Introduction to theoretical foundations IV

(from experimenter's viewpoint)

Fakultät für Physik
K. Rabbertz (ETP)

Karlsruhe Institute of Technology

Summary

- **QED: U(1) gauge transformation → photon exchange**
  - Abelian Lie-Group
- **Weak interactions: $SU(2)_L$ gauge transformation**
  - Non-Abelian Lie-Group
  - Charged currents, $W^\pm$ exchange
  - acts only on doublets of left-handed particles, right-handed anti-particles
- **$SU(2)_L$ requires third neutral boson W_μ^3**
  - mixes with neutral boson B_μ of $U(1)_Y$
  - Weinberg mixing gives physical states of Z_μ and A_μ
  - Z_μ → neutral currents between uncharged particles (neutrinos)
  - A_μ → mediates same electric force between left- and right-handed
  - Elm. coupling derives from g, g', θ_W: $e = \sqrt{g^2 + g'^2} \sin\theta_W \cos\theta_W$

Abelian vs. non-Abelian QFT

Abelian:

$$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{i\alpha}\psi(\vec{x},t)$$

$$\overline{\psi}(\vec{x},t) \to \psi'(\vec{x},t) = \overline{\psi(\vec{x},t)}e^{-i\alpha}$$

$$\partial_\mu \to D_\mu = \partial_\mu \; -ieA_\mu$$

$$D_\mu \to D'_\mu = D_\mu \; -i\partial_\mu\alpha$$

$$A_\mu \to A'_\mu = A_\mu \; +\tfrac{1}{e}\partial_\mu\alpha$$

$$F_{\mu\nu} \equiv [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\mu\nu} \to F'_{\mu\nu} = F_{\mu\nu}$$

$$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Non-Abelian:

$$\psi(\vec{x},t) \to \psi'(\vec{x},t) = e^{i\alpha_\mathrm{a}\mathbf{t}_\mathrm{a}}\psi(\vec{x},t)$$

$$\overline{\psi}(\vec{x},t) \to \psi'(\vec{x},t) = \overline{\psi(\vec{x},t)}e^{-i\alpha_\mathrm{a}\mathbf{t}_\mathrm{a}}$$

$$\partial_\mu \to D_\mu = \partial_\mu \; - igW_{\mu,\mathrm{a}}\mathbf{t}_\mathrm{a}$$

$$D_\mu \to D'_\mu = D_\mu \; - i\left[\alpha_\mathrm{a}\mathbf{t}_\mathbf{a}, D_\mu\right]$$

$$W_\mu \to W'_\mu = W_\mu + i\left[\alpha_\mathrm{a}\mathbf{t}_\mathrm{a}, W_{\mu,\mathrm{a}}\mathbf{t}_\mathrm{a}\right] + \tfrac{1}{g}\partial_\mu\left(\alpha_\mathrm{a}\mathbf{t}_\mathrm{a}\right)$$

$$W_{\mu\nu} \equiv [D_\mu, D_\nu] = \partial_\mu W_\nu - \partial_\nu W_\mu - ig\left[W_\mu, W_\nu\right]$$

$$W_{\mu\nu} \to W'_{\mu\nu} = W_{\mu\nu} - i\left[\vartheta_\mathrm{a}\mathbf{t}_\mathrm{a}, W_{\mu\nu}\right]$$

$$\mathcal{L} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4}W_{\mathrm{a}\mu\nu}W^{\mathrm{a}\mu\nu}$$

Electroweak lagrangian

- **Covariant derivatives:**

$$D_\mu \psi_L = \left(\partial_\mu + ig\frac{t^a}{2} W_\mu^a + ig'\frac{Y}{2}\mathbb{I}_2 B_\mu \right) \psi_L$$

$$D_\mu \psi_R = \left(\partial_\mu + ig'\frac{Y}{2}\mathbb{I}_2 B_\mu \right) \psi_R$$

Note: + or – signs may vary with conventions.

- **Field strength tensors:**

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - \boxed{g\epsilon^{abc} W_\mu^b W_\nu^c}$$

→ **leads to triple (TGC) and quartic (QGC) gauge couplings**

- **Lagrangian of SU(2)$_L$ x U(1)$_Y$:**

$$\mathcal{L}_{\text{EW}} = \overline{\psi_L}(i\gamma^\mu D_\mu)\psi_L + \overline{\psi_R}(i\gamma^\mu D_\mu)\psi_R - \tfrac{1}{4}B^{\mu\nu}B_{\mu\nu} - \tfrac{1}{4}W^{a\,\mu\nu}W_{\mu\nu}^a$$

- **But: Boson mass terms violate SU(2)$_L$ invariance and are forbidden!**

$$m^2 B^\mu B_\mu \qquad m^2 W^{a\,\mu} W_\mu^a$$

Weak reaction status

- Fermi theory corresponds to contact interaction
  - Coupling constant G_F has dimensions $[G_F] = [E]^{-2}$

$$G_F \approx 1.166 \cdot 10^{-5}\mathrm{GeV}^{-2} \longrightarrow \frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$$

  - Cross sections grow beyond all bounds

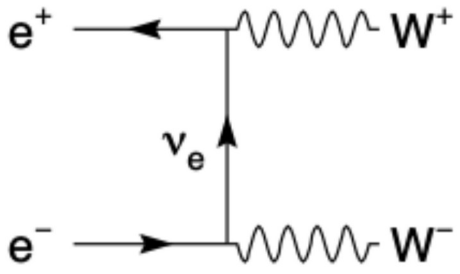
$$\sigma \sim G_F^2 E_{\mathrm{cms}}^2 = G_F^2 \cdot s$$

- Interaction becomes very weak at large distances → still no W mass
- Parity conservation is maximally violated
  - Weak reactions differentiate between left- and right-handed particles → update Fermi model from V to (V – A) interaction
- Particles change charge → charged bosons W^+W^-
- Particles change “flavor” → to be defined

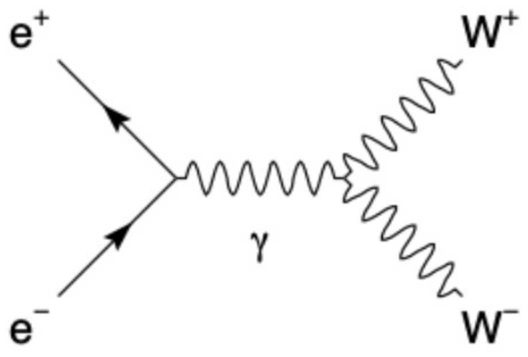
→ Any other issue?

