Karlsruher Institut für Technologie

Vorlesung:

Teilchenphysik I (Particle Physics I)

V14a: Meson-Mixing

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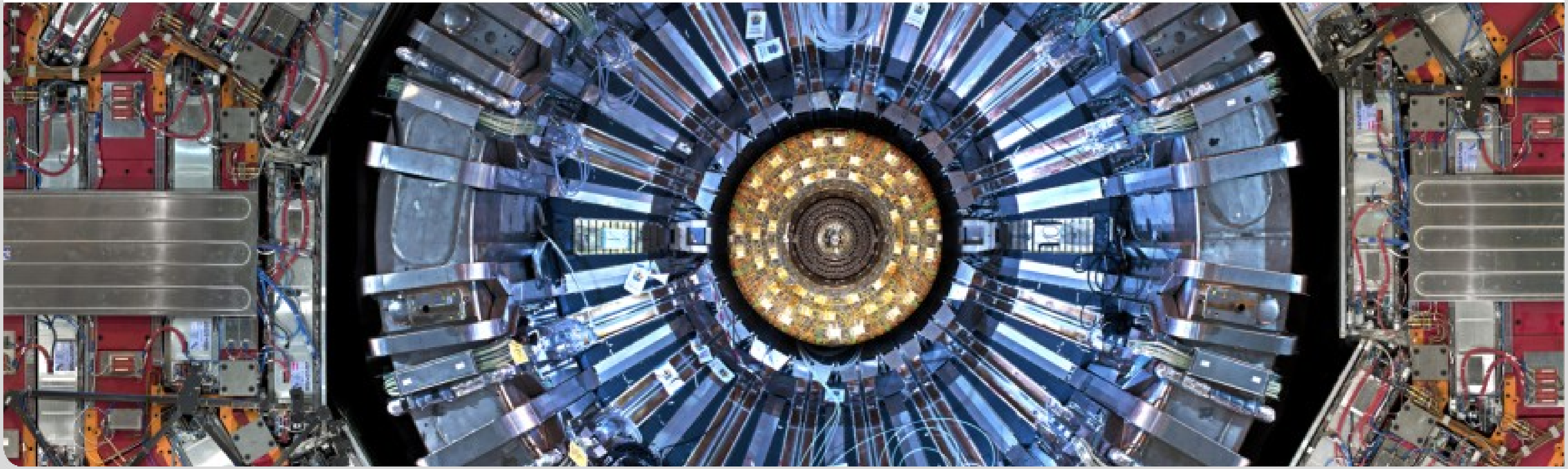

Summary Lecture 13b

Concept of quark mixing

- **Cabbibo:** chared-current couplings for quarks smaller than for leptons
  → u quark couples to linear combination of d and s quarks
- **GIM mechanism:** flavor-chaning neutral currents (K →) suppressed
  → 2x2 mixing matrix, prediction of a new quark, the charm quark
- **KM:** CP violation requires >3x3 mixing matrix
  → CKM matrix, 3rd family postulated

CKM matrix

- Unitary complex 3x3 matrix
- physical degrees of freedom:
  - three magnitudes
  - and one complex phase
- Unitarity relations represended as unitarity triangles
- experimentally: strong hierarchy of elements – diagonal dominant
  → long life times of mesons with heavy quarks

$|V_{CKM}| =$

0.97434	0.22506	0.00357
0.22492	0.97351	0.0411
0.00875	0.0403	0.99915

1. History
2. Basics principles
3. Detectors and Accelerators
4. Theoretical Foundations
    1. Relativistic Quantum Mechanics
    2. Quantum Field Theory and symmetries
    3. Elctroweak Symmetry and Higgs Mechanism
5. QCD and Jets
6. Analysis Chain

7. Flavour Physics
    1. Quark mixing and CKM Matrix
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8. Tests of Electroweak Theory
    1. Discoveries
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Neutal mesons can – and do - mix

Quantum numbers of hadrons:

- Hadrons produced in strong interactions → **strong** eigenstates
- Strong eigenstates: not necessarily **physical** particles (= mass eigenstates) or eigenstates of **weak** interactions → **flavor mixing**
- Flavor-changing weak processes in systems of neutral mesons: particle-antiparticle **oscillations**

