left- and right-handed particles
- Particles change charge
- Particles change “flavor”

$\longrightarrow$ **How to describe something so different?**

Time out

Review on Lie groups

Marius Sophus Lie
(*17. December 1842, † 18. February 1899)

Unitary transformations

- **U(n): Group of unitary transformations in $\mathbb{R}^n$ with properties:**
  - $\mathbf{G} \in U(n)$: $\mathbf{G}^\dagger \mathbf{G} = \mathbb{I}_n$ $\det \mathbf{G} = \pm 1$

    For example: $\psi(x) \to \psi'(x) = e^{i\alpha}\psi(x)$ U(1) phase transformation

- **Splitting a phase from G one can always achieve:** $\det \mathbf{G} = +1$

$$U(n) = U(1) \times SU(n)$$

Unitary transformations

$$\det \mathbf{G} = \pm 1$$

Special unitary transformations

$$\det \mathbf{G} = +1$$

Infinitesimal to finite transformations

- **The SU(n) can be composed from infinitesimal transformations with a continuous parameter:** $\alpha \in \mathbb{R}$

$$\mathbf{G}|_{\text{finite}} = \mathbb{I}_n + i\alpha_{\text{finite}}\mathbf{t} \qquad (\alpha_{\text{finite}} \in \mathbb{R}, \mathbf{t} \in \mathcal{M}(\mathrm{n} \times \mathrm{n}))$$

$$\mathbf{G}|_{\text{finite}} \approx \left(\mathbb{I}_n + i\frac{\alpha_{\text{finite}}}{2}\mathbf{t}\right)^2 = \mathbb{I}_n + 2 \cdot i\frac{\alpha_{\text{finite}}}{2}\mathbf{t} - \frac{\alpha_{\text{finite}}^2}{4}\mathbf{t}^2$$

$$\mathbf{G}|_{\text{finite}} \approx \left(\mathbb{I}_n + i\frac{\alpha_{\text{finite}}}{m}\mathbf{t}\right)^m \xrightarrow{m \to \infty} e^{i\alpha_{\text{finite}} \cdot \mathbf{t}}$$

- **t are the generators of G and define the structure of G**
- **The set of G forms a Lie group**
- **The set of t form the tangential space or the Lie algebra**

Properties of the generators t

- **Hermitian:**

$$\mathbf{G}^\dagger\mathbf{G} = \mathbb{I}_n = \left(\mathbb{I}_n - i\alpha\mathbf{t}^\dagger\right)\left(\mathbb{I}_n + i\alpha\mathbf{t}\right) = \mathbb{I}_n + i\alpha\left(\mathbf{t} - \mathbf{t}^\dagger\right) + O(\alpha^2)$$

$$\rightarrow \qquad \mathbf{t} = \mathbf{t}^\dagger$$

- **Traceless (Ex. SU(n)):**

$$\det\mathbf{G} = \det\left(\mathbb{I}_n + i\alpha\mathbf{t}\right) = 1 + i\alpha\mathrm{Tr}(\mathbf{t}) + O(\alpha^2) = 1$$

$$\rightarrow \qquad \mathrm{Tr}(\mathbf{t}) = 0$$

- **Dimensionality:**
  - n real entries in diagonal
  - ½ n (n-1) complex entries
  - -1 entry for determinant

$$\begin{pmatrix} * & * & * & * & * \\ & * & * & * & * \\ & & * & * & * \\ & & & * & * \\ & & & & * \end{pmatrix}$$

→ n^2 generators of U(n)

n^2-1 generators of SU(n)

Use in standard model

- **U(1) → 1 generator:** $\mathbf{t} = 1 \quad G = e^{i\alpha \cdot 1}$

  → equivalent to rotations in 2-dim., i.e. the orthogonal group O(2)

  - **Commutator:** 0 → Abelian

- **SU(2) → 3 generators:** $\mathbf{t}_a = \frac{1}{2}\sigma_a \qquad (a = 1, 2, 3)$

  Pauli matrices σ_a

  $$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

  → equivalent to rotations in 3-dim., i.e. the orthogonal group O(3)