W^+W^- pair production

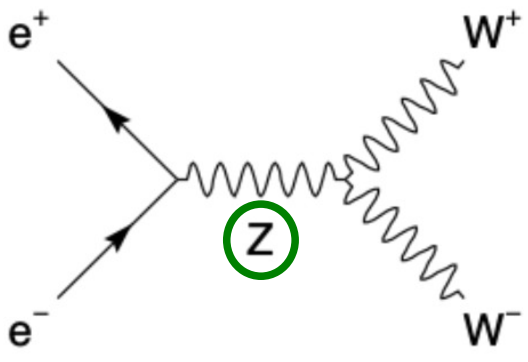
- **Abelian U(1) → uncharged photons → no photon self-coupling**
- **Non-Abelian SU(2) → charged bosons → triple and quartic gauge couplings (TGC, QGC)**
- **Solves problem in e^+e^- → W^+W^- pair production of (V – A) theory**
  - **Neutrino exchange diagram is divergent**
  - **Only combination of all three diagrams gives finite result**
  - **Indirect hint that Z boson must exist!**



Neutrino exchange



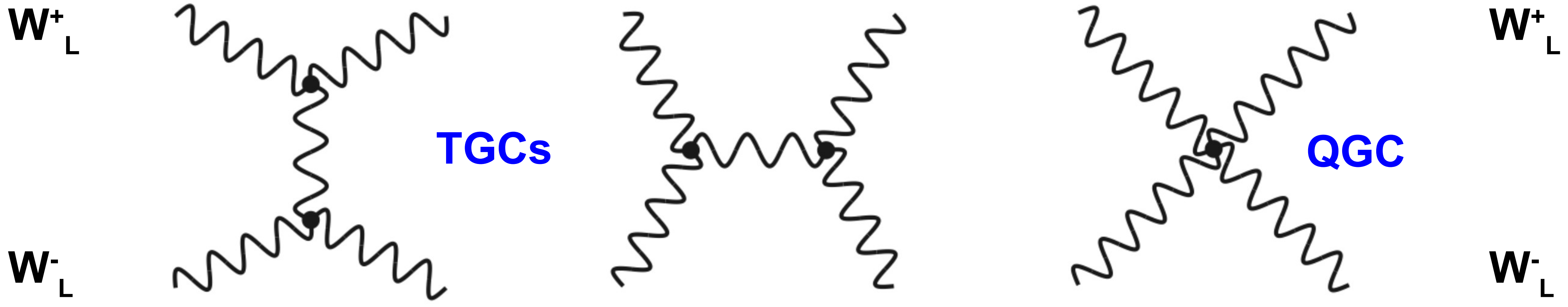
e^+e^- annihilation & γWW TGC



e^+e^- annihilation & ZWW TGC

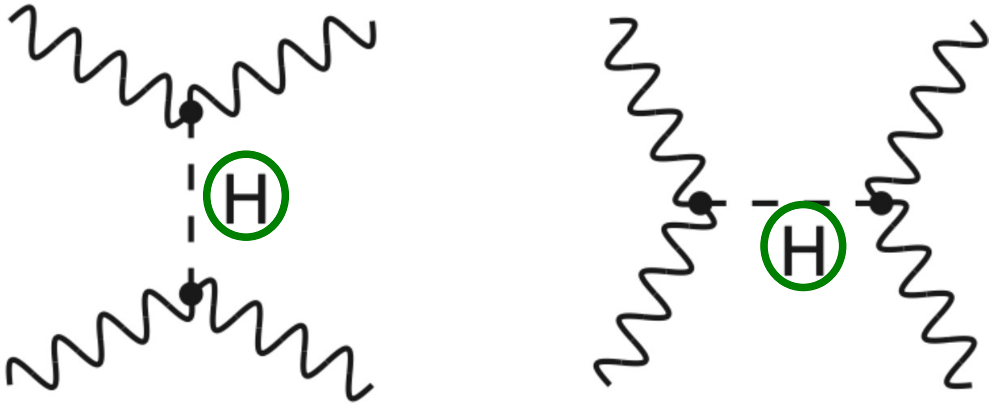
Longitudinal W boson scattering

- Three diagrams for $W_L W_L \rightarrow W_L W_L$ scattering



  - **Not finite!**

- Need to complement with additional diagrams involving scalar H
  - **Indirect hint that Higgs boson must exist!**



Massive gauge bosons

- **Example: Abelian U(1)**
  - **Field transformation:** $A_\mu \rightarrow A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha$
  - **In mass term:** $m_A A_\mu A^{\mu *} \rightarrow m_A A'_\mu A'^{\mu *} =$

$$m_A A_\mu A^{\mu *} + \underbrace{\frac{1}{e} m_A \left(A_\mu \partial^\mu \alpha + A^{\mu *}\partial_\mu \alpha\right) + m_A \frac{1}{e^2}\partial_\mu \alpha \partial^\mu \alpha}$$

  - **Break local invariance**
  - **Fundamental problem for all gauge field theories**

Massive fermions

- **No problem in Abelian U(1)**
  - **Field transformation:** $\psi \rightarrow \psi' = e^{i\alpha}\psi$

    $\overline{\psi} \rightarrow \overline{\psi'} = \overline{\psi}e^{-i\alpha}$
  - **In mass term:** $m_\psi\overline{\psi}\psi \rightarrow m_{\psi'}\overline{\psi'}\psi' = m_\psi\overline{\psi}\psi$
- **Also no issue in non-Abelian SU(3)**
  - **So why problem with SU(2) in the standard model?**

Massive fermions

- **No problem in Abelian U(1)**
  - **Field transformation:** $\psi \to \psi' = e^{i\alpha}\psi$

$$\overline{\psi} \to \overline{\psi'} = \overline{\psi} e^{-i\alpha}$$

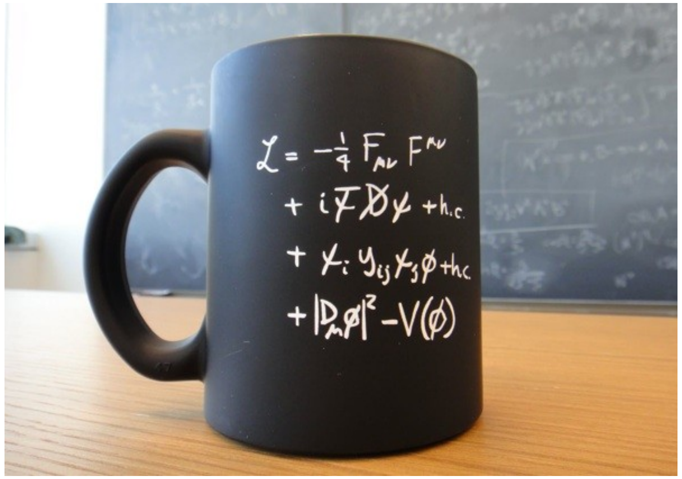
  - **In mass term:** $m_\psi \overline{\psi}\psi \to m_{\psi'} \overline{\psi'}\psi' = m_\psi \overline{\psi}\psi$
- **Also no issue in non-Abelian SU(3)**
  - **So why problem with SU(2) in the standard model?**
  - **Because of distinction between left- and righthanded fermions**

$$\boxed{m_e \bar{e} e = m_e \overline{(e_L + e_R)}(e_L + e_R) = m_e \bar{e}_R e_L + m_e \bar{e}_L e_R}$$