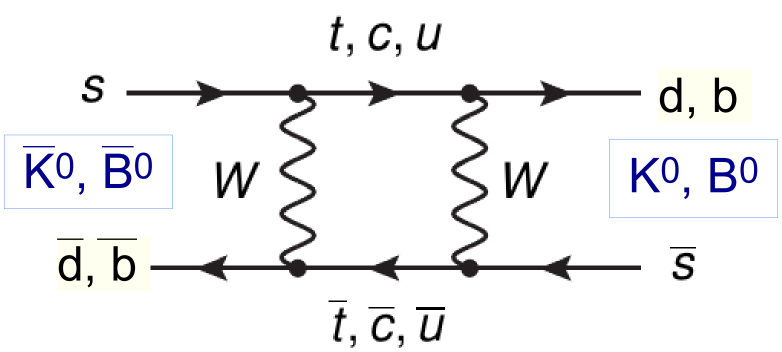
$$|P\rangle \quad \leftrightarrow \quad |\overline{P}\rangle$$

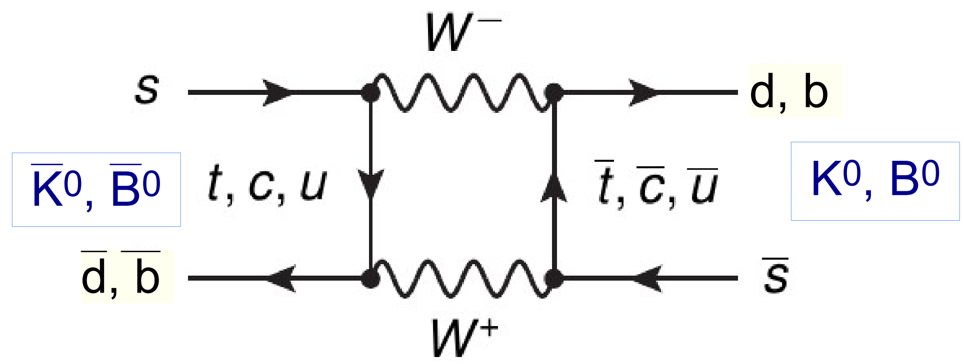
Particle – antiparticle sytems with established mixing:

Neutral kaons:	$\lvert K^0\rangle = \lvert\bar{s}d\rangle$	$\leftrightarrow$	$\lvert\overline{K}^0\rangle = \lvert s\bar{d}\rangle$
Neutral *D* mesons:	$\lvert D^0\rangle = \lvert c\bar{u}\rangle$	$\leftrightarrow$	$\lvert\overline{D}^0\rangle = \lvert\bar{c}u\rangle$
Neutral *B* mesons:	$\lvert B_d^0\rangle = \lvert\bar{b}d\rangle$	$\leftrightarrow$	$\lvert\overline{B}_d^0\rangle = \lvert b\bar{d}\rangle$
	$\lvert B_s^0\rangle = \lvert\bar{b}s\rangle$	$\leftrightarrow$	$\lvert\overline{B}_s^0\rangle = \lvert b\bar{s}\rangle$

Meson Mixing and Box-Diagrams

In the Standard Model, mixing is caused by “box diagrams”





consider **time evolution** of a quantum-mechanical state:

- Starting point: **pure** state $|P\rangle$ or $|\bar{P}\rangle$ created in **strong** interaction
- After time interval Δt: **mixture** of $|P\rangle$ and $|\bar{P}\rangle$ with particle decays superimposed (different lifetimes for different particles)
- Phenomenological description of time evolution: **Schrödinger equation** with “effective Hamilton operator” Σ

Mixing: Time evolution

Formalism:

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}|P(t)\rangle \\ |\overline{P}(t)\rangle\end{pmatrix} = \Sigma\begin{pmatrix}|P(t)\rangle \\ |\overline{P}(t)\rangle\end{pmatrix} = \left(M - i\frac{\Gamma}{2}\right)\begin{pmatrix}|P(t)\rangle \\ |\overline{P}(t)\rangle\end{pmatrix} \text{ with } M^\dagger = M,\ \Gamma^\dagger = \Gamma$$

Mass matrix

Decay width matrix

Components of the effective Hamilton operator:

$$\Sigma = M - i\frac{\Gamma}{2} = \begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix}$$