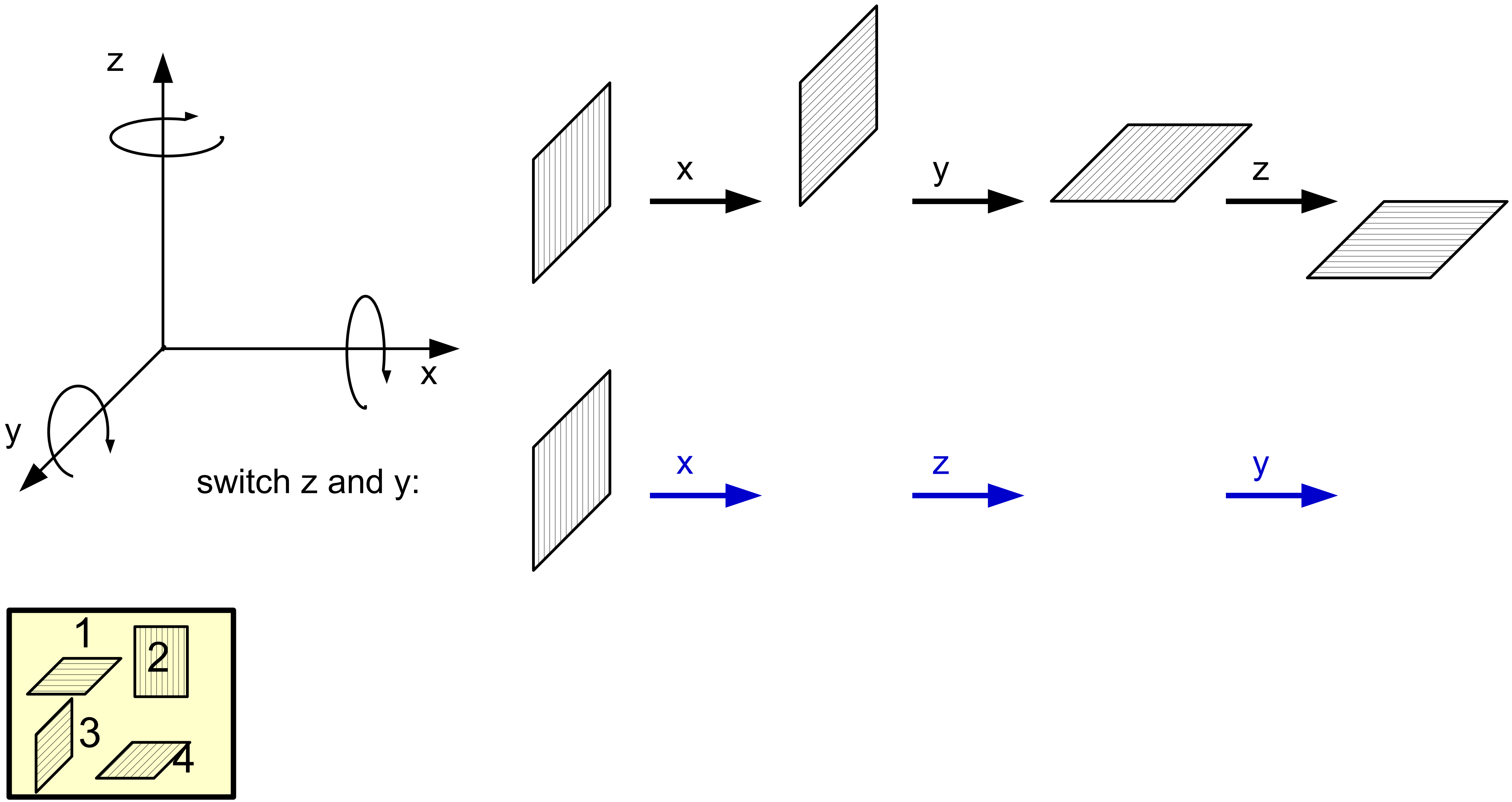
  - **Commutator:** $[\mathbf{t}_a, \mathbf{t}_b] = i\epsilon_{abc}\mathbf{t}_c$

    → Non-Abelian

  - **With structure constants of SU(2):** ϵ_{abc} Levi-Civita tensor

Non-Abelian symmetry transformation

- **Example in O(3): 90° rotations in R^3**



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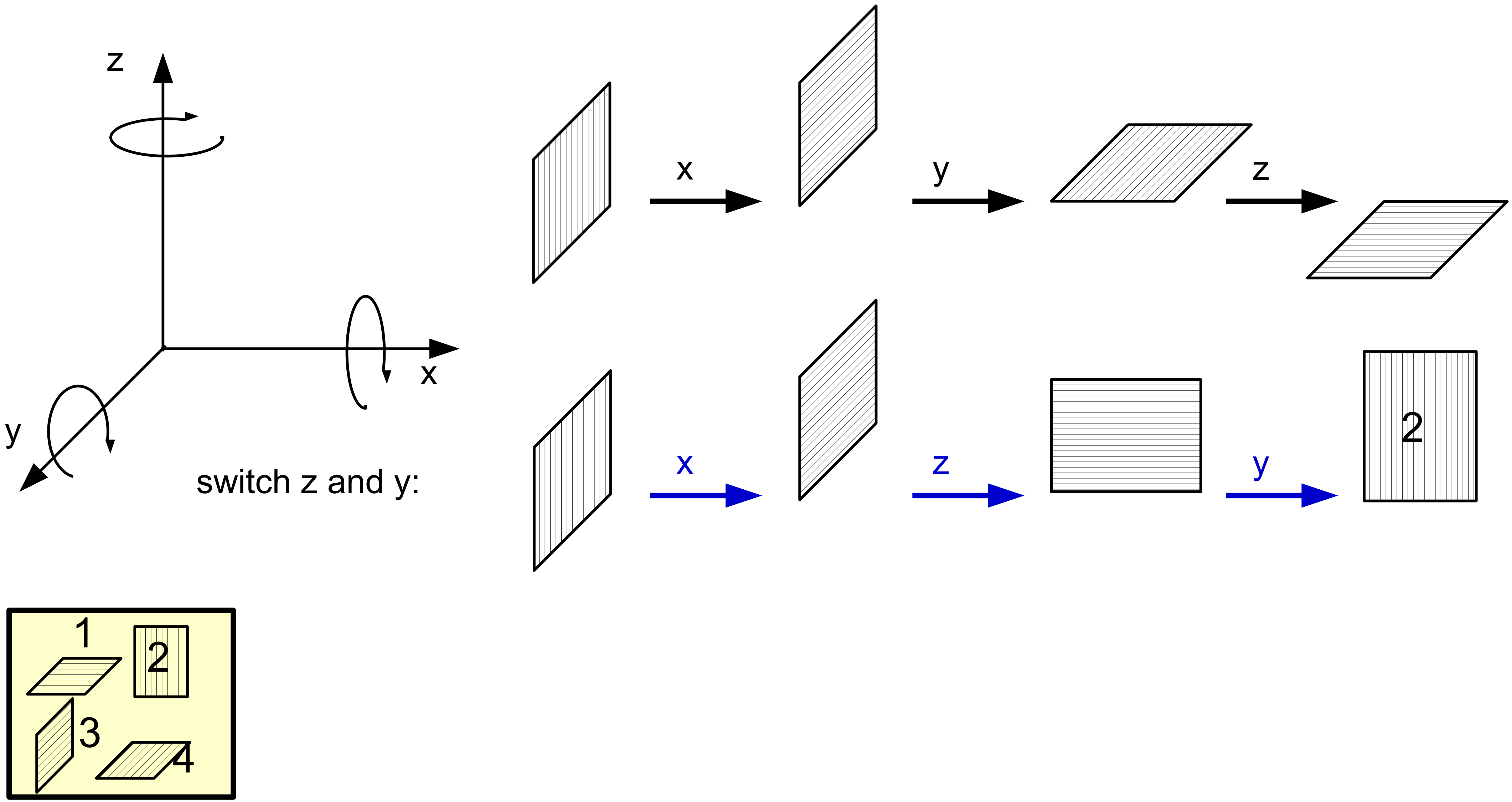