SU(2) singlet ⚡ **lower component of SU(2) doublet**

$SU(2)_L$ x $U(1)_Y$ dilemma

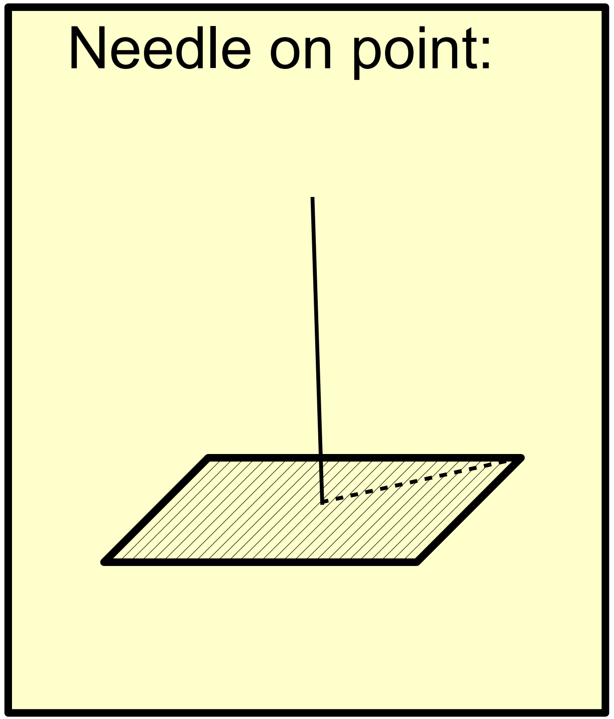
- **Postulation of local gauge symmetry ...**
  - **can motivate all interactions between elementary particles**
  - **gives a geometrical interpretation for the presence of gauge bosons (propagate info on local phases between space points)**
  - **predicts non trivial self-interactions between gauge bosons**
  - **fails miserably including any particle masses**



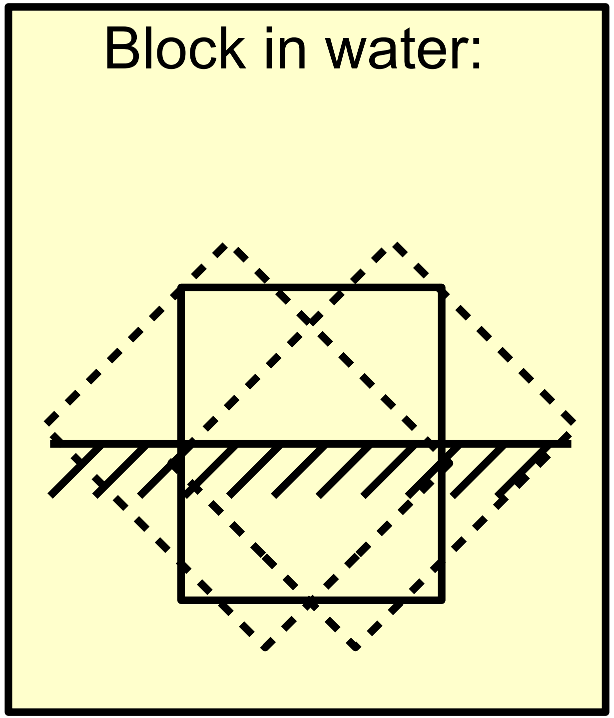
Spontaneous symmetry breaking

- **Symmetry present in system, i.e. the Lagrangian**
- **But is broken in the ground state, i.e. the quantum vacuum**

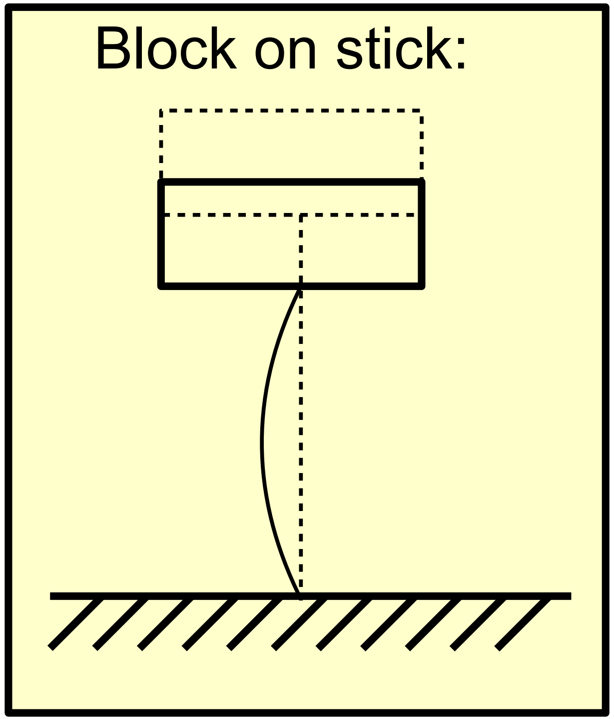
- **Examples from classical mechanics**



φ symmetry



axis-symmetry

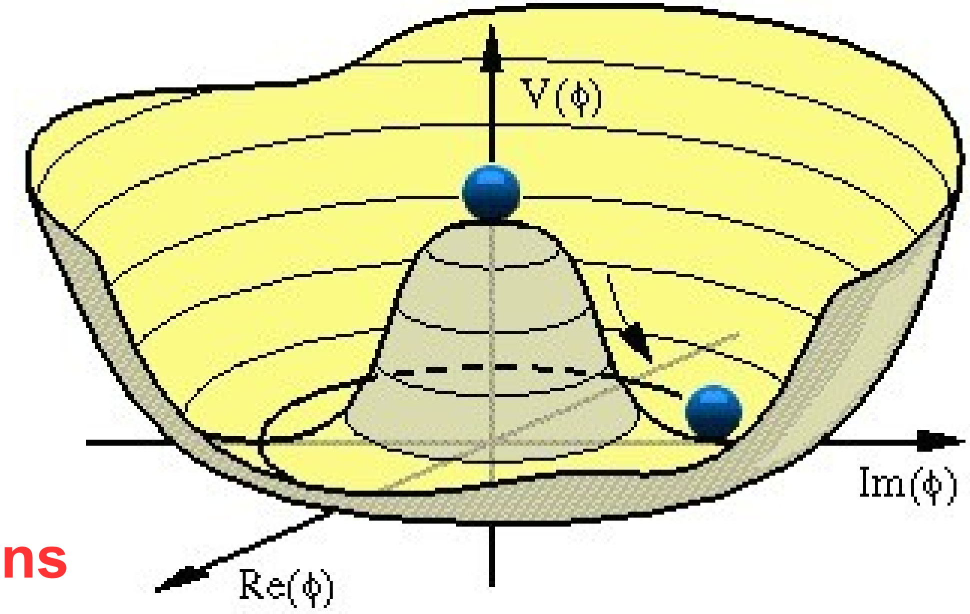


φ symmetry

Application to particle physics

Goldstone potential

$$\begin{aligned} \phi &= \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ V(\phi) &= -\mu^2|\phi|^2 + \lambda|\phi|^4 \\ \mathcal{L}(\phi) &= \partial_\mu\phi\partial^\mu\phi^* - V(\phi) \end{aligned}$$

