Introduction to theoretical foundations V
(from experimenter's viewpoint)

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Summary

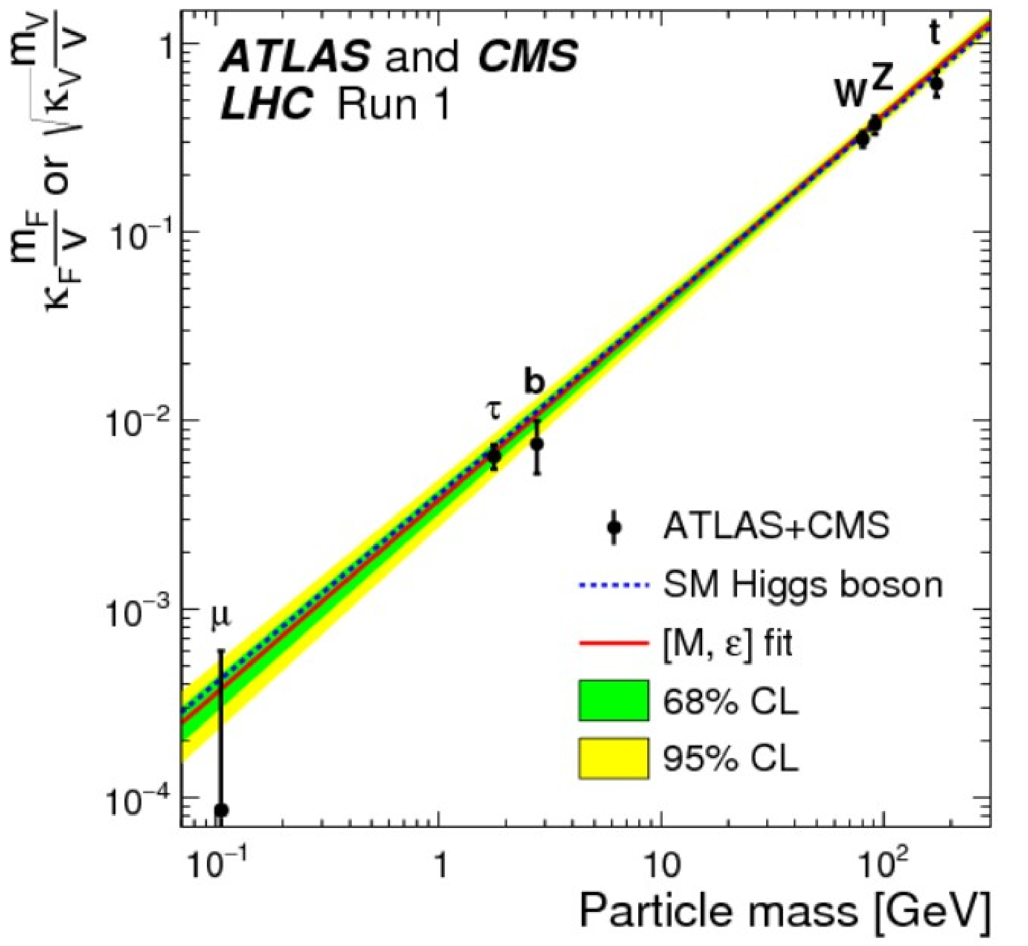
- Electroweak lagrangian: $SU(2)_L \times U(1)_Y$ does not allow for mass terms
- Symmetry in lagrangian gets broken through vacuum ground state
- Adding complex scalar field / doublet and “Mexican hat” potential
  - symmetry spontaneously broken (Brout-Englert-Higgs mechanism BEH)
  - W and Z bosons dynamically acquire mass terms
  - photon remains massless through Weinberg mixing and gauge fixing
  - 4^{th} degree of freedom shows up as new scalar particle: Higgs boson
- Mass terms for leptons become possible as well (Yukawa couplings)

To last week’s question:

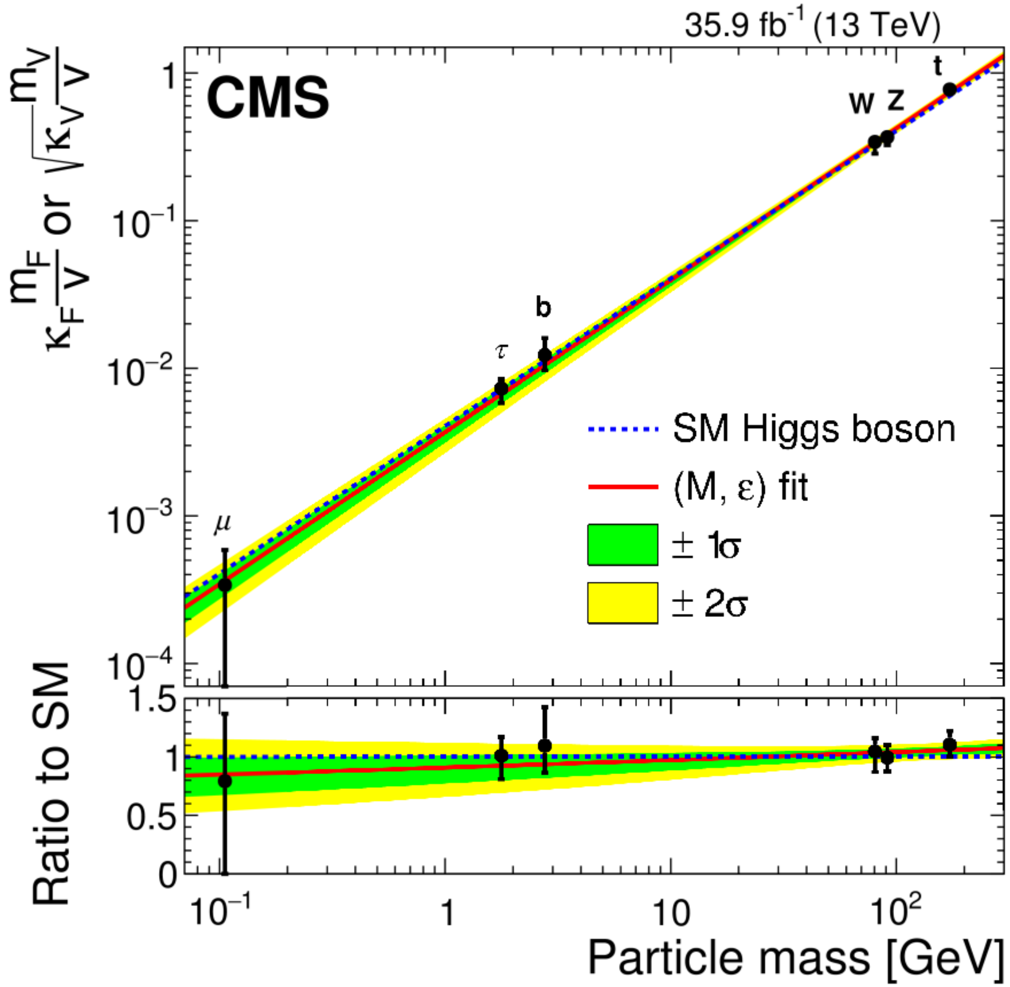
- BEH & BCS: No analogue to Cooper pairs ($m = 2\, m_e$)
  - massive W, Z ↔ massive photon; massive Higgs ↔ plasmon
  - Fraser, D. & Koberinski, A., “The Higgs mechanism and superconductivity: A case study of formal analogies”, Studies in History and Philosophy of Modern Physics, 2016, 55, 72 – 91. Preprint

Higgs couplings so far

Run 1, ATLAS & CMS combined
2 x ~ 25/fb, E_{CMS} = 7 and 8 TeV



Run 2 2016, CMS only
1 x ~ 36/fb, E_{CMS} = 13 TeV



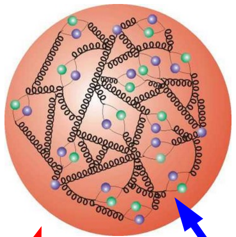
What about hadronic matter?

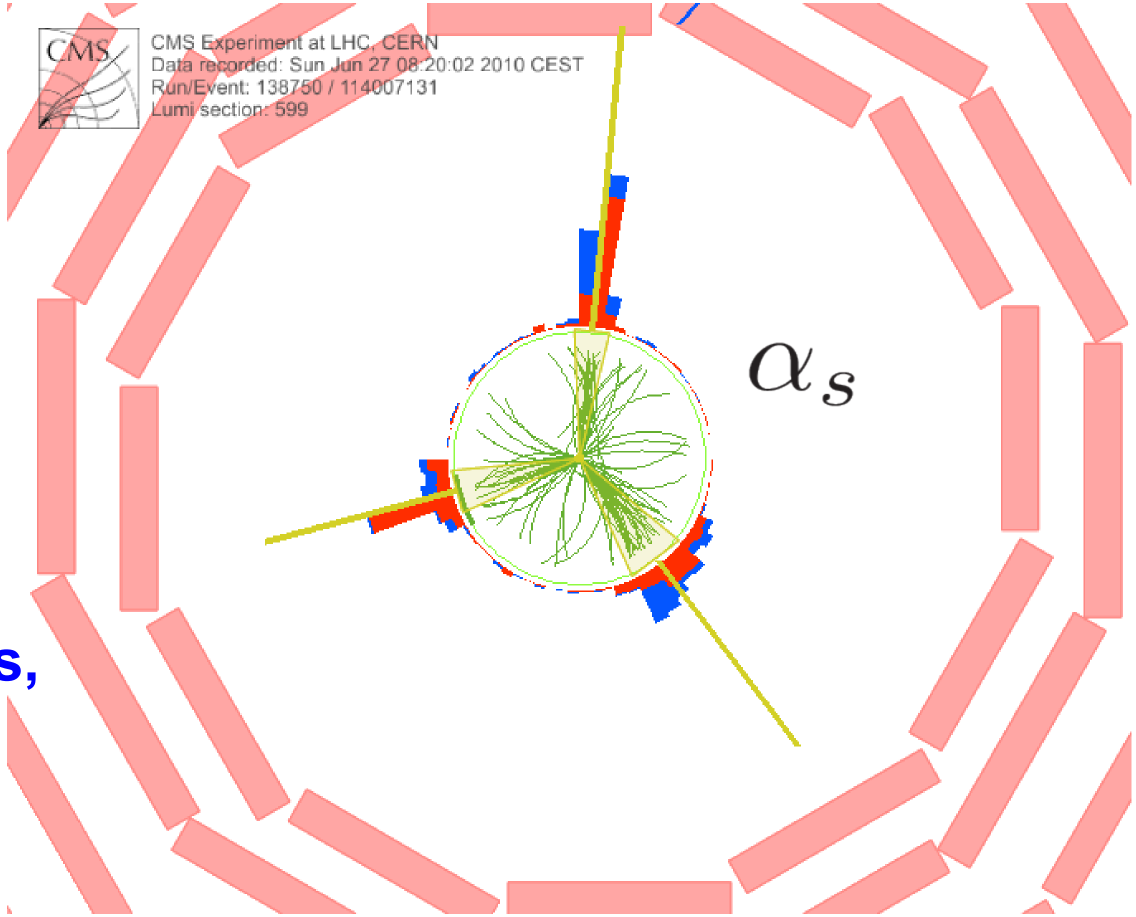
Understanding of the building blocks of matter and the forces between them