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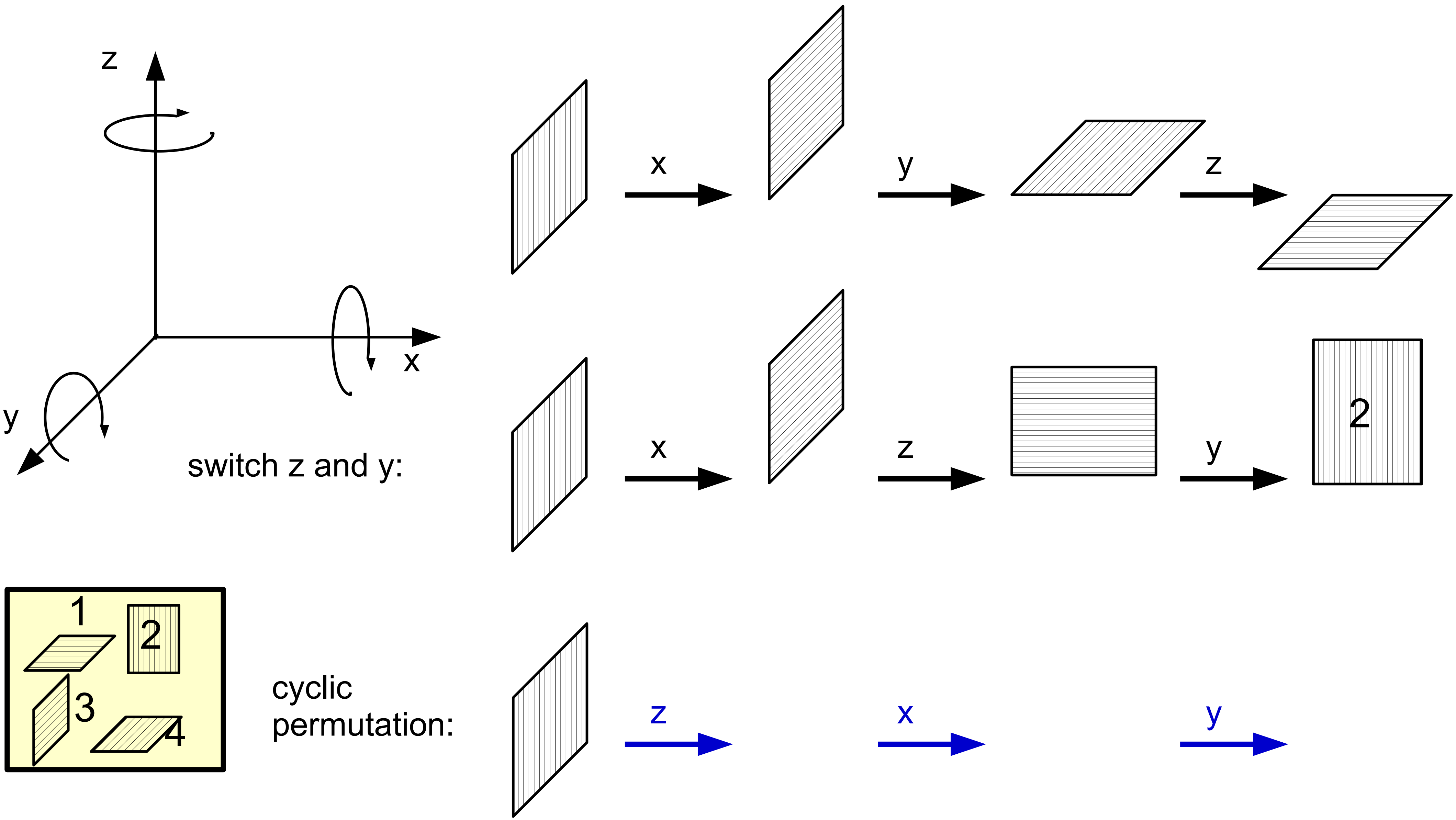

