

Vorlesung 13b

Teilchenphysik I (Particle Physics I)

Flavor Physics

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1. History
2. Basics principles
3. Detectors and Accelerators
4. Theoretical Foundations
 1. Relativistic Quantum Mechanics
 2. Quantum Field Theory and symmetries
 3. Electroweak Symmetry and Higgs Mechanism
5. QCD and Jets
6. Analysis Chain

7. Flavour Physics
 1. Quark mixing and CKM Matrix
 2. Meson-Antimeson Mixing
 3. CP Violation

8. Tests of Electroweak Theory
 1. Discoveries
 2. Precision Physics with Z bosons
 3. W Boson and electroweak fit

Introduction (Quark) Flavour Physics

- Experimentally: **three fermion families** (generations)
→ six different quark and lepton flavors
- Flavor quantum numbers (weak isospin, strangeness, charm, beauty, truth) **conserved** in electromagnetic and strong interactions

Only weak interactions **change flavor**: charged currents (CC)

- **Universal** W -boson coupling to **left-handed** fermions:

$$\mathcal{L} \sim \bar{f} \gamma^\mu \frac{1}{2} (1 - \gamma_5) f'$$

- Transitions within the same $SU(2)_L$ doublet occur in particle **decays** and **flavor oscillations**
- **Quark mixing**: mass eigenstate $\neq$ flavor eigenstate $\neq$ CP eigenstate
→ rich phenomenology: “**flavor physics**”

(Pre-)History of Flavour Physics

- 1947: strangeness (Rochester, Butler)
- 1953: flavor $SU(3)$ (Gell-Mann; Nishijima)
- 1957: parity violation (Lee, Yang; Wu; Goldhaber, Grodzins)
- 1963: quark mixing (Cabibbo)
- 1964: CP violation (Christenson, Cronin, Fitch, Turlay)
- 1964: quark model (Gell-Mann; Zweig)
- 1970: GIM mechanism (Glashow, Iliopoulos, Maiani)
- 1973: three families and CKM matrix (Kobayashi, Maskawa)

Classification of Weak Decays

■ Leptonic decays:

- No transitions between lepton families
- Example muon decay: clean environment (=small theoretical uncertainties) to measure Fermi coupling constant G_F

$$\Gamma(\mu \rightarrow e \bar{\nu}_e \nu_\mu) = \frac{1}{\tau_\mu} = \frac{G_F^2}{192\pi^3} m_\mu^5 (1 + \Delta)$$

radiative
corrections

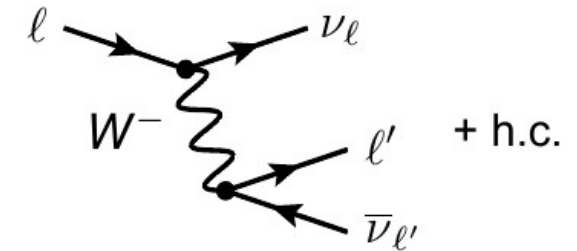
■ Semileptonic decays:

- Final state contains leptons and hadrons
- Example: neutron beta decay

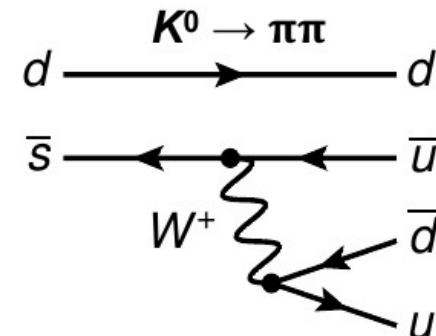
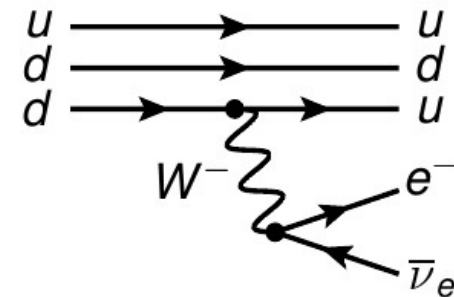
■ Hadronic decays:

- Final state contains only hadrons
- Example: $K^0 \rightarrow \pi\pi$

Leptonic Decay



Neutron Decay



Cabbibo Theory

Experimentally (1960s):

- **Flavor changing** transition of leptons and quarks **similar** but not exactly the same: coupling strength in neutron decays **approx. 97%** of coupling strength in muon decays
- **Strange** particle decays **suppressed**, only about 23% of coupling strength in muon decays

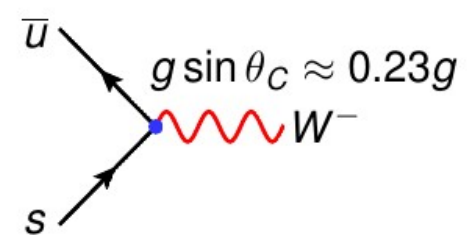
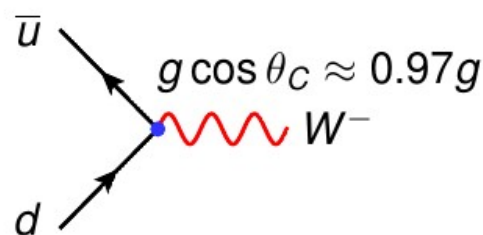
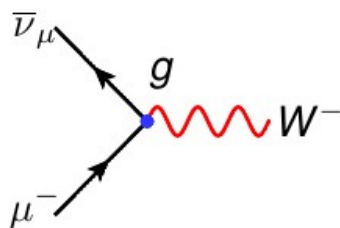
Explanation: **quark mixing** (N. Cabibbo, 1963)
→ eigenstate of weak interaction d' : mixture of mass eigenstates d and s

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d \cos \theta_C + s \sin \theta_C \end{pmatrix} \quad \text{with } \theta_C \approx 13^\circ \text{ Cabibbo angle}$$

N. Cabibbo



cerncourier.com



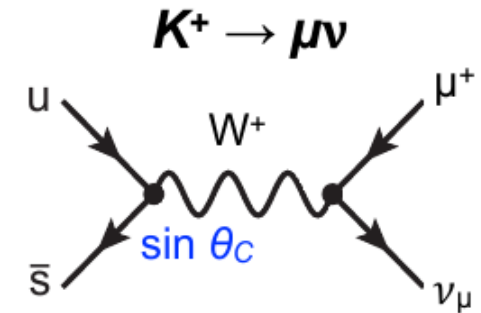
Flavour-Changing Neutral Currents (FCNC)

FCNC were experimentally found to be strongly suppressed:

Compare branching fractions of

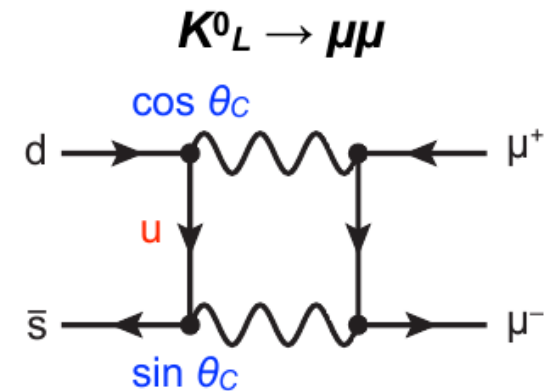
$K^+ \rightarrow \mu^+ \nu_\mu$ and $K^0_L \rightarrow \mu\mu$:

- $K^+ \rightarrow \mu^+ \nu_\mu$: **charged** current with $\Delta S = 1$
- $K^0_L \rightarrow \mu\mu$: **neutral** current with $\Delta S = 1$
→ **flavor-changing** neutral current (FCNC)
- Experimentally: FCNC strongly suppressed,
 $B(K^0 \rightarrow \mu\mu)/B(K^+ \rightarrow \ell\nu) \approx 10^{-9} \rightarrow$ why?



in the quark model:

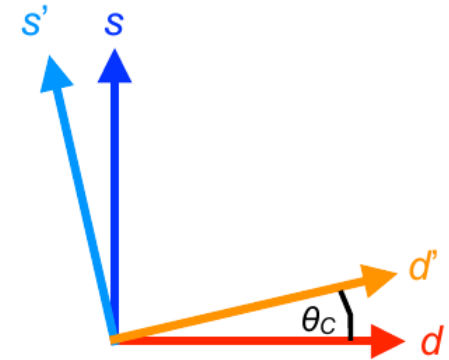
- $K^+ \rightarrow \mu^+ \nu_\mu$: **s-channel exchange** of W boson
- $K^0_L \rightarrow \mu\mu$: **box diagram** with two W bosons
- Model with three quarks (u, d, s): box diagram contains virtual **u quark** → **no explanation** of strong FCNC suppression



Explanation: GIM Mechanism

The **GIM** – Mechanism (S. Glashow, J. Iliopoulos, L. Maiani, PRD 2 (1970) 1285)

- Quarks span “**flavour space**”,
 - states transformed by **flavour rotation**;
 - only relative rotation angle relevant:
 - convention: **rotation of down-type quarks**
- W bosons do not couple to eigenstates of strong interactions u, d, s (the mass eigenstates), but to left-handed component of “rotated” eigenstates u', d', s' (without defined mass)



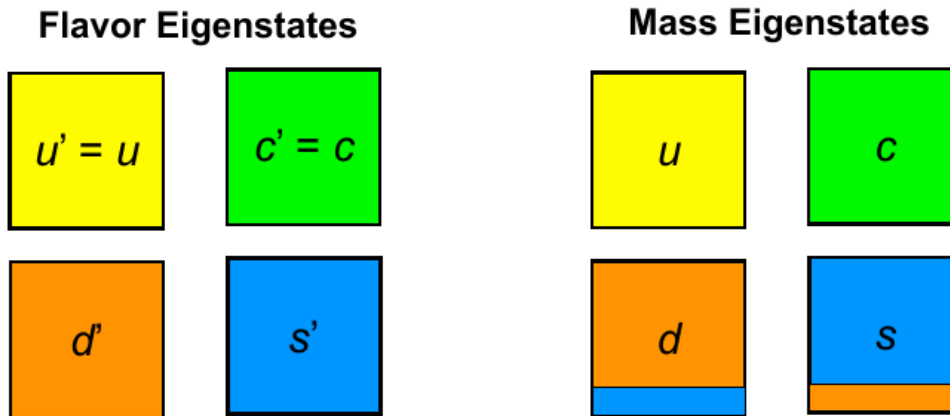
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

2x2 mixing matrix with one real parameter θ_C

??? The d quark has an isospin partner – what about the s quark ?

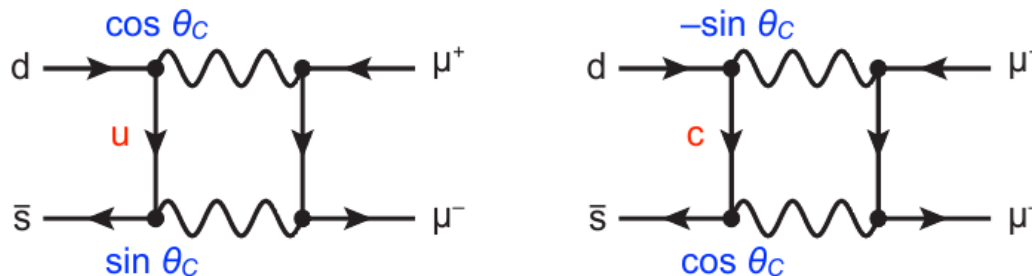
→ **postulate a partner also for the s quark, the charm (c) quark !**

GIM Mechanism: there must be another quark !



The $K^0 \rightarrow \mu^+\mu^-$ process in the GIM-picture:

Box diagrams with virtual u and c quarks $\rightarrow$ **destructive interference**



Negative interference between u and c quark !