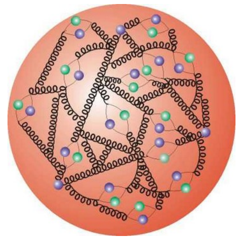
Most successful tool: Investigation of scattering processes!

Dismantling not possible!

But frequent repetitions ...







Inside nucleus, Quark-Gluon-Soup ?

Nucleus, Proton

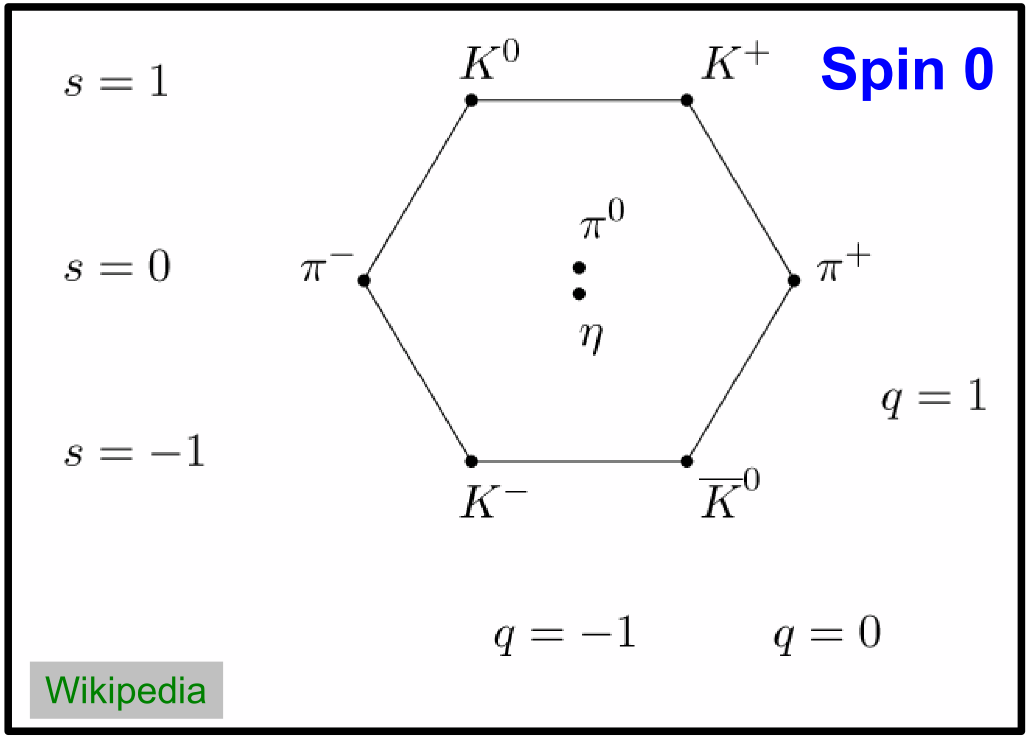
HERA-Proton, DESY

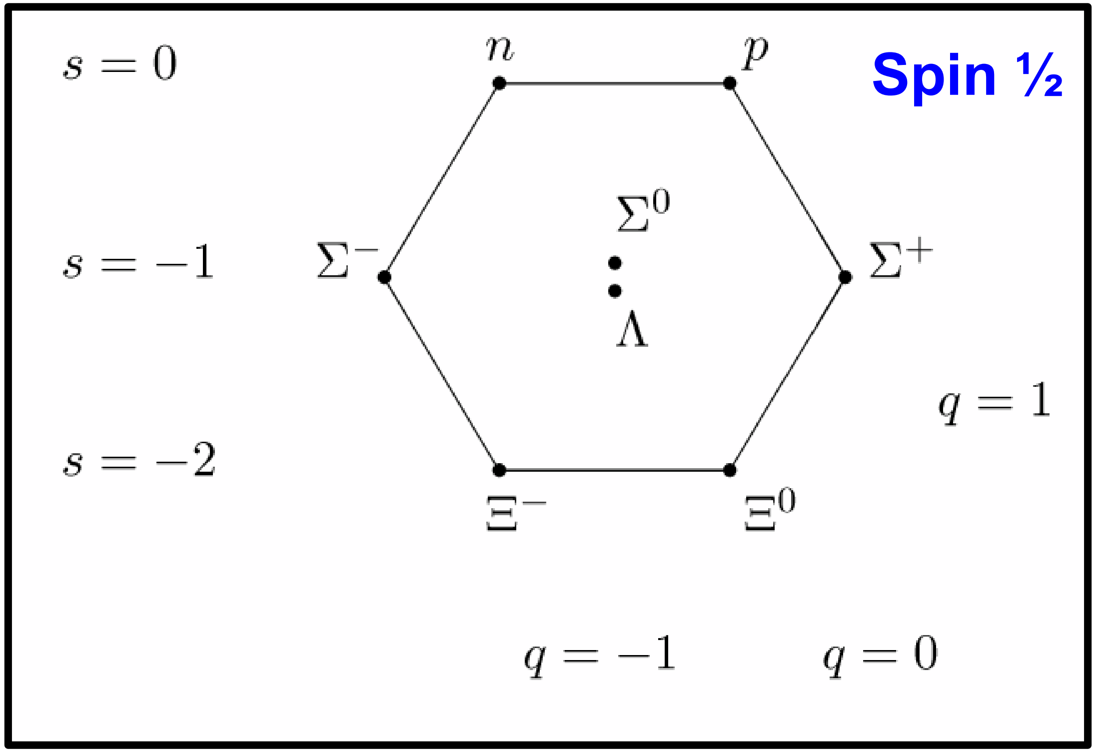
A particle zoo!

Nobel prize 1969

- **Cosmic ray experiments and the development of accelerators lead to the discovery of many new "elementary" particles in the years 1947 – 1970!**
  - **M. Gell-Mann, 1964: Brings first order into the particle zoo via the Eightfold Way: Ordering of the mesons (left) and baryons (right) in schemes according to charge q and "strangeness" s:**

nobelprize.org



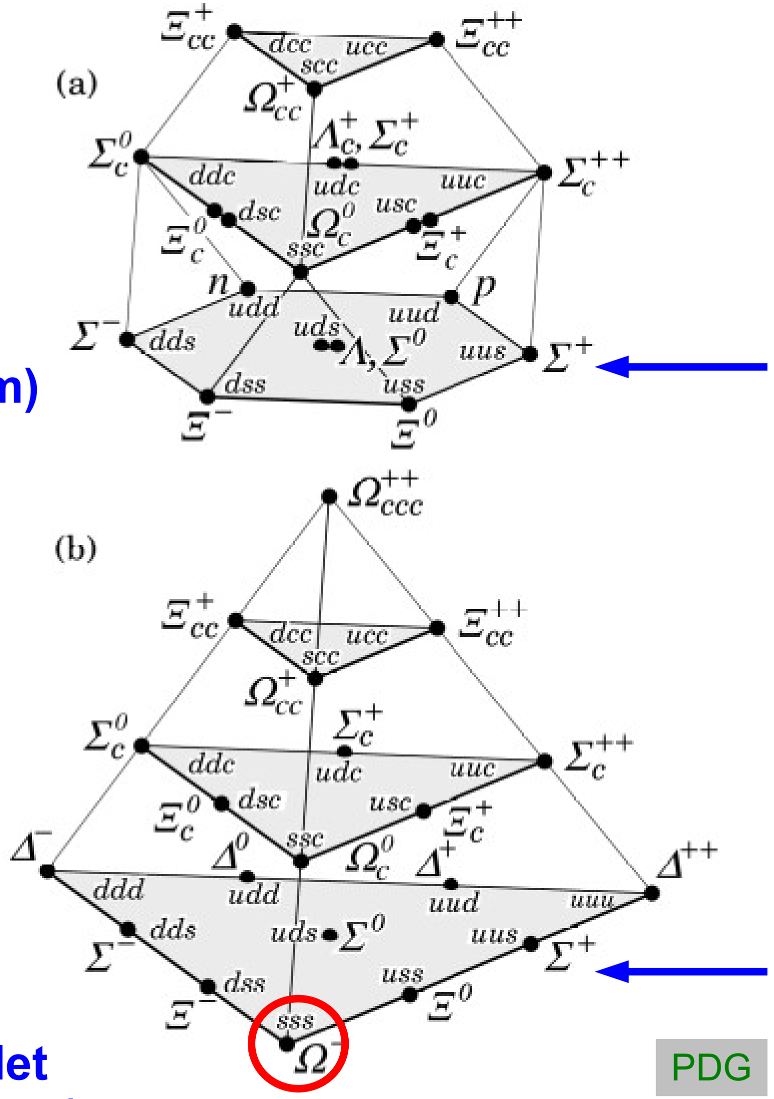


With charm and color

- The J=3/2 baryon dekuplet was not yet known completely at the time of Gell-Mann's idea (neither was charm)
  - Prediction of the Ω^- baryon
- But: Like with the Δ^{++} and Δ^-
  - Spin-statistic-problem!
- J=3/2 fermions with symmetric space, spin and flavor wave function (ground state, 3xspin up, 3 identical quarks) is in contradiction to Pauli's exclusion rule!
  - A way out:
    - O.W. Greenberg, 1964: Additional degree of freedom
    - M. Gell-Mann, 1972: Color = tristate, RedGreenBlue

Originally the names were red, blue & white ...