Non-Abelian symmetry transformation

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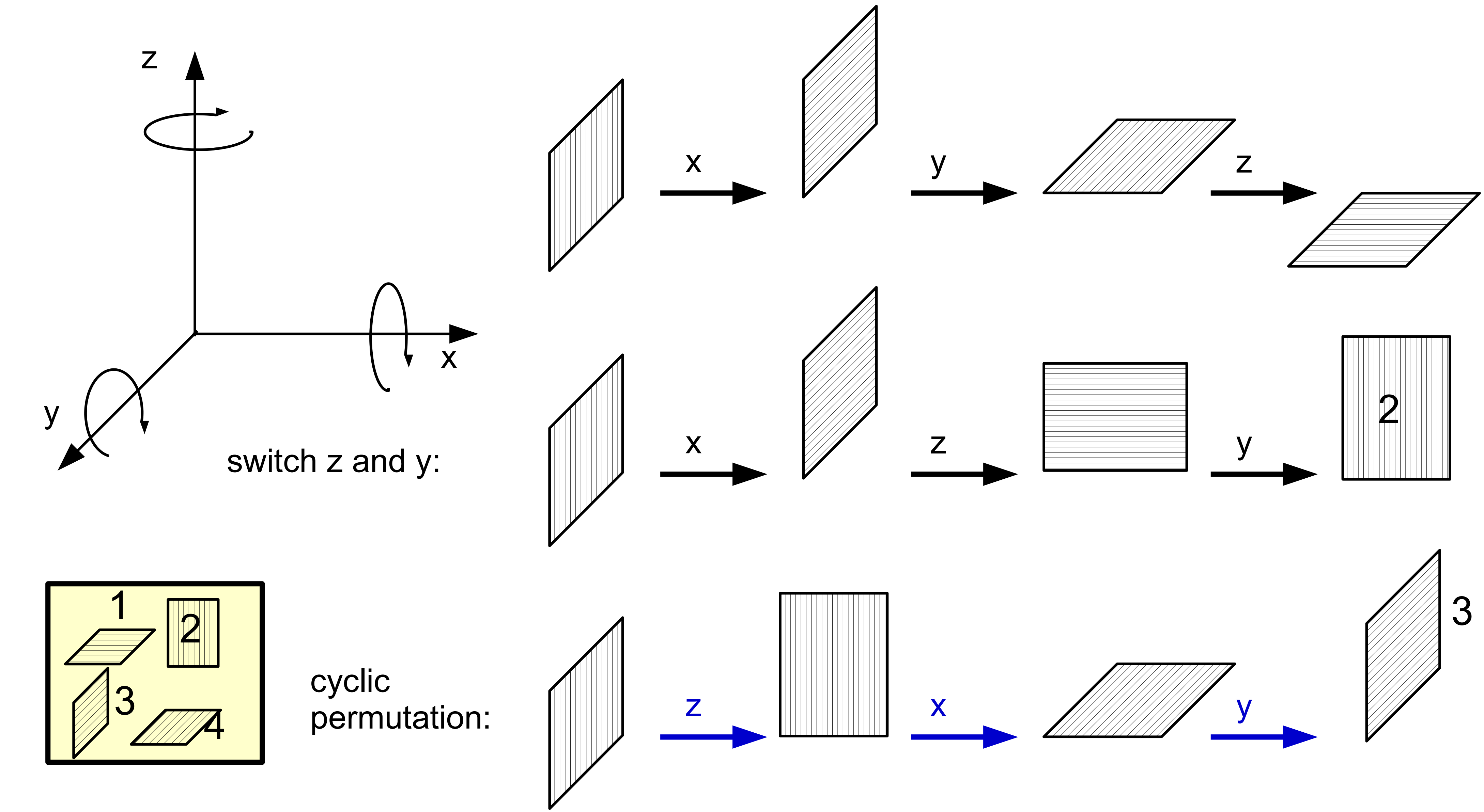
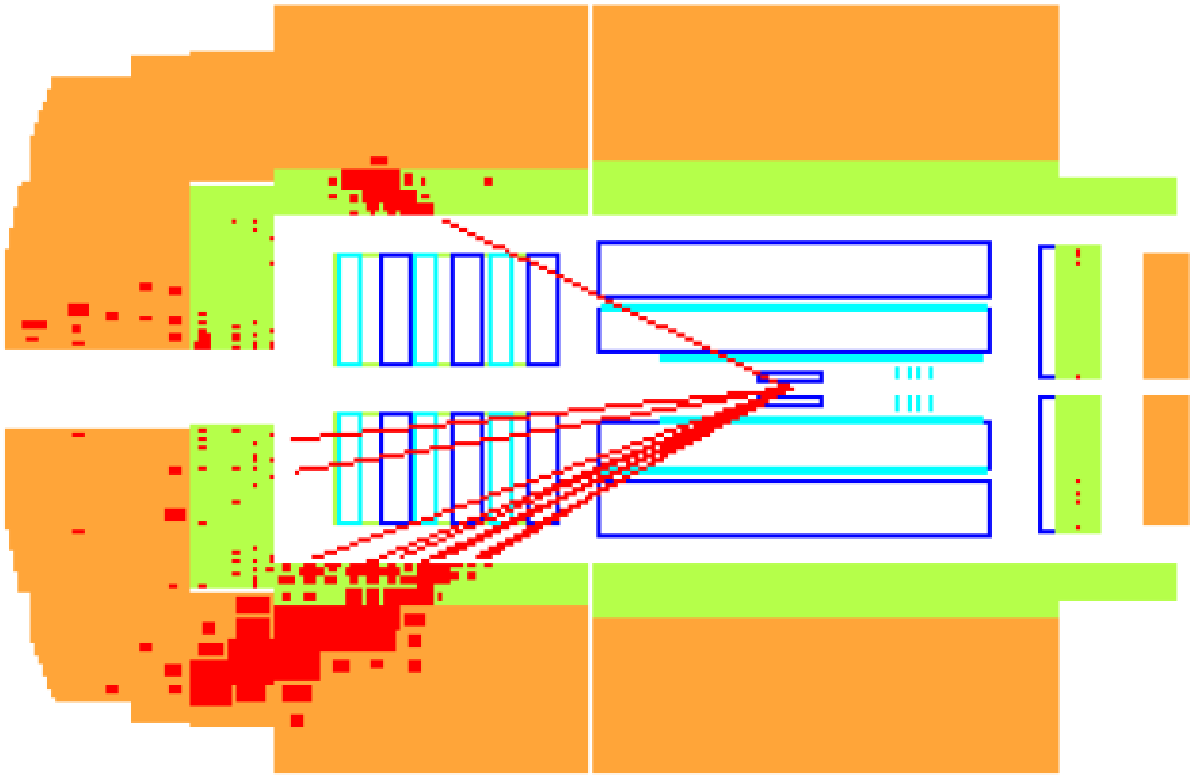