If they had the same mass, the process would be forbidden

$\rightarrow$ **prediction of charm quark mass**

1974: J/ψ discovery $\rightarrow$ interpretation as **charmonium** ($c\bar{c}$) state

Extension to more quark flavours: CKM matrix

An **extension** of the GIM model **to three down-type quark flavours** was theoretically proposed in 1973 by **M. Kobayashi and T. Maskawa**

The proposed 5th quark,
the **b, beauty or bottom quark**, was **discovered** in **1977**

→ **Cabbibo-Kobayashi-Maskawa (CKM) Matrix**

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

→ Charged-Current reaction written as

$$J_{\text{CC}}^{\mu,+} = (\bar{u}, \bar{c}, \bar{t}) \left(\gamma^\mu \frac{1}{2} (1 - \gamma_5) V_{\text{CKM}} \right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- Notes:**
- 3x3 mixing matrix can have one complex phase in addition to three rotation angles → explanation for CP violation ([see later](#))
 - Origin of CKM matrix lies in the Higgs sector, which explains masses of fundamental particles by Yukawa couplings of fermions to the Higgs boson; Yukawa couplings are not flavour-diagonal !

Nobel Prize 2008

one half awarded to



M. Kobayashi



T. Maskawa

“for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature”

Properties of the CKM Matrix

- **Unitary** complex 3×3 matrix: $V_{\text{CKM}}^\dagger V_{\text{CKM}} = V_{\text{CKM}} V_{\text{CKM}}^\dagger = \mathbb{1}_3$
 - Complex 3×3 matrix: 18 parameters (9 magnitudes, 9 phases)
 - Number of **physical** degrees of freedom (d.o.f): 4 (3 magnitudes, 1 phase)
- Quantum-mechanical **phase**: 6 phases can be absorbed in definitions of quark fields, one total phase cannot be observed → number of d.o.f. **reduced by 5**
- CKM **unitarity**: number of d.o.f. **further reduced by 9**

- Three real equations: 3 d.o.f.

$$\sum_{i=1}^3 V_{ij} V_{ij}^* = 1 \text{ for } j = 1 \dots 3$$

- Three complex equations: 6 d.o.f.

$$\sum_{i=1}^3 V_{ij} V_{ik}^* = 0 \text{ for } j, k = 1 \dots 3, k > j$$

$$\begin{array}{|c|c|c|} \hline \blacksquare & \blacksquare & \blacksquare \\ \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} \times \begin{array}{|c|c|c|} \hline \blacksquare & \square & \square \\ \hline \blacksquare & \square & \square \\ \hline \blacksquare & \square & \square \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 1 & \square & \square \\ \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array}$$

$$V_{\text{CKM}}^\dagger \times V_{\text{CKM}} = \mathbf{1}_3$$

$$\begin{array}{|c|c|c|} \hline \blacksquare & \blacksquare & \blacksquare \\ \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} \times \begin{array}{|c|c|c|} \hline \square & \blacksquare & \square \\ \hline \square & \blacksquare & \square \\ \hline \square & \blacksquare & \square \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \square & 0 & \square \\ \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array}$$

Parametrizations of CKM Matrix

Standard parameterization: three Euler angles θ_{ij} , one phase δ
(Kobayashi-Maskawa phase $\rightarrow$ CP violation)

$$V_{\text{CKM}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{s-b \text{ mixing}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{d-b \text{ mixing} + \text{phase}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{d-s \text{ mixing}}$$

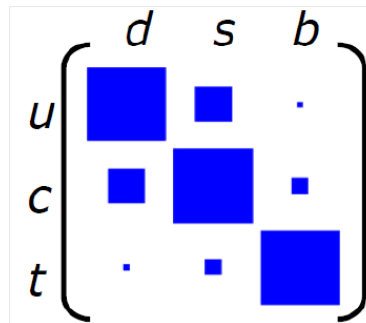
with $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

CKM Matrix: numerical Values

Experimentally: **magnitudes** of CKM matrix elements (PDG 2016)