With strangeness and charm!



J = ½ baryon octet with C = 0 (i.e. no charm)

J = 3/2 baryon dekuplet with C = 0 (i.e. no charm)

PDG

Quark-Parton-Model

- **M. Gell-Mann:** Composition of mesons out of a quark-antiquark-pair and baryons out of three quarks (named after a citation from J. Joyce “Finnegan's Wake”: “Three quarks for Muster Mark.”)
- **G. Zweig:** Analogous idea, his naming “aces” for the hypothetical constituents did not stick.
  - Quarks/Aces seen only as hypothetical mathematical constructs; charges coming in thirds like required here were never observed
- **R. Feynman:** Explanation of the measurements of deep-inelastic electron-proton scattering at the SLAC-MIT experiment with point-like scattering centres inside the protons: Partons
  - Later: Identification of the partons with (anti-)quarks and gluons

Sakurai prize 2015

Scienceworld

nobelprize.org

Nobel prize 1965 for QED with J. Schwinger, S.-I. Tomonaga

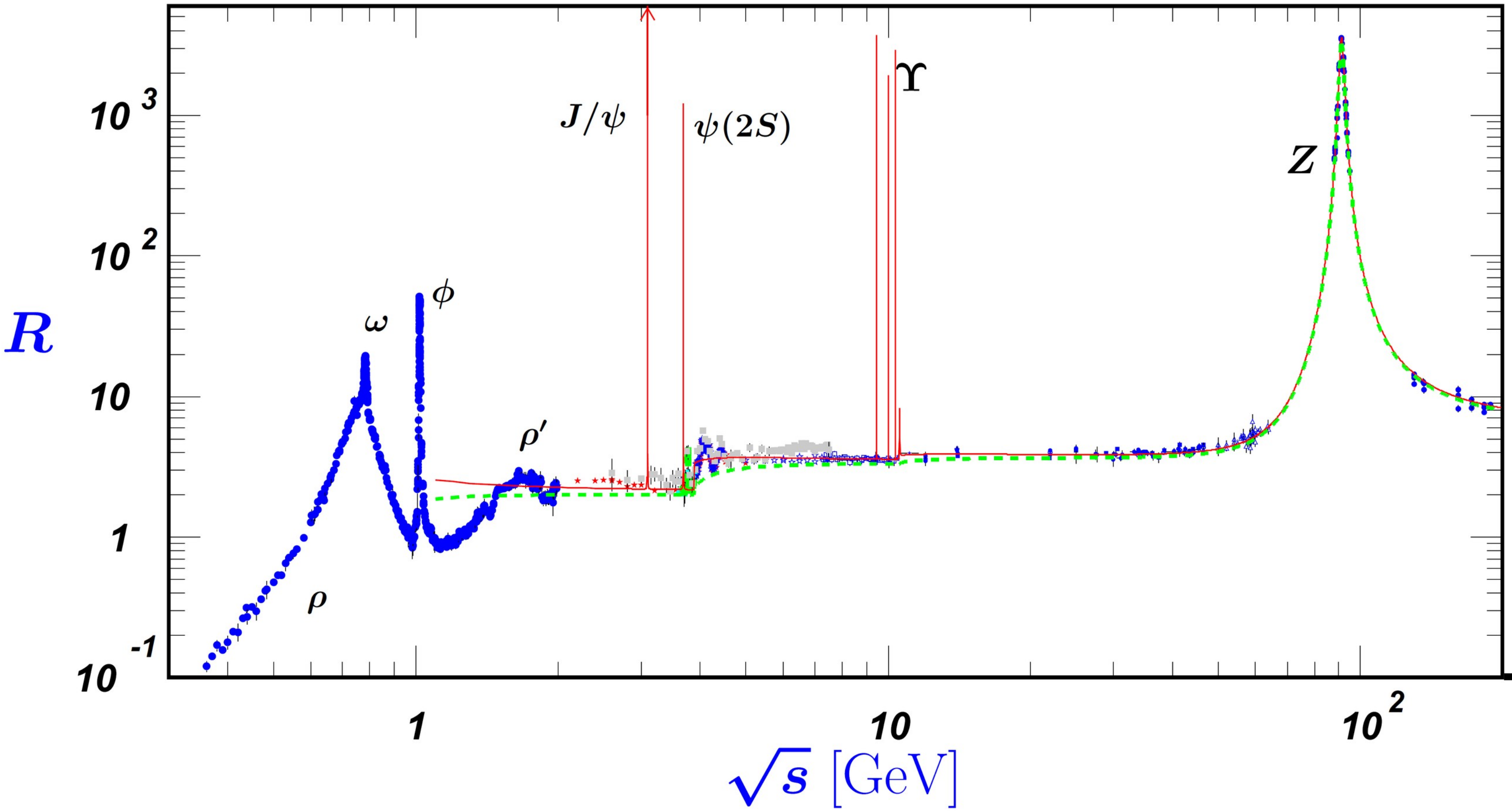
Historically interesting:
Abraham Pais, “Inward bound”, Clarendon Press, Oxford 1986.

→ “lepton”, “baryon”

More evidence for "color"

Hadronic branching ratio in elektron-positron annihilation

$$R(s) = \frac{\sigma(e^+e^- \to hadrons, s)}{\sigma(e^+e^- \to \mu^+\mu^-, s)}$$



Higgs, top quark

PDG

More evidence for "color"

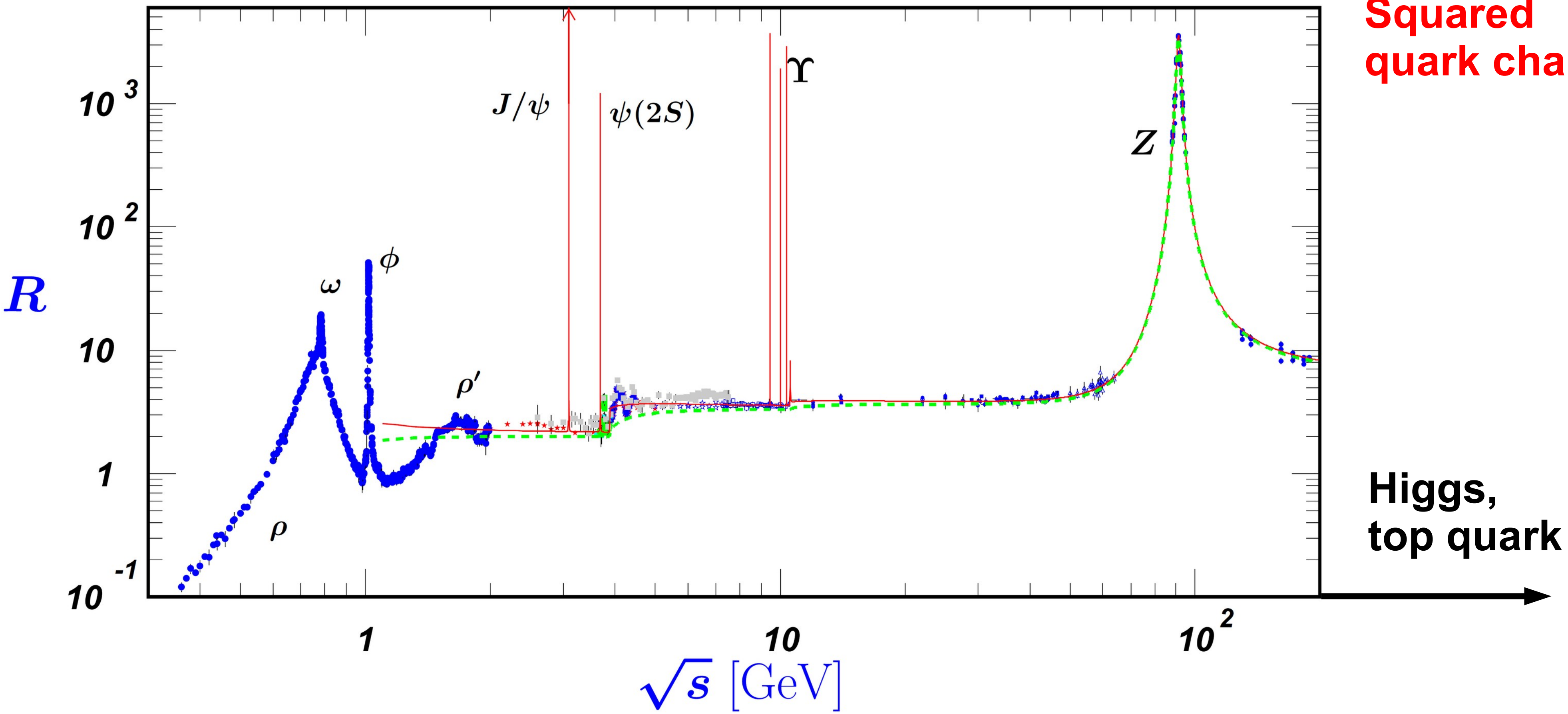
Hadronic branching ratio in elektron-positron annihilation

$$R(s) = \frac{\sigma(e^+e^- \to hadrons, s)}{\sigma(e^+e^- \to \mu^+\mu^-, s)} = 3 \cdot \sum_q Q_q^2$$