- M_{11}, M_{22}: quark masses and binding energies given by **strong** interactions $\rightarrow$ **no** oscillations
- Γ_{11}, Γ_{22}, M_{12}, Γ_{12}: **oscillation** and **decay** through **weak** processes
- CPT symmetry: particle and antiparticles have the **same masses and decay widths** $\rightarrow M_{11} = M_{22} = m$, $\Gamma_{11} = \Gamma_{22} = \Gamma$

Mixing: Time evolution (2)

Diagonalize effective Hamilton operators Σ
$\rightarrow$ masses and decay widths of **physical** particles

- Ansatz: consider two linear combinations of $|P\rangle$ and $|\bar{P}\rangle$

$$|P_L\rangle = p\,|P\rangle + q\,|\overline{P}\rangle\,, \quad |P_H\rangle = p\,|P\rangle - q\,|\overline{P}\rangle$$

  with $|P_L\rangle$ **"light"** and $|P_H\rangle$ **"heavy"** mass eigenstate
  and p, q complex coefficients with normalization condition $|p|^2 + |q|^2 = 1$

- **Time evolution** of **physical** particles $|P_L\rangle$ und $|P_H\rangle$:

$$|P_{L,H}(t)\rangle = \exp\left[-iM_{L,H}t - \frac{\Gamma_{L,H}}{2}t\right]|P_{L,H}\rangle$$

- Time evolution of **strong** eigenstates $|P\rangle$ and $|\bar{P}\rangle$:
  transformation using matrix of eigenvectors (p, q) and (p, $-q$)

$$\begin{pmatrix}|P(t)\rangle\\ |\overline{P}(t)\rangle\end{pmatrix} = \begin{pmatrix}p & p\\ q & -q\end{pmatrix}\begin{pmatrix}\exp\left[-iM_Lt - \frac{\Gamma_L}{2}t\right] & 0\\ 0 & \exp\left[-iM_Ht - \frac{\Gamma_H}{2}t\right]\end{pmatrix}\begin{pmatrix}p & p\\ q & -q\end{pmatrix}^{-1}\begin{pmatrix}|P\rangle\\ |\overline{P}\rangle\end{pmatrix}$$

Mixing: Time evolution (3)

Result of calulation:

$$\begin{pmatrix} |P(t)\rangle \\ |\overline{P}(t)\rangle \end{pmatrix} = \begin{pmatrix} g_+(t) & \frac{p}{q} g_-(t) \\ \frac{q}{p} g_-(t) & g_+(t) \end{pmatrix} \begin{pmatrix} |P\rangle \\ |\overline{P}\rangle \end{pmatrix}$$

$$\text{with } g_\pm(t) = \frac{1}{2}\left(\exp\left[-iM_L t - \frac{\Gamma_L}{2} t\right] \pm \exp\left[-iM_H t - \frac{\Gamma_H}{2} t\right]\right)$$

Interpretation as **transition probabilities** :

- $|g_+(t)|^2$: probability for $|P\rangle$ ($|\bar{P}\rangle$) to remain in the same state
- $|q/p|^2 |g_-(t)|^2$: probability for $|P\rangle$ to oscillate to $|\bar{P}\rangle$ after time interval t
- $|p/q|^2 |g_-(t)|^2$: probability for $|\bar{P}\rangle$ to oscillate to $|P\rangle$ after time interval t

Remark: **indirect CP violation** if $p \neq q$ (more later)

Mixing: Time evolution (4)

Usual convention: express masses and widths of heavy and light mass eigenstates by **average values and differences**

$$m = M_{11} = M_{22} = \frac{1}{2}(M_H + M_L) \qquad \Gamma = \Gamma_{11} = \Gamma_{22} = \frac{1}{2}(\Gamma_L + \Gamma_H)$$

$$\Delta m = M_H - M_L \qquad \Delta\Gamma = \Gamma_L - \Gamma_H$$

sometimes also: $x = \frac{\Delta m}{\Gamma}$

Transition probabilities as a function of Γ, $\Delta\Gamma$, Δm:

$$|g_\pm(t)|^2 = \frac{\exp[-\Gamma t]}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m\, t)\right]$$