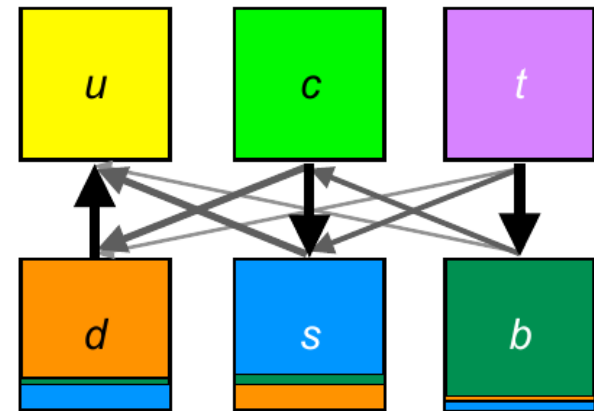
$$|V_{\text{CKM}}| = \begin{pmatrix} 0.97434^{+0.00011}_{-0.00012} & 0.22506 \pm 0.00050 & 0.00357 \pm 0.00015 \\ 0.22492 \pm 0.00050 & 0.97351 \pm 0.00013 & 0.0411 \pm 0.0013 \\ 0.00875^{+0.00032}_{-0.00033} & 0.0403 \pm 0.0013 & 0.99915 \pm 0.00005 \end{pmatrix}$$



graphical representation, size of boxes
representing size of elements

There is a clear hierarchy:

- dominating diagonal elements → transitions within family most likely
- transitions between 1st and 3rd family strongly suppressed



Note: smallness of the off-diagonal elements is the reason for the long life times of mesons with heavy quarks !

Convenient: Wolfenstein Parametrisation

Starting from the standard parametrisation

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

with the choice of parameters

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}$$

$$s_{23} = \lambda \left| \frac{V_{cb}}{V_{us}} \right| = A\lambda^2$$

$$s_{13}e^{i\delta} = V_{ub}^* = A\lambda^3(\rho + i\eta)$$

and expanding up to $\lambda^3 \rightarrow$

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

The hierarchy of CKM Matrix elements, e.g. $|V_{ud}| > |V_{us}| > |V_{cb}| > |V_{ub}|$ is clearly expressed in this parametrisation.

Unitarity Triangles

Unitarity of the matrix **dictates relations**

or $\sum_{i=1}^3 V_{ij} V_{ik}^* = 0$ for $j, k = 1 \dots 3, k > j$ matrix elements:

Geometrically, the sum of three complex numbers = 0 represents a **triangle** in the complex plane

C. Jarlskog showed that the area of all such “unitarity triangles” is the same and proportional to the level of CP violation

Note: Need >4 measurements to over-constrain the 4 free elements of the CKM matrix.

First and second* column:

$$V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0 \\ \cong \mathcal{O}(\lambda) + \mathcal{O}(\lambda) + \mathcal{O}(\lambda^5)$$

$$V_{ts}^* V_{td} \quad \text{[Diagram: A very thin yellow triangle with a long horizontal base and a small vertical height, representing a small area.]}$$

First and third* column:

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \\ \cong \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3)$$

$$\begin{array}{c} V_{ub}^* V_{ud} \quad V_{tb}^* V_{td} \\ \text{[Diagram: A yellow triangle with a horizontal base and two slanted sides, representing a larger area than the first triangle.]} \\ V_{cb}^* V_{cd} \end{array}$$

Second and third* column

$$V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} = 0 \\ \cong \mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^2)$$

$$V_{ub}^* V_{us} \quad \text{[Diagram: A very thin yellow triangle with a long horizontal base and a small vertical height, representing a small area.]}$$

(not to scale)