Color factor N_c

Squared quark charges

$$R_{\text{uds}} = 3 \cdot \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2$$



Higgs,
top quark

PDG

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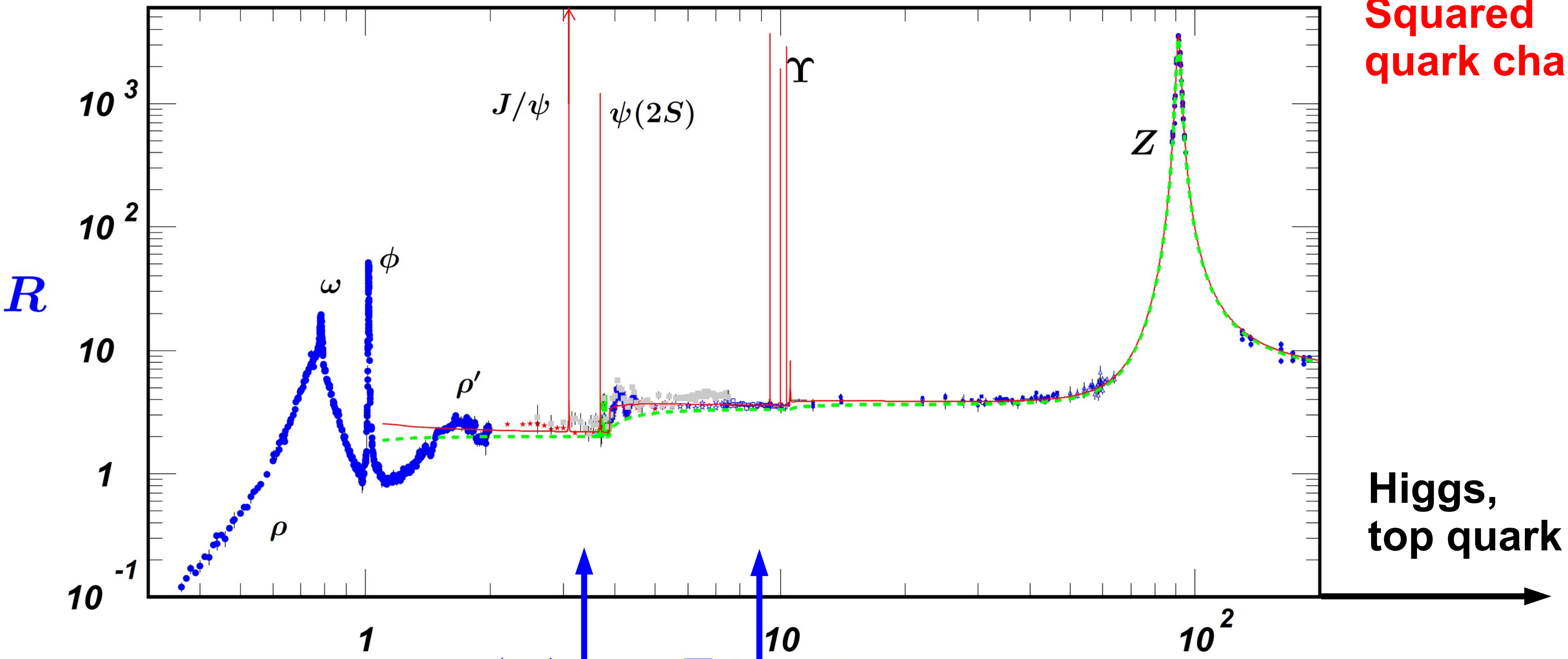


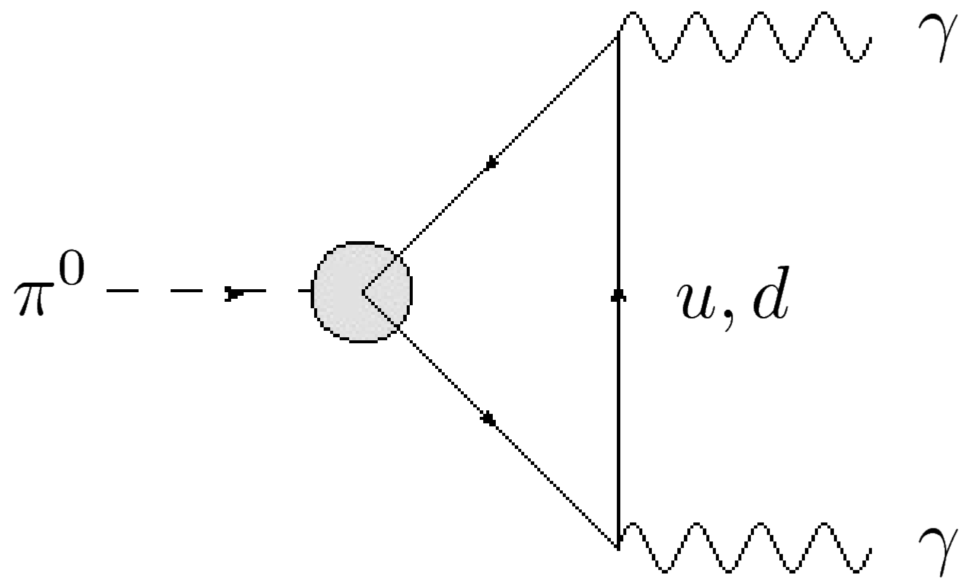
Higgs, top quark

charm threshold $(c\bar{c})$ $R \to \frac{10}{3}$

$R \to \frac{11}{3}$ $(b\bar{b})$ **bottom threshold**

PDG

Pion decay rate into two photons



LO amplitude of the decay

Color factor N_c

mass

$$\Gamma(\pi^0 \to \gamma\gamma) = N_c^2 (Q_u^2 - Q_d^2)^2 \frac{\alpha^2 m_\pi^3}{64\pi^3 f_\pi^2}$$

Squared quark charges

Decay constant (from charged pions)

Evaluation from independent measurements of other observables:

$$\Gamma(\pi^0 \to \gamma\gamma) = 7.33\,\mathrm{eV} \left(\frac{N_c}{3}\right)^2$$

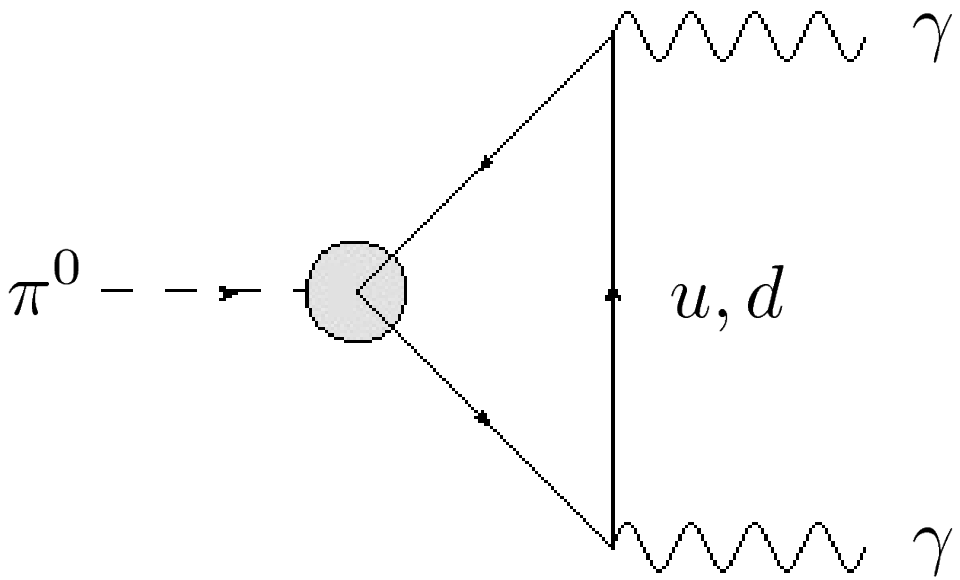
Measurement:

$$\Gamma(\pi^0 \to \gamma\gamma) = 7.84 \pm 0.56\,\mathrm{eV}$$

PDG

More evidence for “color”

Pion decay rate into two photons



LO amplitude of the decay

$$\Gamma(\pi^0 \to \gamma\gamma) = N_c^2 (Q_u^2 - Q_d^2)^2 \frac{\alpha^2 m_\pi^3}{64\pi^3 f_\pi^2}$$

Attention, not the only choice!
$N_C = 1$, $Q_u = 1$, $Q_d = 0$ ...