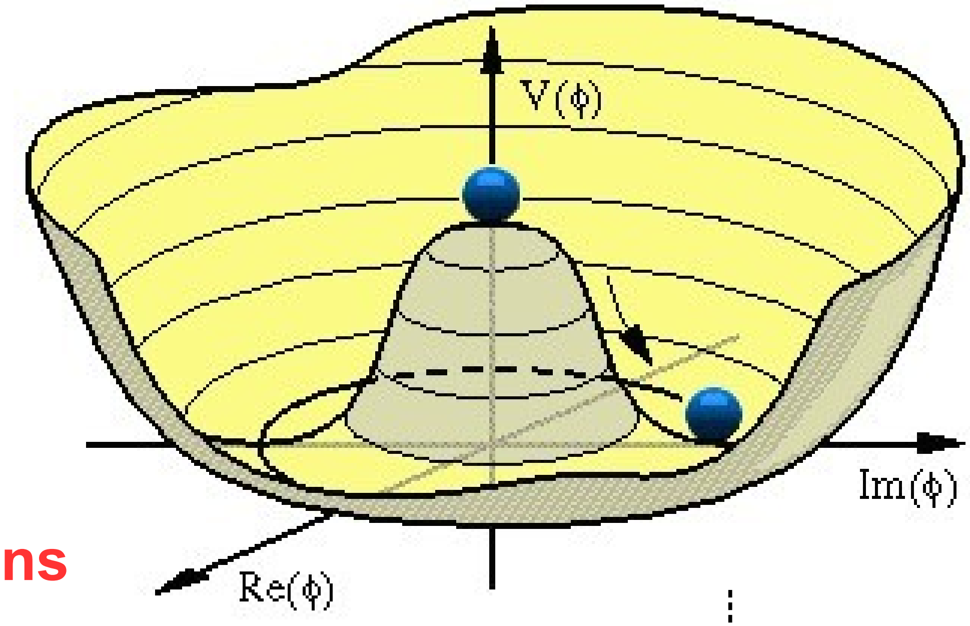


- **invariant under U(1) transformations (i.e. Φ symmetric)**
- **metastable in Φ = 0**
- **ground state breaks U(1) symmetry, BUT at the same time all ground states are indistinguishable in Φ**

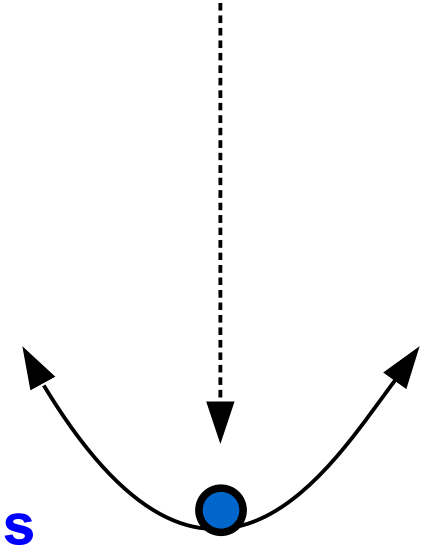
Application to particle physics

- **Goldstone potential**

$$\begin{aligned} \phi &= \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ V(\phi) &= -\mu^2|\phi|^2 + \lambda|\phi|^4 \\ \mathcal{L}(\phi) &= \partial_\mu\phi\partial^\mu\phi^* - V(\phi) \end{aligned}$$



  - **invariant under U(1) transformations (i.e. Φ symmetric)**
  - **metastable in Φ = 0**
  - **ground state breaks U(1) symmetry, BUT at the same time all ground states are indistinguishable in Φ**

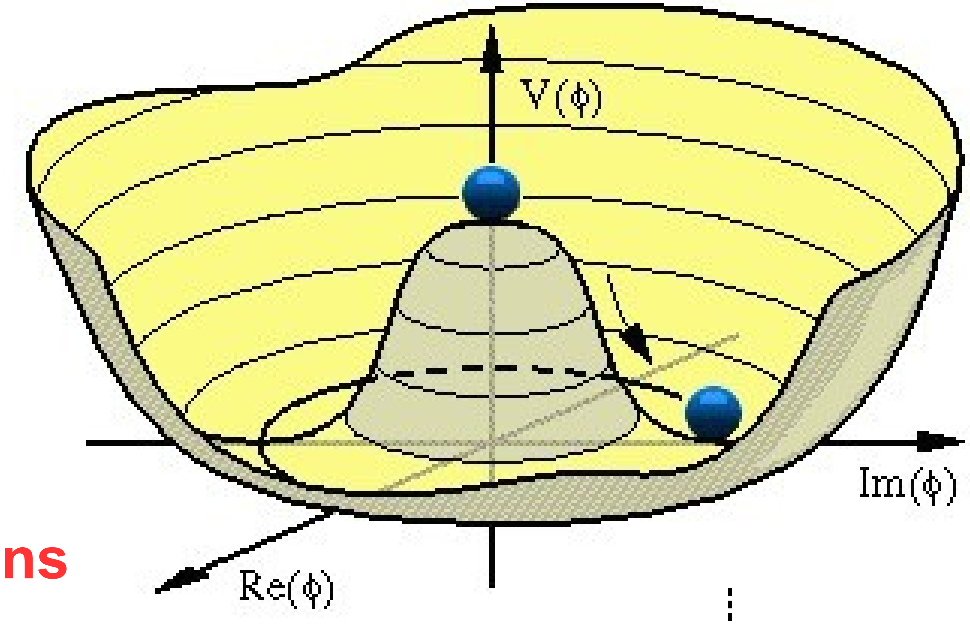


Φ has radial excitations in potential V(Φ)

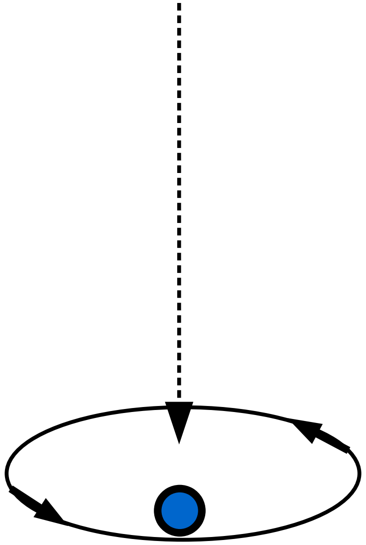
Application to particle physics

- **Goldstone potential**

$$\begin{aligned} \phi &= \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ V(\phi) &= -\mu^2|\phi|^2 + \lambda|\phi|^4 \\ \mathcal{L}(\phi) &= \partial_\mu\phi\partial^\mu\phi^* - V(\phi) \end{aligned}$$



- **invariant under U(1) transformations (i.e. Φ symmetric)**
- **metastable in Φ = 0**
- **ground state breaks U(1) symmetry, BUT at the same time all ground states are indistinguishable in Φ**