Decay

Oscillation due to decay width difference

Oscillation due to mass difference

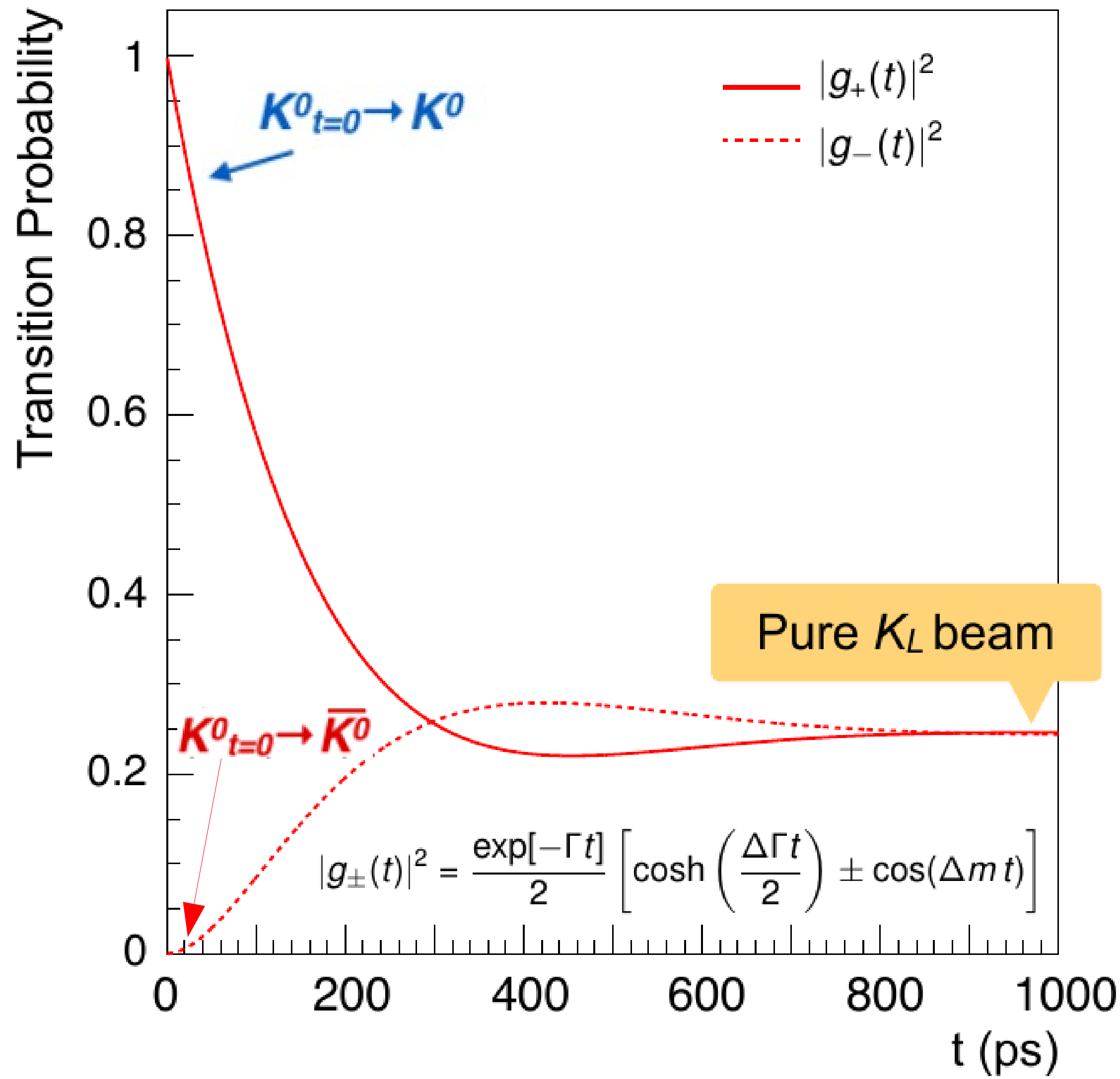
Example: Neutral Kaon Mixing

Historically, mass eigenstate were distinguished by lifetime (K-short / K-long) instead of mass (light / heavy)

$$|P_L\rangle = |K_S^0\rangle \,,\; |P_H\rangle = |K_L^0\rangle$$

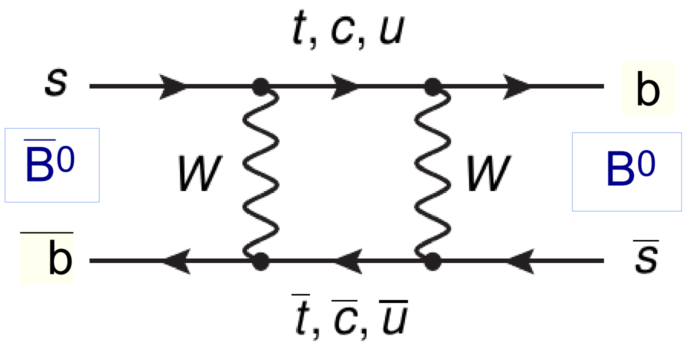
Kaon oscillation parameters:

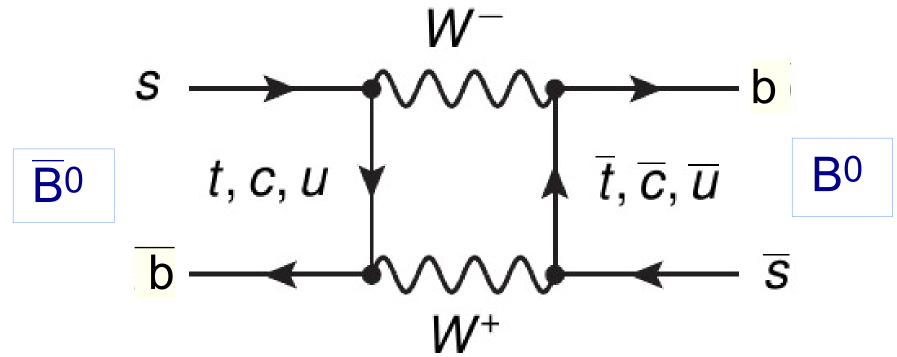
- $\Gamma = \dfrac{1}{178.8\ \text{ps}}$
- $\Delta\Gamma \approx \Gamma$
- $\Delta m = 0.0053\ \text{ps}^{-1}$
- mass and life time differences are of same order of magnitude



Example2: Neutral B-Meson Mixing

Mass difference from box diagrams:





approximation: m_t ist the only relevant quark mas, $V_{tb} \approx 1$

- Result: $\Delta m_{b,s} \sim (V^*_{td,ts} V_{tb})^2$
- **$|V_{td}|$** and **$|V_{ts}|$** can be determined from **oscillation frequency**

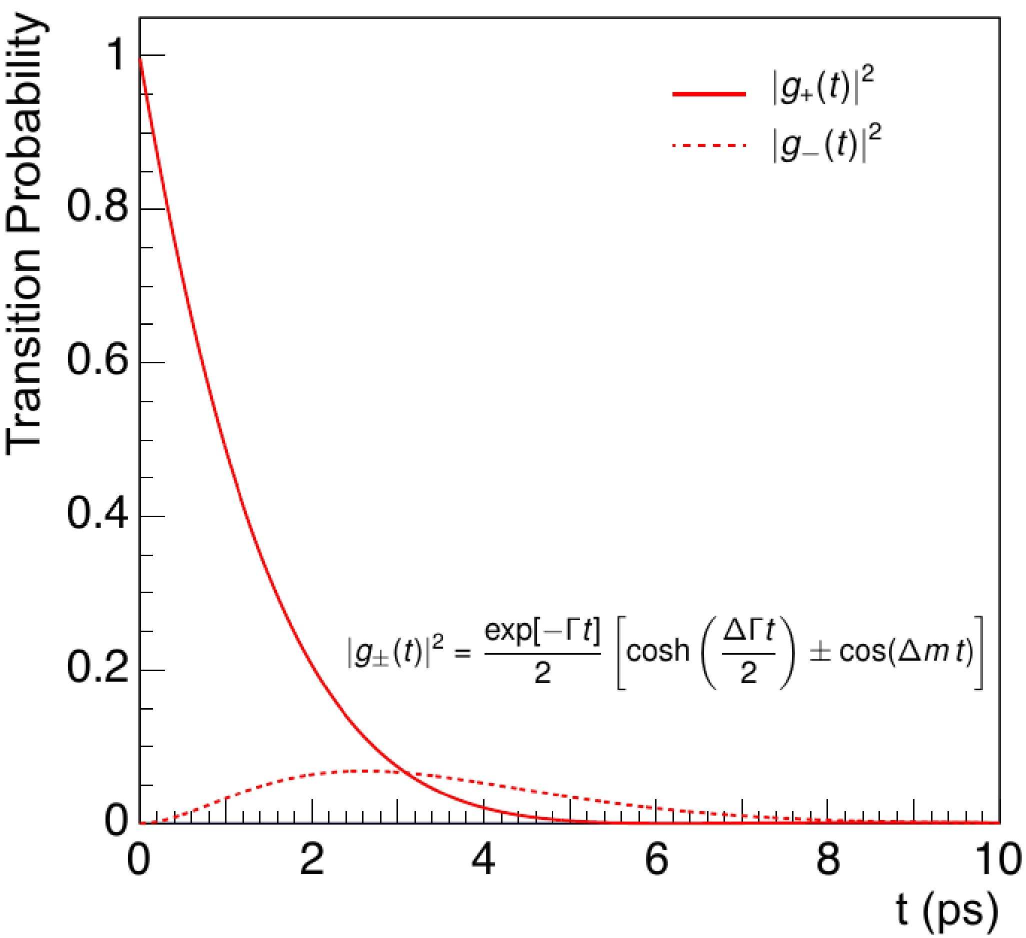
Measurements of oscillation frequency:

- B_d system: ARGUS (DESY),UA1 (CERN) 1987 ($\rightarrow$ large value of Δm_d = indirect hint: top quark very heavy), *B* factories at SLAC & KEK (more later)
- B_s system: Tevatron, LHCb

Neutral Bd – Meson Mixing

Bd oscillation parameters:

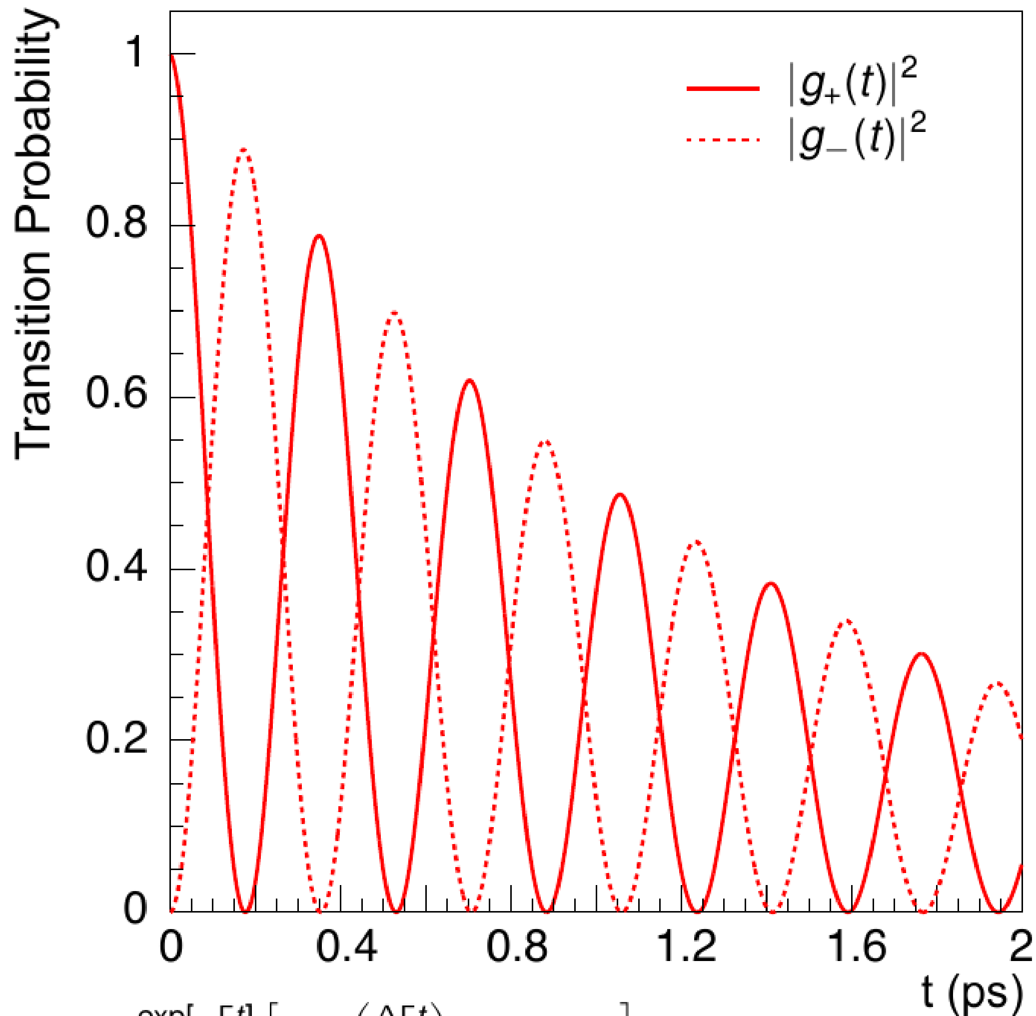
- $\Gamma_d = \frac{1}{1.53\,\text{ps}}$
- $\Delta\Gamma_d \approx 0$
- $\Delta m_d = 0.507\,\text{ps}^{-1} \approx \Gamma_d$
- life time approx.
  one oscillation period
- dominant effect:
  oscillation due to
  mass difference Δmd