Evaluation from independent measurements of other observables:

$$\Gamma(\pi^0 \to \gamma\gamma) = 7.33\,\mathrm{eV} \left(\frac{N_c}{3}\right)^2$$

Measurement:

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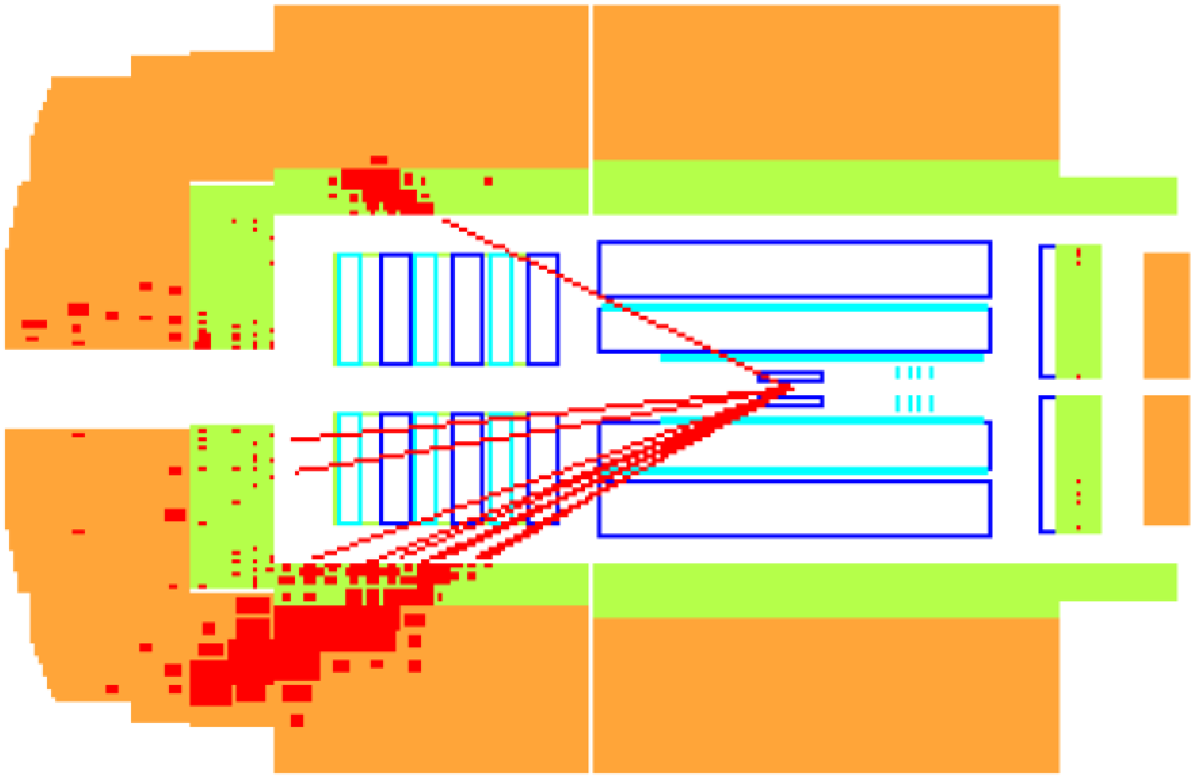
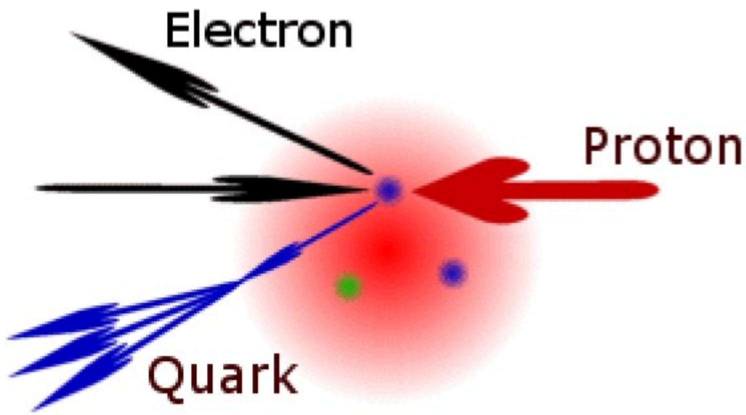
PDG

Electromagnetic reaction:

Backscattering of electron off charged proton constituent

H1 Detector

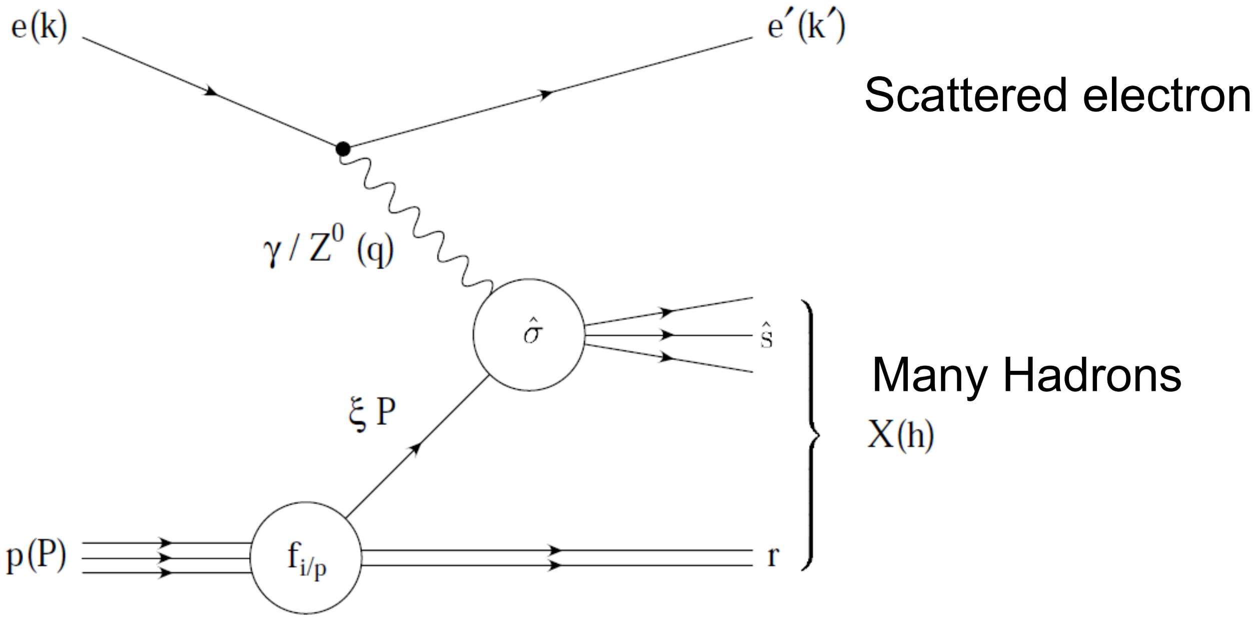




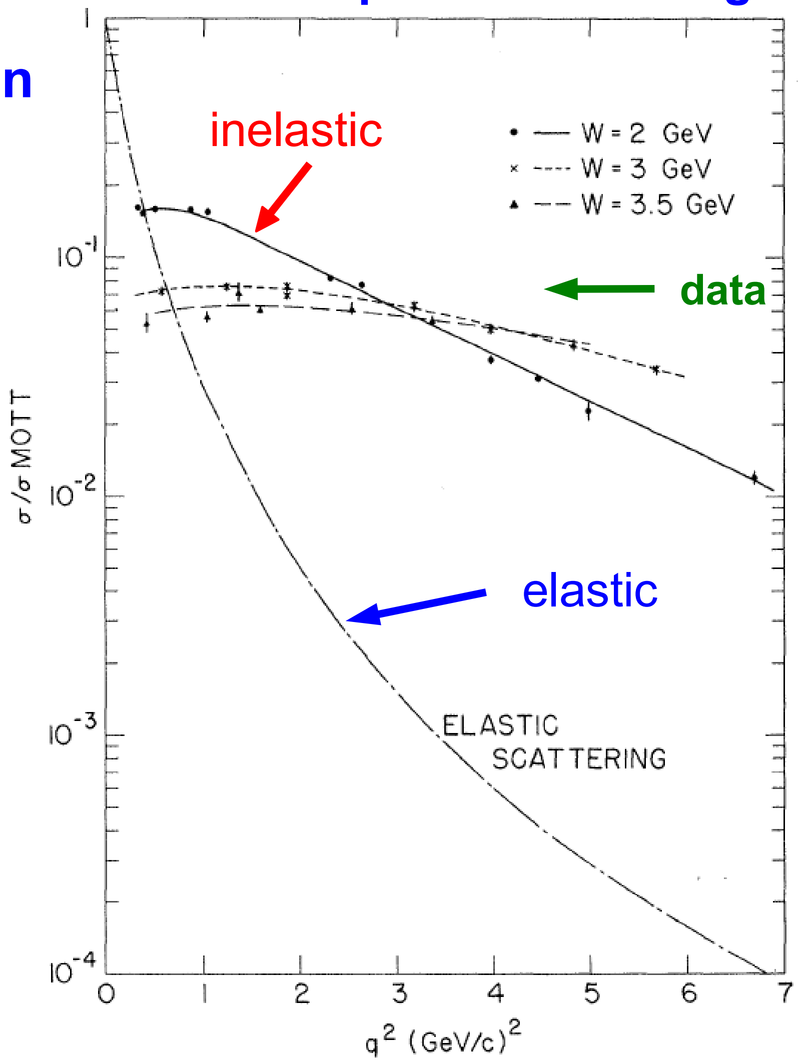
H1 Event Tutorial, J Meyer, DESY (2005)

- **Inelastic >> elastic cross section**
- **Inelastic cross section ~ const. * Mott x section**
  - **approximately independent of resolution ~ q^2**
  - **scale invariant, i.e. no natural length scale**
  - **like scattering at point-like objects**

Deep-inelastic scattering (DIS)



electron-proton scattering



PRL 23 (1969) 935.