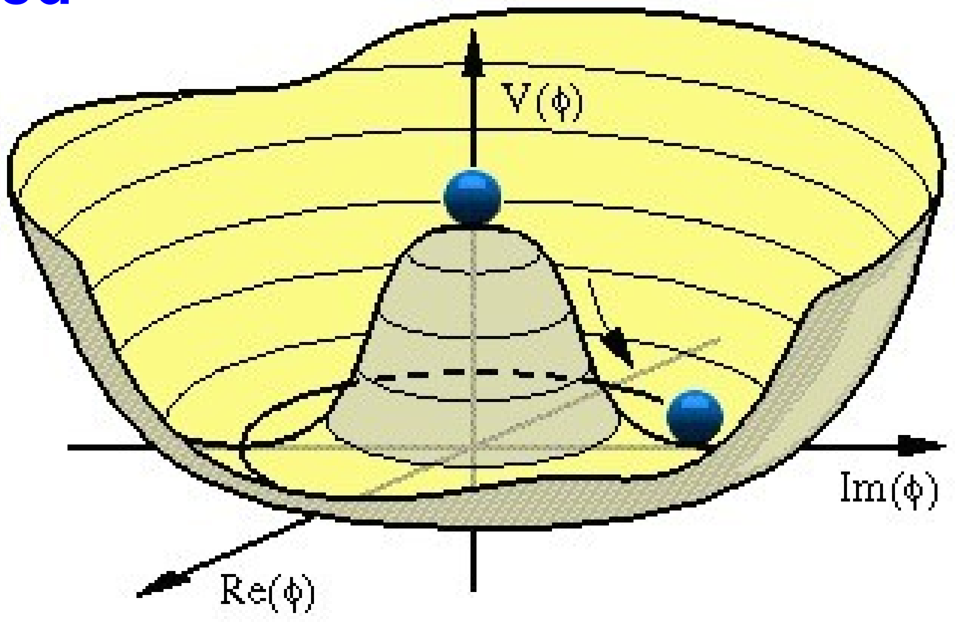


Φ can move freely in the circle corresponding to the minimum in V(Φ)

Goldstone theorem

- **In particle physics this is formalized in the *Goldstone* Theorem:**

In a relativistic covariant quantum field theory with spontaneously broken symmetries massless particles (=*Goldstone* bosons) are created.



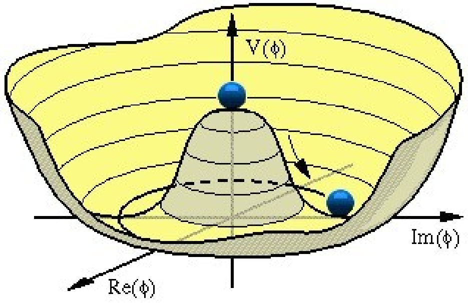
- **Goldstone Bosons can be:**
  - **Elementary fields, which are already part of the theory**
  - **Unphysical or gauge degrees of freedom**
  - **Bound states, which are created by the theory (Cooper-pairs, ...)**

Ground state

- The energy ground state is where the Hamiltonian is minimal

$$\mathcal{H} = \frac{\partial L}{\partial\left(\partial_\mu \phi\right)}\partial_\mu\phi + \frac{\partial L}{\partial\left(\partial^\mu \phi^*\right)}\partial^\mu\phi^* - \mathcal{L} = \partial_\mu\phi\partial^\mu\phi^* + V(\phi)$$

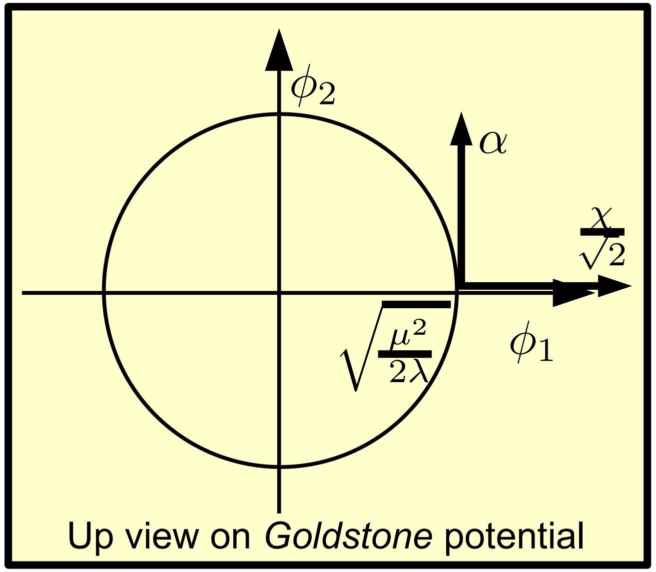
$\longrightarrow$ This happens at $|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$



- To analyse ground state expand system anywhere around point on circle

$$|\phi| = \sqrt{\frac{\mu^2}{2\lambda}}$$

$$\phi(\chi,\alpha) = e^{i\alpha}\left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right)$$



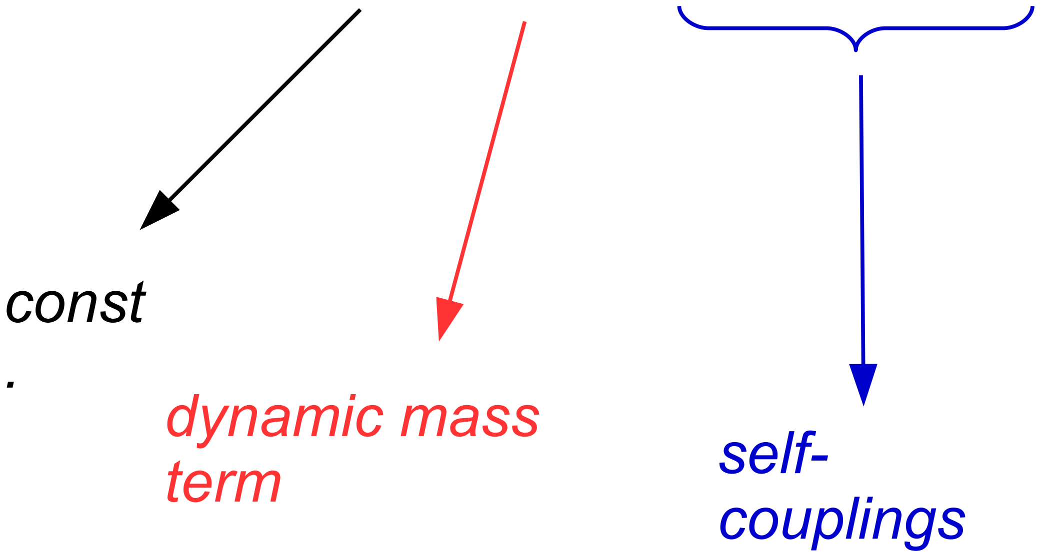
Up view on *Goldstone* potential

Dynamic mass terms

- **An expansion around ground state in cylindrical coordinates gives**

$$\mathcal{L} = \Big[\partial_\mu \phi \partial^\mu \phi^* - V(\phi)\Big]_{\phi(\chi,\alpha)} = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi + \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right)^2 \partial_\mu \alpha \partial^\mu \alpha - V'(\chi)$$

$$\boxed{V'(\chi) = \Big[-\mu^2|\phi|^2 + \lambda|\phi|^4\Big]_{\phi(\chi)} = -\frac{\mu^4}{4\lambda} + \mu^2\chi^2 + \underbrace{\mu\sqrt{\lambda}\chi^3 + \frac{\lambda}{4}\chi^4}}$$



const

.