"The" Unitarity Triangle

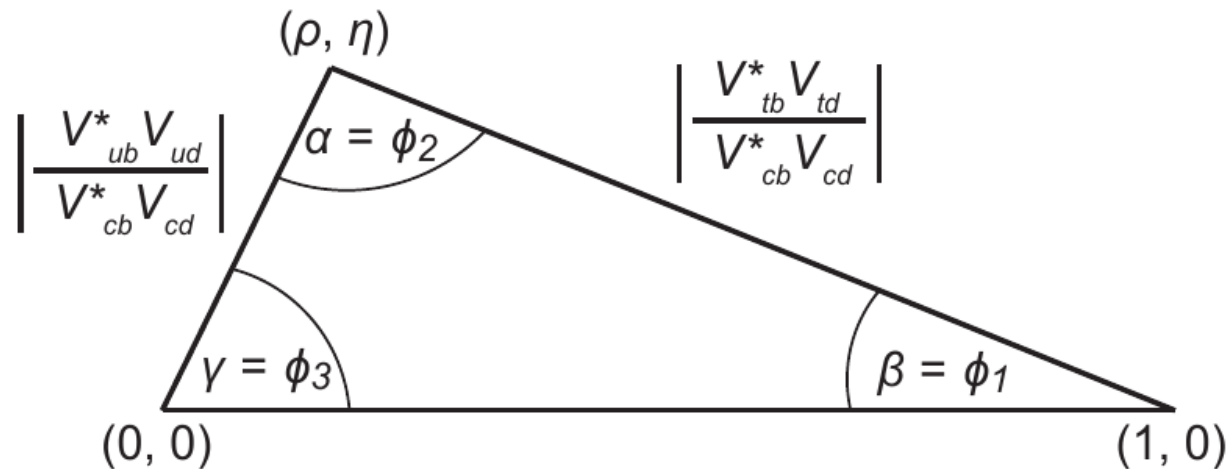
Best choice: unitarity triangle from **first column and third column***

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

- Side length of **same order**, in Wolfenstein parameterization:

$$|V_{ub}^* V_{ud}| = \mathcal{O}(\lambda^3), \quad |V_{cb}^* V_{cd}| = \mathcal{O}(\lambda^3), \quad |V_{tb}^* V_{td}| = \mathcal{O}(\lambda^3)$$

- Base length **normalized to 1**: "the" unitarity triangle



$$\frac{1}{A\lambda^3} (A\lambda^3(\rho + i\eta) - A\lambda^3 + A\lambda^3(1 - \rho - i\eta) + \mathcal{O}(\lambda^5)) = 0$$

Determination of CKM Matrix Elements

The experimental side: which processes are sensitive to V_{ij} ?

- nuclear β decays
- semileptonic decays of mesons containing flavor i or j in initial or final state
- meson - antimeson oscillations
- single top production and $t\bar{t}$ decays

$$V = \begin{pmatrix} \begin{array}{c} \text{u} \\ \text{c} \\ \text{t} \end{array} & \begin{array}{c} \text{d} \\ \text{s} \\ \text{b} \end{array} & \begin{array}{c} \text{e}^- \bar{\nu} \\ \ell^- \bar{\nu} \\ \ell^- \bar{\nu} \end{array} & \begin{array}{c} n \rightarrow p \\ K \rightarrow \pi \\ B \rightarrow \pi \end{array} \\ & \begin{array}{c} \ell^- \bar{\nu} \\ \ell^- \bar{\nu} \\ \ell^- \bar{\nu} \end{array} & \begin{array}{c} D \rightarrow \pi \\ D \rightarrow K \\ B \rightarrow D \end{array} \\ & \begin{array}{c} B^0 \rightarrow \bar{B}^0 \\ B_s \rightarrow \bar{B}_s \\ t \rightarrow b W \end{array} \end{pmatrix}$$

P. Goldenzweig

V_{ij} and V_{ij}^* in Lagrange Density

Example:

weak current for
 $u \rightarrow d$ transition

$$j_{ud} = \bar{d} \underbrace{\left[-i \frac{g_W}{\sqrt{2}} V_{ud}^* \gamma^u \frac{1}{2} (1 - \gamma^5) \right]}_{\text{vertex factor}} u$$

weak current for
 $d \rightarrow u$ transition

$$j_{du} = \bar{u} \left[-i \frac{g_W}{\sqrt{2}} \gamma^u V_{ud} \frac{1}{2} (1 - \gamma^5) \right] d$$

CKM element enters either as V_{ud}^* or V_{ud} , depending on the order of the interaction, $u \rightarrow d$ or $d \rightarrow u$

(other flavour analogous !)