Previously: U(1) and SU(2)

- **U(1) → 1 generator:** $\mathbf{t} = 1 \quad G = e^{i\alpha\cdot 1}$

  → **equivalent to rotations in 2-dim., i.e. the orthogonal group O(2)**

  - **Commutator:** 0 → Abelian

- **SU(2) → 3 generators:** $\mathbf{t}_a = \frac{1}{2}\sigma_a \qquad (a = 1, 2, 3)$

  Pauli matrices σ_a

  $$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

  → **equivalent to rotations in 3-dim., i.e. the orthogonal group O(3)**

  - **Commutator:** $[\mathbf{t}_a, \mathbf{t}_b] = i\epsilon_{abc}\mathbf{t}_c$

    → Non-Abelian

  - **With structure constants of SU(2):** ϵ_{abc} Levi-Civita tensor

Now: SU(3)

- **SU(3) → 8 generators:** $\tau_A = \frac{1}{2}\lambda_A \qquad (A = 1, \ldots, 8)$

  Gell-Mann matrices λ_A

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- **Commutator:** $[\tau_A, \tau_B] = i f_{ABC} \tau_C$ → Non-Abelian
- **With structure constants SU(3):** $f_{123} = 1$

Not showing index permutations!

$$f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2} \qquad f_{458} = f_{678} = \frac{\sqrt{3}}{2}$$

QCD Lagrangian

- **Covariant derivative:** $(D_\mu)_{ab} = \partial_\mu \delta_{ab} + i g_s \tau^A_{ab} \mathcal{A}^A_\mu$

  Color indices of octet representation A,... = 1,…,8

  Color indices of triplet representation a,b,c = 1,2,3

  Gluon field $\mathcal{A}^A_\mu$

- **Field strength tensors:**

$$\mathcal{G}^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu - \boxed{g_s f^{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu}$$

→ leads to triple (TGC) and quartic (QGC) gauge couplings

- **Lagrangian of $SU(3)_C$:**

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \overline{\psi}_a (i\gamma^\mu (D_\mu)_{ab} - m_q)\psi_b - \frac{1}{4}\mathcal{G}^A_{\mu\nu}\mathcal{G}^{\mu\nu}_A$$

Not showing gauge fixing or ghost terms ...

- **The gluon remains massless → $SU(3)_C$ exact symmetry of nature!**

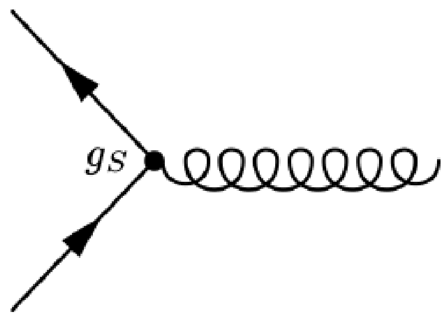
QCD Lagrangian

Invariance under local $SU(3)_c$ transformations

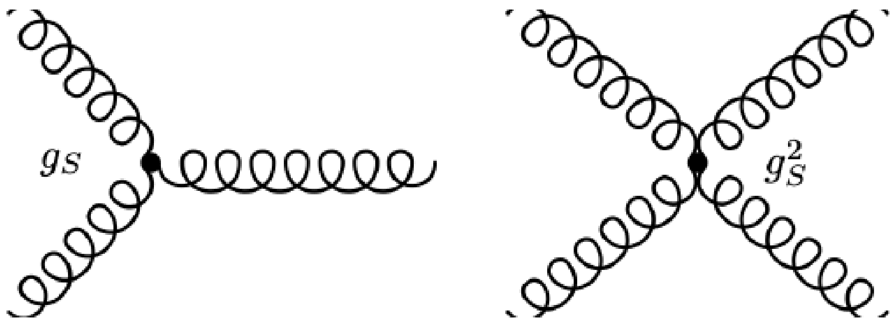
- **Three color charges a = 1, 2, 3 → Red, Green, Blue (as analogue to electric charge in QED)**
- **Eight vector fields (gluons) $\mathcal{A}_\mu^A$ carry color charge and color anti-charge**
- **The gluons are massless**
  → exact symmetry
  → in principal infinite range of strong force

$$\mathcal{G}_{\mu\nu}^A = \partial_\mu \mathcal{A}_\nu^A - \partial_\nu \mathcal{A}_\mu^A - \boxed{g_s f^{ABC} \mathcal{A}_\mu^B \mathcal{A}_\nu^C}$$

- **Non-zero commutator leads to gluon self-interactions via triple and quartic gauge couplings**



quark-gluon interaction



gluon self-interaction

Yang-Mills Theories

- Extension of the gauge principle to ***non-Abelian groups***
  - Standard Model: in particular SU(2) and SU(3)
- SU(n) transformations $\psi \rightarrow \exp[i\frac{1}{2}g\beta^a(x)\tau^a]\psi$
  - $n^2 - 1$ ***generators*** $\boldsymbol{\tau^a}$
  - Non-Abelian algebra $[\tau^a, \tau^b] = if^{abc}\tau^c$ with ***structure constants*** $\boldsymbol{f^{abc}}$
- Analogue to QED: ***invariance under local SU(n) transformations by introducing covariant derivative and field-strength tensor***

$$D_\mu = \partial_\mu + ig\tau^a A^a_\mu$$

with

$$A^a_\mu \rightarrow A^a_\mu + \frac{1}{g}\partial_\mu\beta^a(x) + f^{abc}\beta^b(x)A^c_\mu$$

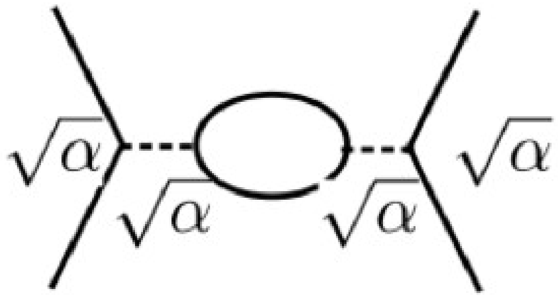
$$[D_\mu, D_\nu]^a = igF^a_{\mu\nu}, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

- ***Non-zero structure constants lead to gauge boson self-interaction***
- NB: above relations also hold for U(1)

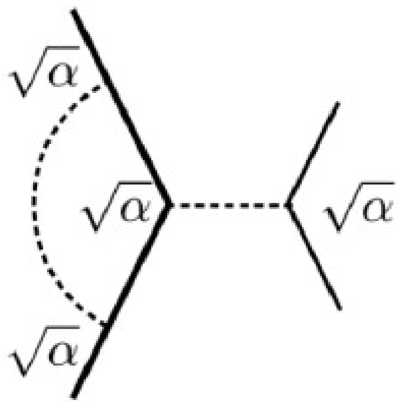
Quantum corrections

- Dynamics of a theory not entirely described by Lagrange density
  - Fields are quantised: ***effects due to quantum corrections*** occur
  - Taken into account in perturbation series
- 'Good' quantum-field theories, like the Standard Model, are
  - ***Anomaly free***: symmetries of the Lagrangian not destroyed by quantum corrections
  - ***Renormalizable***: divergencies in quantum corrections absorbed in redefined parameters of the Lagrangian

Veltman, 't Hooft: Nobel prize 1999



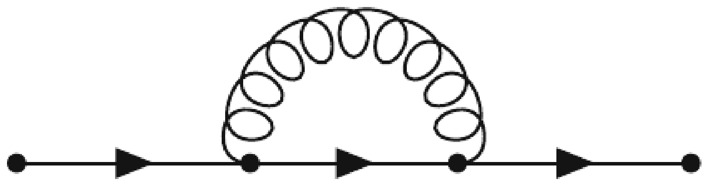
Modifies effective particle masses ('running masses')

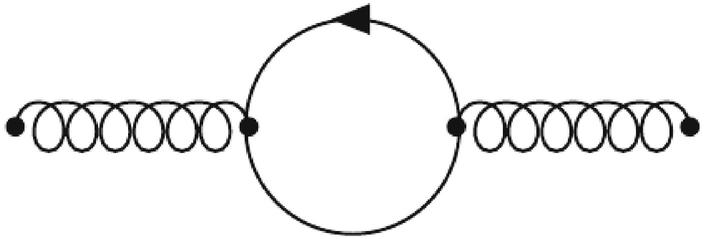


Modifies effective couplings ('running couplings')

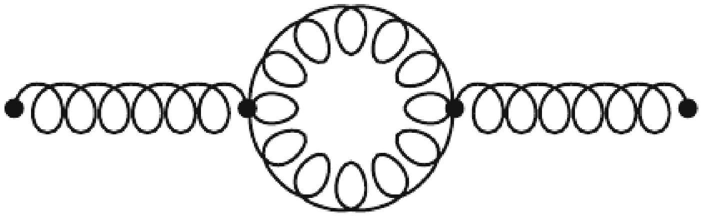
Quantum corrections

- **Quark (left) and gluon (middle & right) self-energy corrections:**

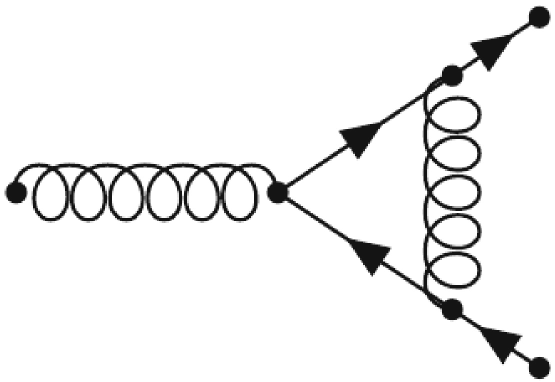


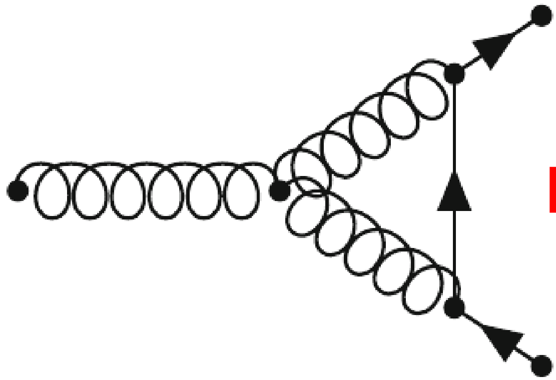


Not in QED!