dynamic mass term

self-couplings

Dynamic mass terms

- **An expansion around ground state in cylindrical coordinates gives**

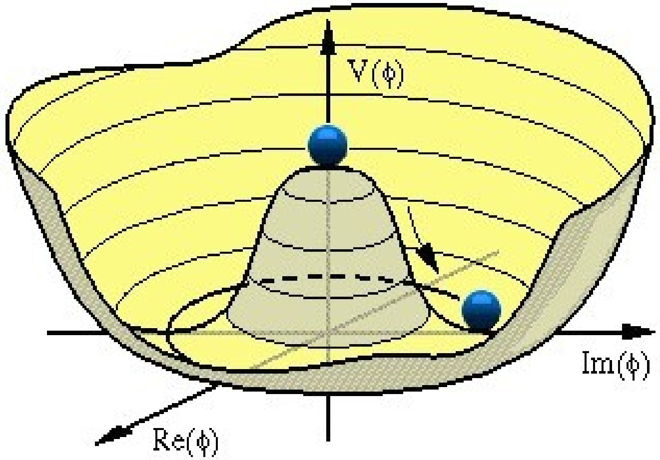
$$\mathcal{L} = \left[\partial_\mu\phi\partial^\mu\phi^* - V(\phi)\right]_{\phi(\chi,\alpha)} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \left(\sqrt{\frac{\mu^2}{2\lambda}} + \frac{\chi}{\sqrt{2}}\right)^2 \partial_\mu\alpha\partial^\mu\alpha - V'(\chi)$$

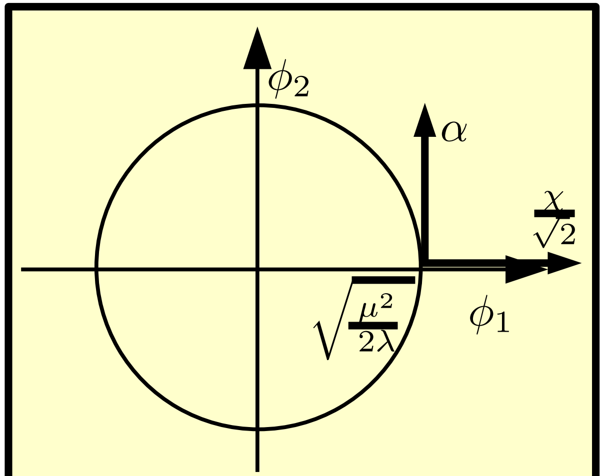
$$\boxed{V'(\chi) = \left[-\mu^2|\phi|^2 + \lambda|\phi|^4\right]_{\phi(\chi)} = -\frac{\mu^4}{4\lambda} + \mu^2\chi^2 + \mu\sqrt{\lambda}\chi^3 + \frac{\lambda}{4}\chi^4}$$

- **Remarks: Expansion around minimum**
  - **no linear term for field χ**
  - **mass term for field χ along radial excitation independent of exact shape of potential at minimum**
  - **field α staying in degenerate minimum does not acquire mass term → Goldstone boson**

U(1) example for symmetry breaking

- *Goldstone* potential and expansion of $\phi \rightarrow \phi(\chi, \alpha)$ around energy ground state generates a mass term $\frac{e^2 \mu^2}{2\lambda} A_\mu A^{\mu *}$ for gauge field A_μ from bare coupling $e^2 \left|\phi\right|^2 A_\mu A^{\mu *}$
- Originally postulated complex scalar field Φ → 2 degrees of freedom
- Χ is only real field; α has been absorbed into A_μ
- The “lost” degree of freedom shows up as additional helicity state 0 of massive vector boson not possible for e.g. massless photon!





Up view on *Goldstone* potential

Choice of potential

- **Observations for the choice of the Goldstone potential:**
  - **leads to spontaneous symmetry breaking**
  - **does not distinguish any direction in space, only depends on Φ**
  - **bound from below, i.e. no negative energy states → stable theory**
  - **simplest potential with these features**
  - **no uneven powers in Φ**
  - **Powers larger than 4 lead to couplings with dimensions and not renormalisable theories**

Now let’s try to get 3 boson masses ...

Complex isospin doublet

- Ansatz: introduce **simplest scalar field** that fulfills requirements:
  - Symmetric under $SU(2)_L \times U(1)_Y$ gauge group
  - Three **massive** gauge bosons, but **massless** photons
- Solution: **isospin doublet** of two complex-valued fields $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$
  - **Four** degrees of freedom → later: **three** degrees of freedom to make gauge bosons massive, **one** physical Higgs boson
  - Quantum numbers of field: $I = 1/2$, $I_3 = \pm 1/2$, $Y = 1$
  - Covariant derivative:

$$D_\mu \Phi = \left(\partial_\mu - ig \frac{\tau^a}{2} W_\mu^a - ig' \frac{Y}{2} \mathbb{1}_2 B_\mu \right) \Phi$$

  - Additional terms in electroweak Lagrangian:

$$\boxed{\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2} \quad \text{with } \mu^2 < 0,\ \lambda > 0$$

Vacuum expectation value

- Choose **vacuum expectation value** (VEV) of Higgs field after SSB:

$$\langle 0|\Phi|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \text{ with } v = \sqrt{\frac{-\mu^2}{2\lambda}}$$

- Physical Higgs field:
  - **Expansion** around VEV for particular gauge choice (“unitary gauge”):

$$\Phi(x) \to \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h(x)\end{pmatrix}$$

  - Choose neutral component of Higgs field ϕ^0 such that subgroup $U(1)_{EM}$ of $SU(2)_L \times U(1)_Y$ remains unbroken → photon **massless**

- In Lagrangian: replace Higgs fields by first terms of expansion
  - Terms with Higgs VEV v → gauge boson **masses**
  - $h(x)$ → physical **Higgs boson** = charge-neutral spin-0 particle

Gauge boson masses

- **Kinetic term** of Higgs Lagrangian: mass terms quadratic in fields

$$\mathcal{L}_{\text{mass}} = \frac{g^2 v^2}{4} W_\mu^+ W^{\mu,-} + (B^\mu, W^{\mu,0}) \frac{v^2}{8} \begin{pmatrix} g'^2 & -gg' \\ -gg' & g^2 \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^0 \end{pmatrix}$$

- **Diagonalize** mass matrix to obtain physical states A_μ (photon) and Z_μ (new: Z boson)

$$\begin{pmatrix} B_\mu \\ W_\mu^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} \quad \text{with } \cos\theta_W := \frac{g}{\sqrt{g^2 + g'^2}}$$

  - θ_W: **weak mixing angle** (also: Weinberg angle)
  - Measured value: $\sin^2\theta_W \approx 0.23$