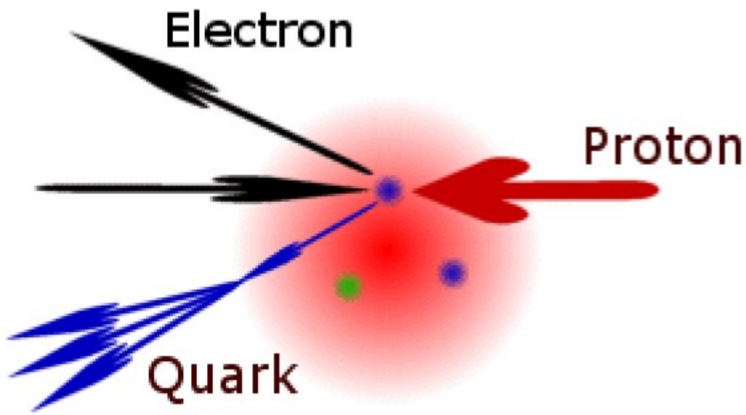
Time out

Electromagnetic reaction:

Backscattering of electron off charged proton constituent

H1 Detector



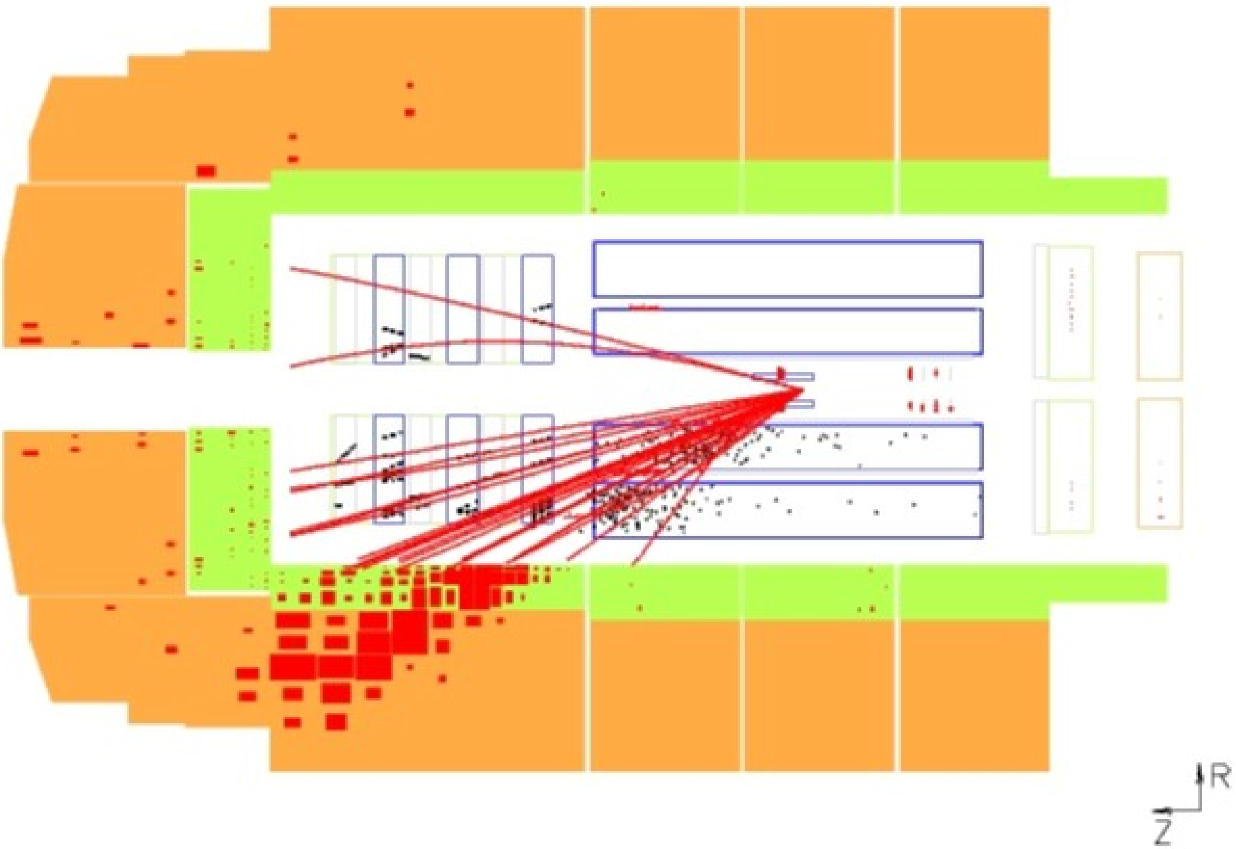


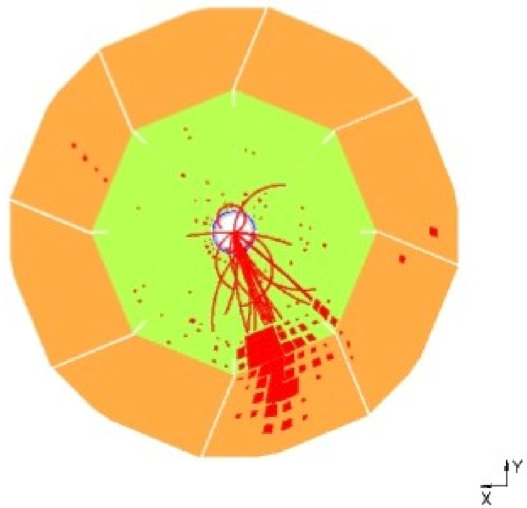
H1 Event Tutorial, J Meyer, DESY (2005)