- **Quark-gluon vertex corrections:**





Not in QED!

Beta functions

- **In (renormalisable) QFT the beta function encodes the dependence of the coupling parameter g on the energy (or distance) scale μ:**

$$\alpha_i := \frac{g_i^2}{4\pi} \qquad \beta(g) = \frac{\partial g}{\partial \log(\mu^2)}$$

- **Beta function of QED (1-loop):** $\beta(\alpha) = \frac{1}{3\pi}\alpha^2$
  - **The coupling increases with energy scale**
  - **The coupling decreases with larger distances**
    - **Infinite range, Coulomb potential:** $V(r) \propto \frac{1}{r}$

- **Beta function of QCD (1-loop):** $\beta(\alpha_s) = -\left(\frac{11N_C - 2N_f}{12\pi}\right)\alpha_s^2$
  - **The coupling decreases with energy scale, if** $N_C = 3, \quad N_f \leq 16$
    - **Asymptotic freedom**
  - **The coupling increases with larger distances**
    - **Confinement, string potential:** $V(r) \approx \sigma \cdot r$ **with tension** $\sigma \approx 1\,\mathrm{GeV/fm}$

QCD and asymptotic freedom

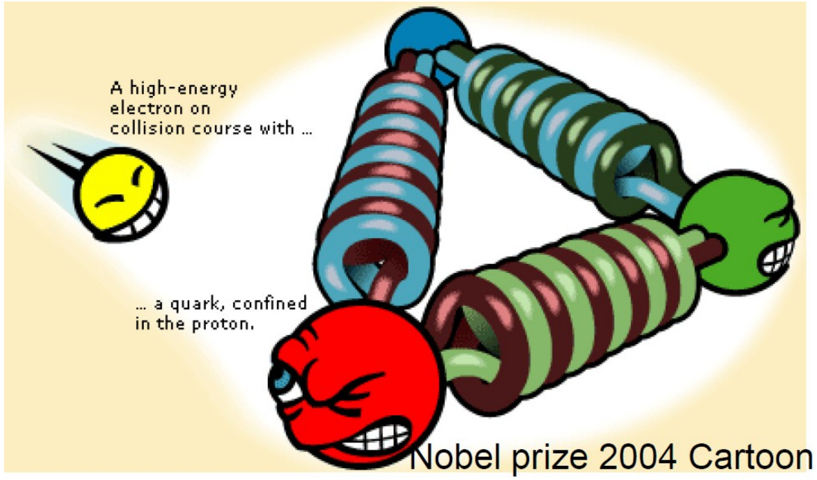
Nobel prize 2004

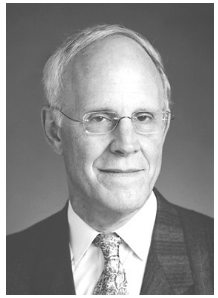
- **Theory:**
  - **Renormalisation group equation (RGE)**
  - **Solution of 1-loop equation**
  - **Running coupling constant**

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\beta_0 \ln\left(\frac{Q^2}{\mu^2}\right)}$$

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)}$$

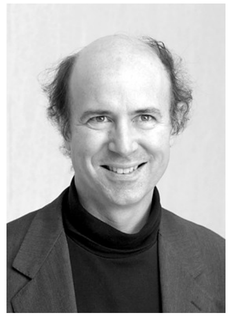
- **What happens at large distances?**
  - **$Q^2 \rightarrow 0$?**
  - **Cannot be answered here!**
    For $Q^2 \rightarrow \Lambda^2$ perturbation theory not applicable anymore!





D. Gross

D. Politzer



F. Wilczek

- **'Strong' coupling weak for $Q^2 \rightarrow \infty$, i.e. small distances**
- **Asymptotic freedom**
- **Perturbative methods usable**