- Gauge boson **mass terms**:

$$m_\gamma^2 = 0, \quad m_W^2 = \frac{g^2 v^2}{4}, \quad m_Z^2 = \frac{v^2}{4}(g^2 + g'^2) \quad \rightarrow \varrho_0 = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$$

(testable prediction of standard model: $\varrho_0 = 1$)

Lepton masses

- Mass terms of QED: $m_f\bar{\psi}\psi = m_f\,(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$
  - Symmetric in left-handed and right-handed components
  - **Not invariant** under $SU(2)_L$ gauge symmetry

- Solution using fields of Brout–Englert–Higgs mechanism:
  - Fermion masses generated with **same scalar field** as gauge boson masses
  - Postulate **fermion couplings to Higgs field**, gauge invariant under $SU(2)_L{\times}U(1)_Y$
  - Coupling type: **Yukawa coupling** $\bar{\psi}\phi\psi$ (coupling term to create Yukawa potential for scalar particles)

- Yukawa coupling for electrons:

$$\mathcal{L}_{\text{Yukawa}} = -\sqrt{2}f\left[\bar{L}\,\Phi\,R + \bar{R}\,\Phi^{T*}\,L\right] \qquad \text{with } L = \begin{pmatrix}\nu_e\\ e\end{pmatrix}_L, \quad R = e_R$$

$$\overset{\text{SSB}}{=} -f\left[\bar{L}\begin{pmatrix}0\\ v\end{pmatrix}R + \bar{R}(0,v)L\right] = -fv\,\bar{e}e = -m_e\,\bar{e}e$$

Lepton masses

- Yukawa coupling for leptons:

$$\mathcal{L}_{\text{Yukawa}} = -\sqrt{2}f\left[\overline{L}\Phi R + \overline{R}(\Phi^T)^* L\right] \overset{\text{SSB}}{=} -f\left[\overline{L}\begin{pmatrix}0\\ v\end{pmatrix}R + \overline{R}(0,v)L\right] = -fv\,\overline{e}e = -m_e\overline{e}e$$

  - Mass equivalent to **mixing** of left-handed and right-handed components
  - Mass term exists only for electrons, neutrinos stay **massless**
  - Yukawa couplings for quarks more complicated → later

- Remark: left-handed and right-handed couplings
  - Reminder: chirality **projection operators**:

$$P_R = \frac{1}{2}(1+\gamma_5), \quad P_L = \frac{1}{2}(1-\gamma_5)$$

  - Application to electron spinor:

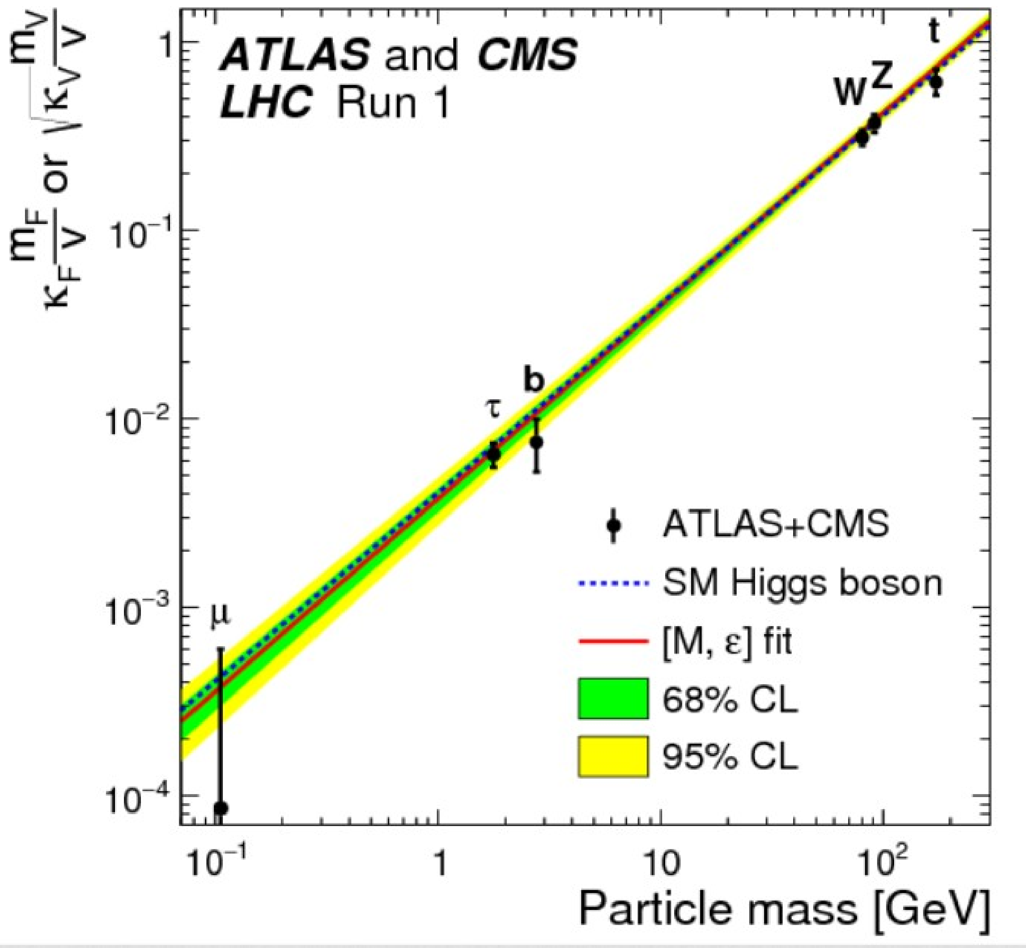
$$e_L = \frac{1}{2}(1-\gamma_5)e, \quad e_R = \frac{1}{2}(1+\gamma_5)e, \quad \overline{e}_L = \overline{e}\frac{1}{2}(1+\gamma_5), \quad \overline{e}_R = \overline{e}\frac{1}{2}(1-\gamma_5)$$