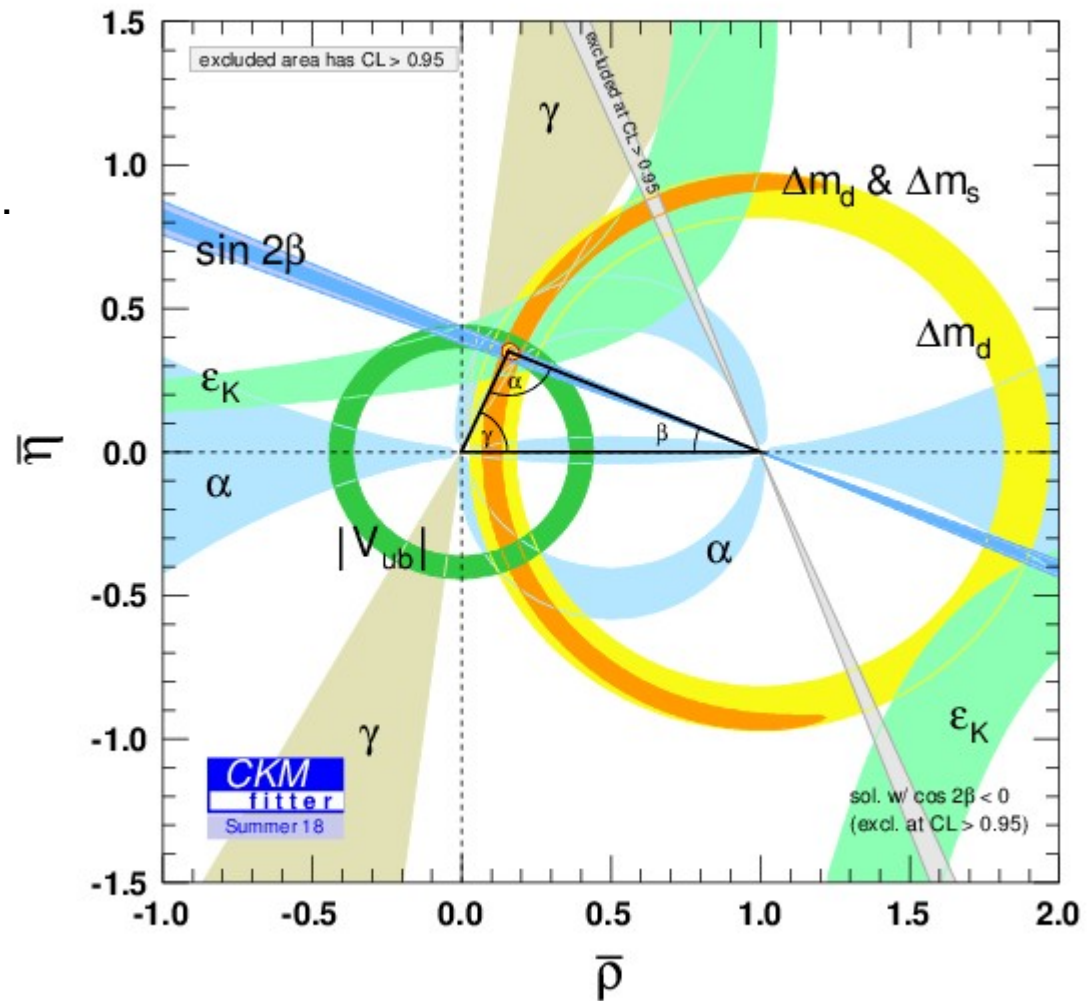
Constraining the CKM Triangle

All available measurements are taken into account in an overall fit (e.g. **CKM fitter** group)

The system is over-constrained.

Most important inputs:

- Magnitudes of CKM matrix elements
- Rare B decays
- Flavor oscillations
- CP violation in neutral kaon and B -meson decays (see later)
- Heavy quark masses



http://ckmfitter.in2p3.fr/www/results/plots_summer18/

for more detail see

- latest PDG review <http://pdg.lbl.gov/2019/reviews/rpp2018-rev-ckm-matrix.pdf>
- Lecture Teilchenphysik II: Flavor Physics