$$\beta_0 = \frac{33 - 2 \cdot N_f}{12\pi}$$

Physik Journal 3 (2004) Nr. 12

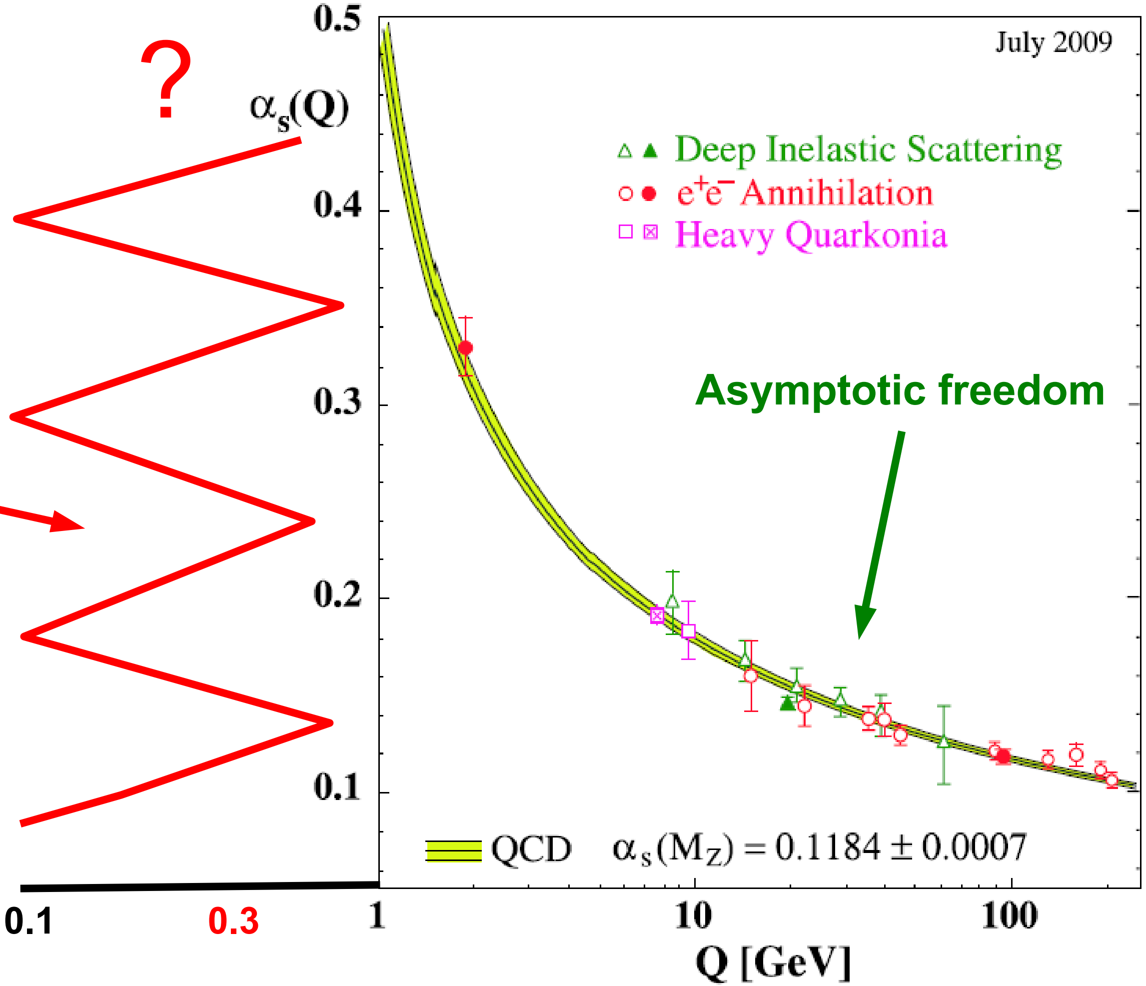
nobelprize.org

Running coupling constant

2009

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)}$$

with Λ typically ≈ 200 – 300 MeV



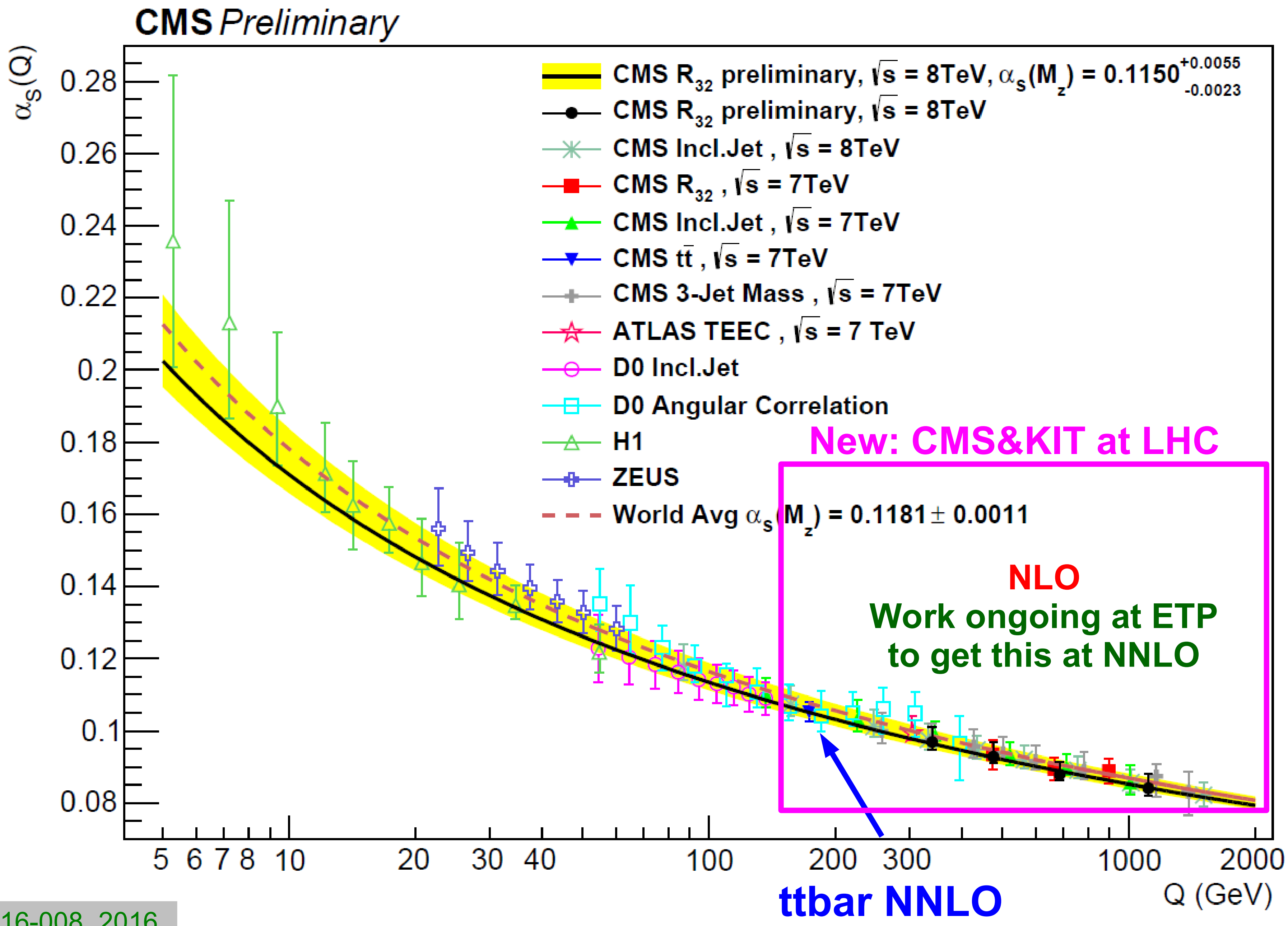
S. Bethke, EPJC 64 (2009).

QCD potential grows linearly with larger distances:

$$V = \sigma \cdot r \approx 1\text{GeV}/\text{fm} \cdot r$$

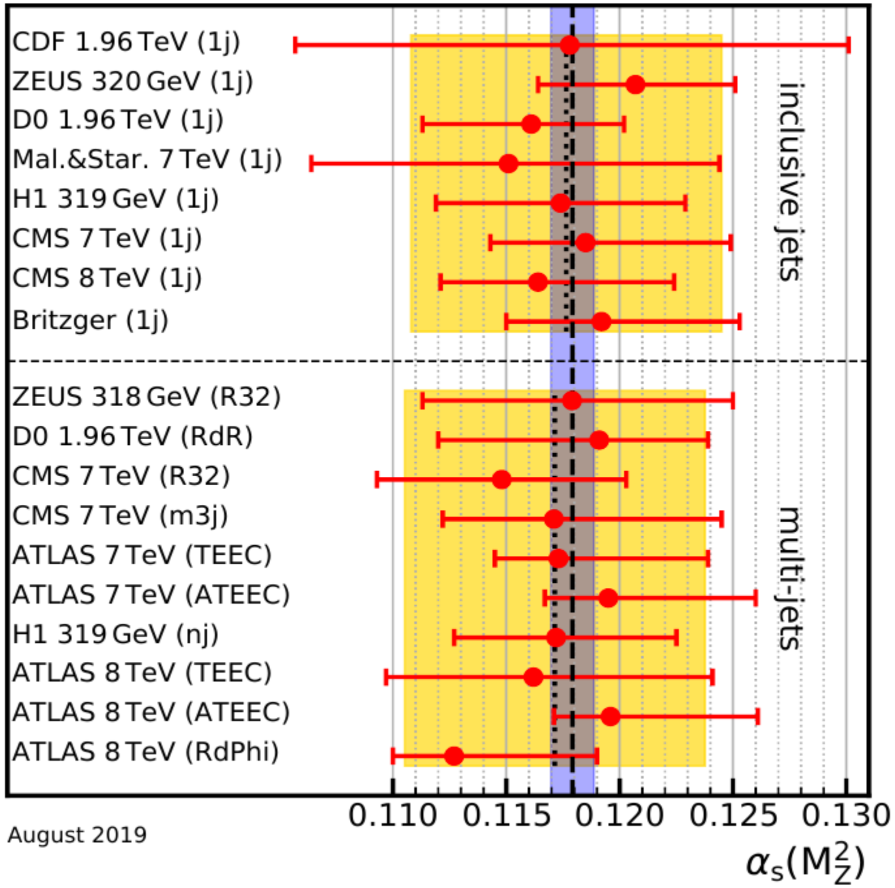
→ No free quarks (or gluons)
→ Confinement

Running coupling from CMS



CMS, SMP-16-008, 2016.

Running coupling from PDG

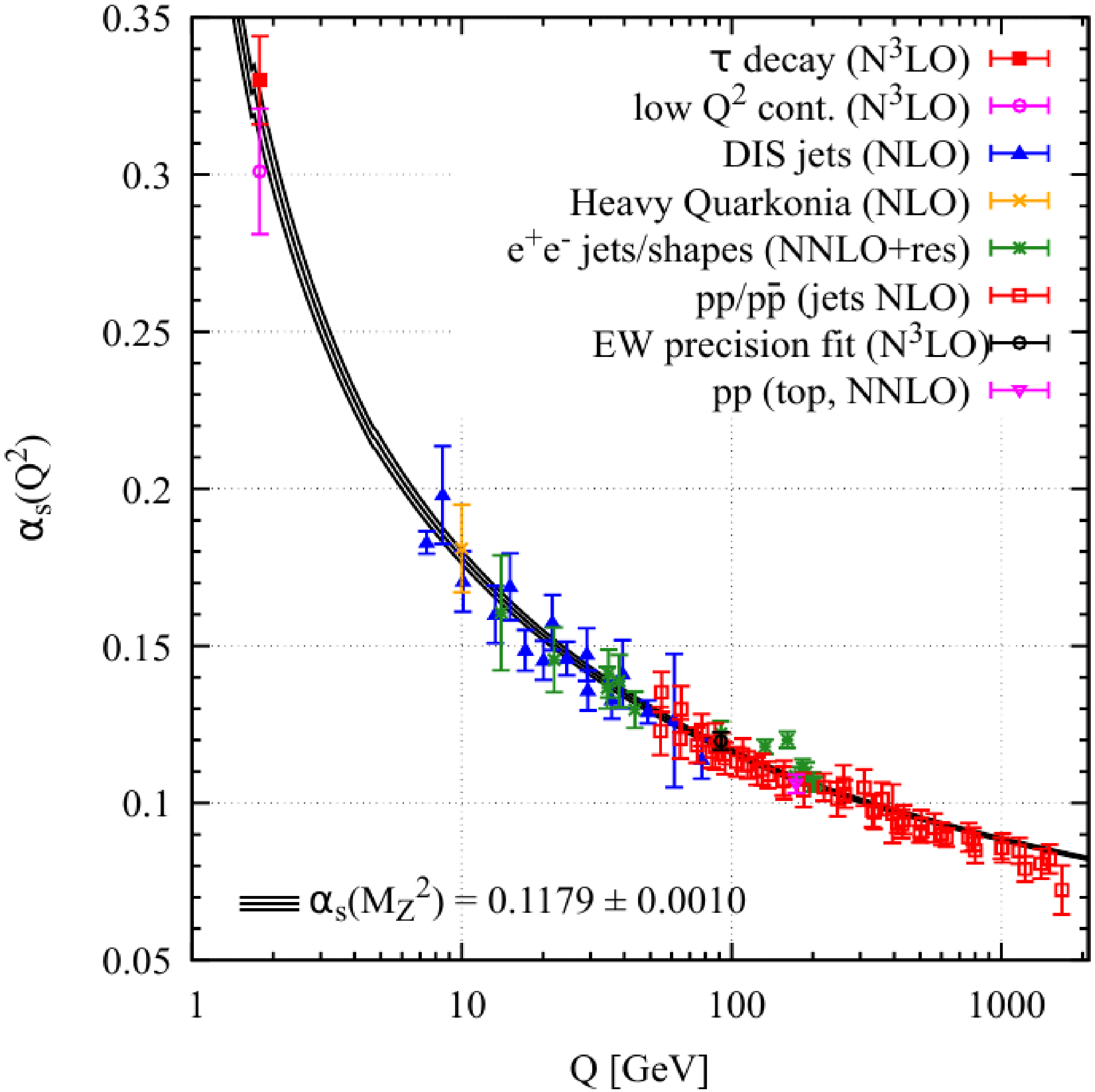


Not yet in world average – need NNLO

$$\alpha_s(M_Z) = 0.1179 \pm 0.0010$$

Most precise "single" result from lattice gauge theory.

2019



Particle Data Group, Review of Particle Physics 2020, QCD ch., Huston, Rabbertz, Zanderighi

Summary

- Strongly interacting particles (“hadrons”) are composite objects.
- The pattern of hadrons is best described by introducing a new three-valued quantum number: “color”
- The constituents carrying color charges are named “quarks”.
- Originally, two types of quarks, “up” and “down” with electrical charges +2/3 and -1/3 (never observed in nature freely …)
- Complemented with further quark types: strange, charm, bottom, top
- Hadrons come in two types:
  - Mesons are made of one quark and one anti-quark
  - (Anti-)Baryons are made of three (anti-)quarks
- Strong interactions are derived from local gauge invariance of color SU(3)
- Eight massless, self-interacting gluons are the carriers of the strong force
- In contrast to QED, quantum corrections lead to color forces decreasing with energy (asymptotic freedom) and increasing with distance (confinement)