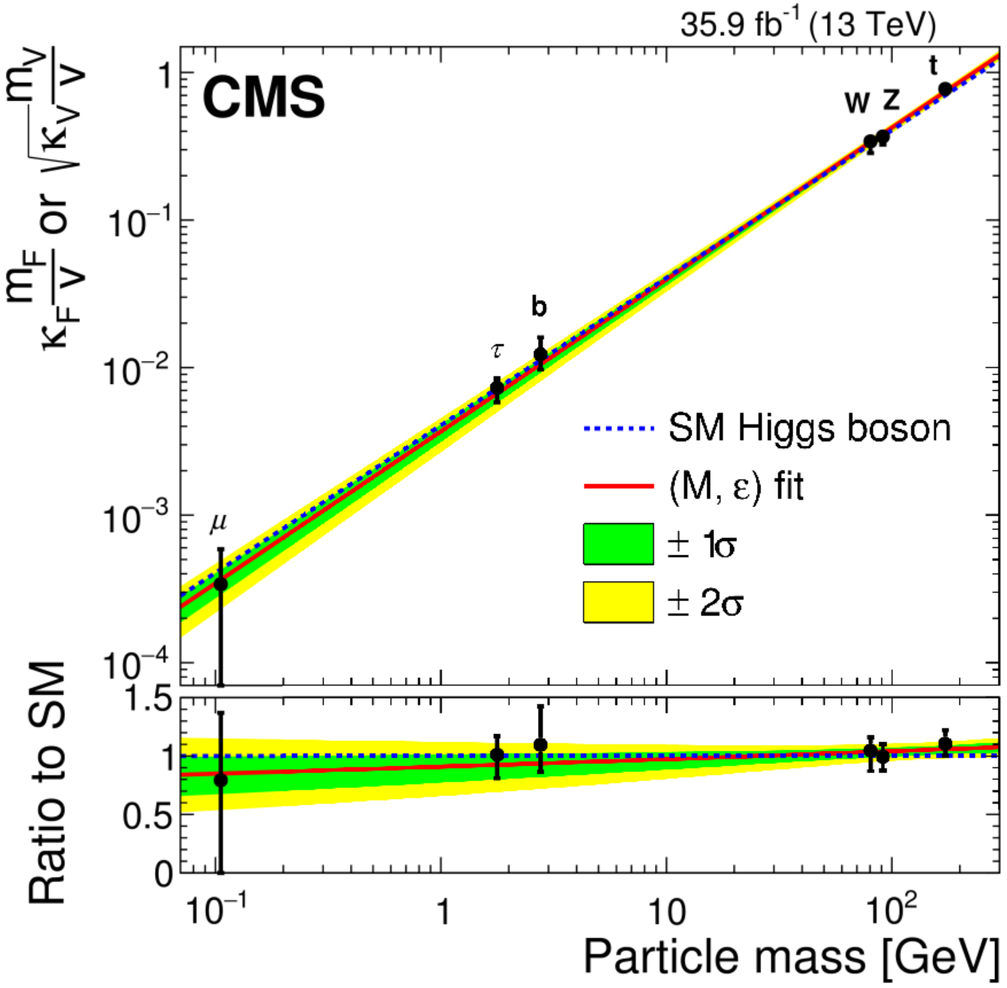
  - Form scalar bilinear forms, e.g. in mass term: $\overline{e}e = \overline{e}_L e_R + \overline{e}_R e_L$

Higgs couplings so far

Run 1, ATLAS & CMS combined
2 x ~ 25/fb, E_{CMS} = 7 and 8 TeV

Run 2 2016, CMS only
1 x ~ 36/fb, E_{CMS} = 13 TeV





Summary

- Electroweak lagrangian: $SU(2)_L \times U(1)_Y$ does not allow for mass terms
- Symmetry in lagrangian gets broken through vacuum ground state
- Adding complex scalar field / doublet and “Mexican hat” potential
  - symmetry spontaneously broken (Brout-Englert-Higgs mechanism BEH)
  - W and Z bosons dynamically acquire mass terms
  - photon remains massless through gauge fixing
  - 4th degree of freedom shows up as new scalar particle: Higgs boson
- Mass terms for leptons become possible as well (Yukawa couplings)

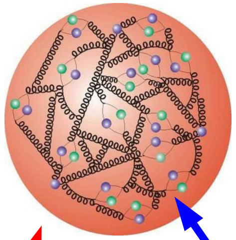
What about hadronic matter?

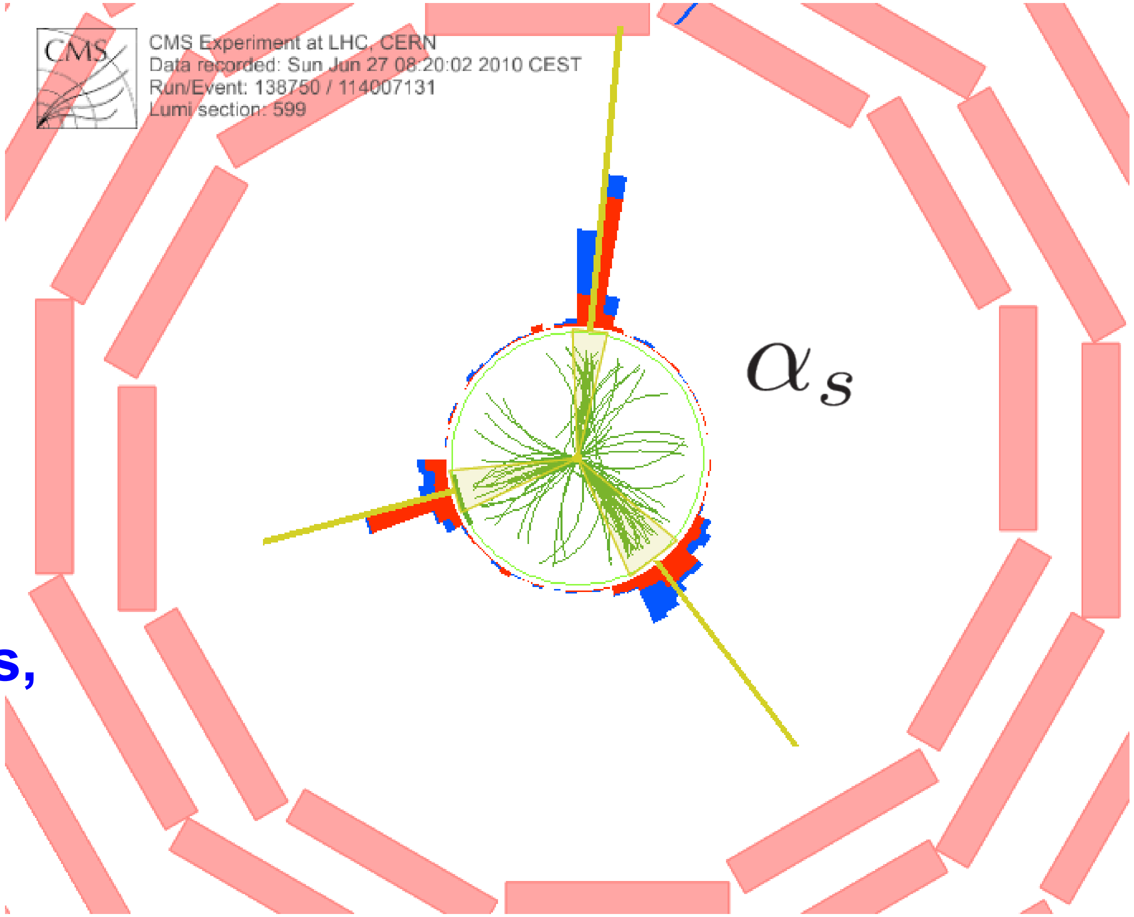
Understanding of the building blocks of matter and the forces between them

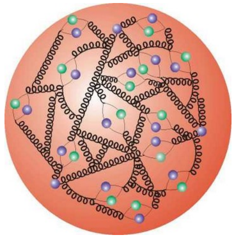
Most successful tool: Investigation of scattering processes!

Dismantling not possible!

But frequent repetitions ...







Inside nucleus, Quark-Gluon-Soup ?

Nucleus, Proton

HERA-Proton, DESY