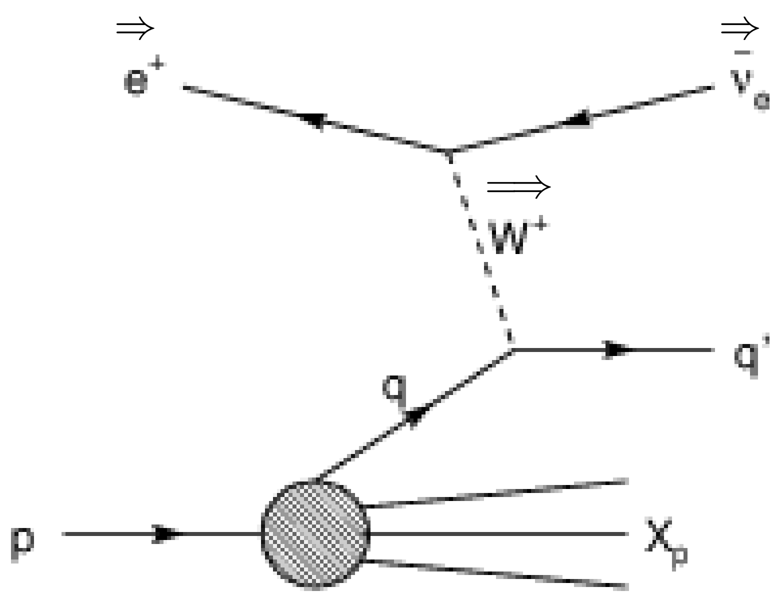
Weak reaction:

Electron → neutrino ~ missing transverse momentum

H1 Detector







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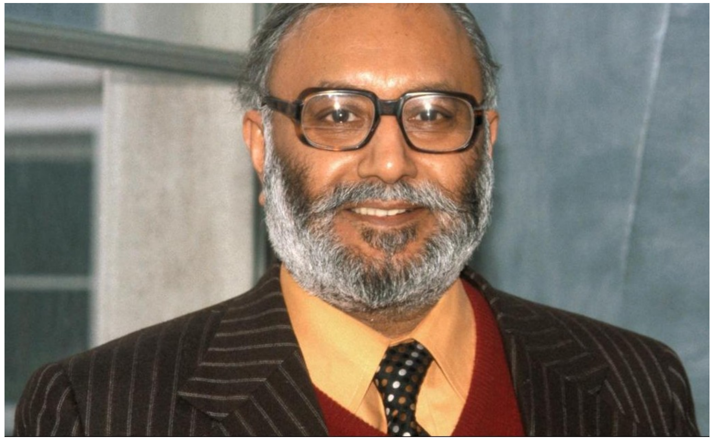
- Interaction becomes very weak at large distances (low energies)
- Parity conservation is maximally violated
  - Weak reactions differentiate between left- and right-handed particles → update Fermi model from V to (V – A) interaction
- Particles change charge
- Particles change "flavor"

→ How to describe something so different?

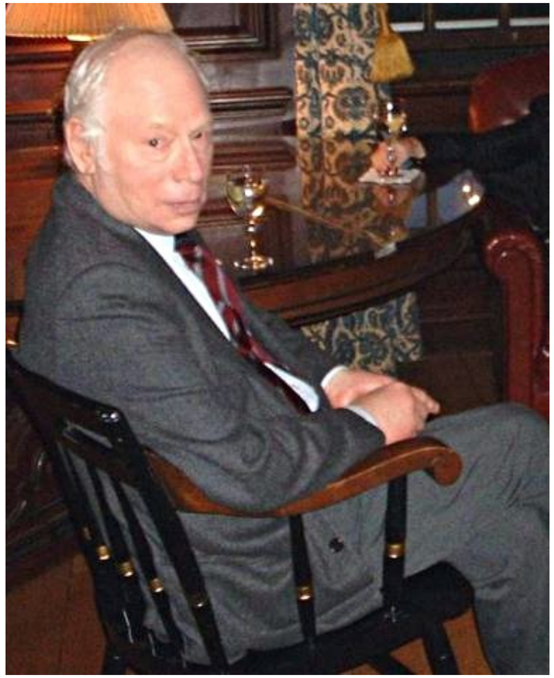
GSW model for the weak interaction

Nobel prize 1979

Sheldon Glashow
(*5. December 1932)



Abdus Salam
(29. January 1926 –
21. November 1996)



Steven Weinberg
(*3. Mai 1933)

SU(2) and weak isospin

- **Postulation of local SU(2) gauge symmetry**
- **Acts only on left-handed particles (right-handed anti-particles)**
  - **Left-handed particles: Doublets of weak isospin** $\psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad I_3 = \pm\frac{1}{2}$
  - **Right-handed particles: Singlets of weak isospin** $\psi_R = e_R \qquad I_3 = 0$

  Projection operators: $P_{R/L} = \frac{1 \pm \gamma^5}{2}$

$$e = e_L + e_R \quad \begin{cases} e_L = \left(\frac{1-\gamma^5}{2}\right) e \\ e_R = \left(\frac{1+\gamma^5}{2}\right) e \end{cases} \qquad \bar{e}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right) \nu = \bar{e}_L \gamma^\mu \nu_L$$

  - **Parity conservation maximally violated**
- **Conserved quantum number of weak isospin I_3**
- **Massive electrically(!) charged vector bosons $W^\pm$**
  - **Need to combine with electromagnetic interactions ...** $\mathrm{SU}(2)_L \times U(1)?$

Time out

Covariant derivative

- **Covariant derivative of SU(2) acts on isospin doublet only → SU(2)$_L$**
- **Interaction lagrangian of SU(2)$_L$ x U(1)**

$$\mathcal{L}_{IA}^{SU(2)\times U(1)} = \overline{\psi}_L \gamma^\mu \left(\partial_\mu \qquad + igW_\mu^a \mathbf{t}^a\right) \psi_L \cdots$$

Covariant derivative

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$$\mathbf{t}^+ = \mathbf{t}_1 + i\,\mathbf{t}_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$ (ascending operator)

$$\mathbf{t}^- = \mathbf{t}_1 - i\,\mathbf{t}_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$ (descending operator)

$$\mathbf{t}^3 = {}^1\!/_2 \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

?

$$W_\mu^a \mathbf{t}^a = \frac{1}{\sqrt{2}} \left(W_\mu^+ \mathbf{t}^+ + W_\mu^- \mathbf{t}^- \right) + W_\mu^3 \mathbf{t}^3$$

3 generators of SU(2)!?

Covariant derivative

- Covariant derivative of U(1) acts on isospin doublet and singlet
- Interaction lagrangian of $SU(2)_L$ x $U(1)_Y$

$$\mathcal{L}_{IA}^{SU(2)\times U(1)} = \overline{\psi}_L \gamma^\mu \boxed{\left(\partial_\mu + i\frac{g'}{2} Y_L B_\mu + igW_\mu^a \mathbf{t}^a\right)} \psi_L + \bar{e}_R \gamma^\mu \boxed{\left(\partial_\mu + i\frac{g'}{2} Y_R B_\mu\right)} e_R$$

- Two new coupling constants g and g'
- Requires 3 weak bosons with one neutral W_μ^3
- U(1) different from electromagnetic …
- Quantum number of weak hypercharge Y for U(1) → $U(1)_Y$

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- Two new coupling constants g and g'
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$SU(2)\times U(1)$ Hypercharges			
Particle	$Y_{R/L}$	I_3	Q
ν	-1	$+1/2$	0
e_L	-1	$-1/2$	-1
e_R	-2	0	-1

$$Q = I_3 + \frac{Y}{2}$$

(Gell-Mann—Nishijima Formula)

Electroweak lagrangian

- **Covariant derivatives:**

$$D_\mu \psi_L = \left(\partial_\mu + ig\frac{t^a}{2} W^a_\mu + ig'\frac{Y}{2}\mathbb{I}_2 B_\mu \right) \psi_L$$

$$D_\mu \psi_R = \left(\partial_\mu + ig'\frac{Y}{2}\mathbb{I}_2 B_\mu \right) \psi_R$$

- **Field strength tensors:**

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon^{abc} W^b_\mu W^c_\nu$$

- **Lagrangian of $SU(2)_L$ x $U(1)_Y$:**

$$\mathcal{L}_{\mathrm{EW}} = \overline{\psi_L}(i\gamma^\mu D_\mu)\psi_L + \overline{\psi_R}(i\gamma^\mu D_\mu)\psi_R - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}W^{a\,\mu\nu}W^a_{\mu\nu}$$

- **But: Boson mass terms violate $SU(2)_L$ invariance and are forbidden!**

$$m^2 B^\mu B_\mu \qquad m^2 W^{a\,\mu} W^a_\mu$$

$SU(2)_L$ x $U(1)$ interactions

- **Charged current interaction**

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\left[\underbrace{\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L}_{e\to\nu} + \underbrace{\overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu}_{\nu\to e}\right]$$

t^+ t^-

- **Neutral current interaction**

$$\mathcal{L}_{IA}^{NC} = -\underbrace{\left(\frac{g}{2}W_\mu^3 - \frac{g'}{2}B_\mu\right)}_{\propto Z_\mu}\left(\overline{\nu}\gamma^\mu\nu\right) + \left(\frac{g}{2}W_\mu^3 + \frac{g'}{2}B_\mu\right)\left(\overline{e}_L\gamma^\mu e_L\right) + \frac{g'}{2}B_\mu\left(\overline{e}_R\gamma^\mu e_R\right)$$

t^3

- **Mix of neutral bosons W_μ^3 & B_μ**
  - **Weinberg rotation**
  - **Physical states of Z_μ and A_μ**

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \qquad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$SU(2)_L \times U(1)$ interactions

- **Charged current interaction**

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\Big[\underbrace{\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L}_{e\to\nu} + \underbrace{\overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu}_{\nu\to e}\Big]$$

- **Neutral current interaction with Z_μ, A_μ →**

$$\mathcal{L}_{IA}^{NC} = -\frac{\sqrt{g^2+g'^2}}{2}\boxed{Z_\mu\left(\overline{\nu}\gamma_\mu\nu\right)}$$

Z_μ couples to neutral particles

$$+\frac{\sqrt{g^2+g'^2}}{2}\left[\left(\cos^2\theta_W - \sin^2\theta_W\right)Z_\mu + 2\sin\theta_W\cos\theta_W A_\mu\right]\left(\overline{e_L}\gamma_\mu e_L\right)$$

$$+\frac{\sqrt{g^2+g'^2}}{2}\left[\qquad -2\sin^2\theta_W Z_\mu + 2\sin\theta_W\cos\theta_W A_\mu\right]\left(\overline{e_R}\gamma_\mu e_R\right)$$

$SU(2)_L$ x $U(1)$ interactions

Charged current interaction

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Neutral current interaction with Z_μ, A_μ →

$$\begin{aligned}\mathcal{L}_{IA}^{NC} = &-\frac{\sqrt{g^2+g'^2}}{2}Z_\mu\left(\overline{\nu}\gamma_\mu\nu\right)\\ &+\frac{\sqrt{g^2+g'^2}}{2}\left[\left(\cos^2\theta_W-\sin^2\theta_W\right)Z_\mu + 2\sin\theta_W\cos\theta_W A_\mu\right]\left(\overline{e_L}\gamma_\mu e_L\right)\\ &+\frac{\sqrt{g^2+g'^2}}{2}\left[-2\sin^2\theta_W Z_\mu + 2\sin\theta_W\cos\theta_W A_\mu\right]\left(\overline{e_R}\gamma_\mu e_R\right)\end{aligned}$$

Same coupling of A_μ to e_L, e_R

$SU(2)_L$ x $U(1)$ interactions

- **Charged current interaction**

$$\mathcal{L}_{IA}^{CC} = -\frac{g}{\sqrt{2}}\left[\underbrace{\overline{\nu}\left(W_\mu^+\gamma^\mu\right)e_L}_{e\to\nu} + \underbrace{\overline{e}_L\left(W_\mu^-\gamma^\mu\right)\nu}_{\nu\to e}\right]$$

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- **→ electromagnetic coupling** $\quad e = \sqrt{g^2+g'^2}\sin\theta_W\cos\theta_W$

Summary

- **QED: U(1) gauge transformation → photon exchange**
  - Abelian Lie-Group
- **Weak interactions: $SU(2)_L$ gauge transformation**
  - non-Abelian Lie-Group
  - charged currents, $W^{\pm}$ exchange
  - acts only on doublets of left-handed particles, right-handed anti-particles
- **$SU(2)_L$ requires third neutral boson W_μ^3**
  - mixes with neutral boson B_μ of $U(1)_Y$
  - Weinberg mixing gives physical states of Z_μ and A_μ
  - Z_μ → neutral currents between uncharged particles (neutrinos)
  - A_μ → mediates same electric force between left- and right-handed
  - elm. coupling derives from g, g', θ_W: $e = \sqrt{g^2 + g'^2} \sin\theta_W \cos\theta_W$

Particles and quantum numbers

Fermion	Chirality	Isospin (I, I_3)	Hypercharge Y	Charge Q (e)
Neutrinos: ν_e, ν_μ, ν_τ	L	(1/2, +1/2)	–1	0
	R	Not part of the standard model		
Charged leptons: e, μ, τ	L	(1/2, –1/2)	–1	–1
	R	(0, 0)	–2	–1
up-type quarks: u, c, t	L	(1/2, +1/2)	+1/3	+2/3
	R	(0, 0)	+4/3	+2/3
down-type quarks: d, s, b	L	(1/2, –1/2)	+1/3	–1/3
	R	(0, 0)	–2/3	–1/3

Abelian vs. non-Abelian QFT

Abelian:

$$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{i\alpha}\psi(\vec{x},t)$$
$$\overline{\psi}(\vec{x},t) \to \psi'(\vec{x},t) = \overline{\psi(\vec{x},t)}e^{-i\alpha}$$

$$\partial_\mu \to D_\mu = \partial_\mu \; -ieA_\mu$$
$$D_\mu \to D'_\mu = D_\mu \; -i\partial_\mu\alpha$$
$$A_\mu \to A'_\mu = A_\mu \; +\tfrac{1}{e}\partial_\mu\alpha$$

$$F_{\mu\nu} \equiv [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\mu\nu} \to F'_{\mu\nu} = F_{\mu\nu}$$

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Non-Abelian:

$$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{i\alpha_a \mathbf{t}_a}\psi(\vec{x},t)$$
$$\overline{\psi}(\vec{x},t) \to \psi'(\vec{x},t) = \overline{\psi(\vec{x},t)}e^{-i\alpha_a \mathbf{t}_a}$$

$$\partial_\mu \to D_\mu = \partial_\mu \; - igW_{\mu,a}\mathbf{t}_a$$
$$D_\mu \to D'_\mu = D_\mu \; - i\,[\alpha_a \mathbf{t}_a, D_\mu]$$
$$W_\mu \to W'_\mu = W_\mu + i\,[\alpha_a \mathbf{t}_a, W_{\mu,a}\mathbf{t}_a] + \tfrac{1}{g}\partial_\mu(\alpha_a \mathbf{t}_a)$$
$$W_{\mu\nu} \equiv [D_\mu, D_\nu] = \partial_\mu W_\nu - \partial_\nu W_\mu - ig\,[W_\mu, W_\nu]$$

$$W_{\mu\nu} \to W'_{\mu\nu} = W_{\mu\nu} - i\,[\vartheta_a \mathbf{t}_a, W_{\mu\nu}]$$

$$\mathcal{L} = \overline{\psi}\,(i\gamma^\mu D_\mu - m)\,\psi - \tfrac{1}{4}W_{a\mu\nu}W^{a\mu\nu}$$