Neutral Bs – Meson Mixing

Bs oscillation parameters:

- $\Gamma_s = \frac{1}{1.47\ \mathrm{ps}}$
- $\Delta\Gamma_s \approx 0$
- $\Delta m_s = 17.77\ \mathrm{ps}^{-1}$
- Fast oscillation, many periods before decay
- dominant effect:
  oscillation tue to
  mass difference Δm_s



$$|g_\pm(t)|^2 = \frac{\exp[-\Gamma t]}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t)\right]$$

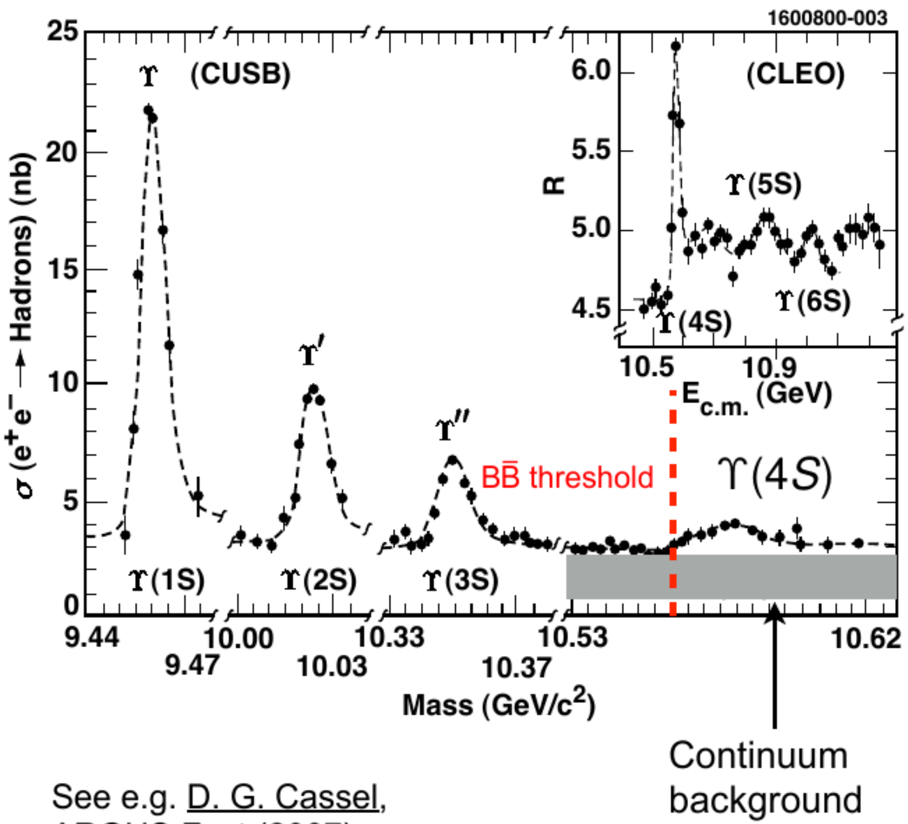
Experimental challenge:
resolution of fast oscillation !

B Factories

Large number of B-Mesons needed to perform (precision) measurements

- use **bottonium resonances** in e^+e^- colliders (DORIS, CESR, PEP-II, KEKb, SupderKEKb, experiments Argus, Cleo, BaBar, Belle (II))
- best choice **$\Upsilon(4S)$ resonance** just above the energy to produce 2 B_d mesons
- approx 50% $B^0\bar{B}^0$ and 50% B^+B^-

example ARGUS experiment at DORIS (DESY)



See e.g. D. G. Cassel, ARGUS Fest (2007)

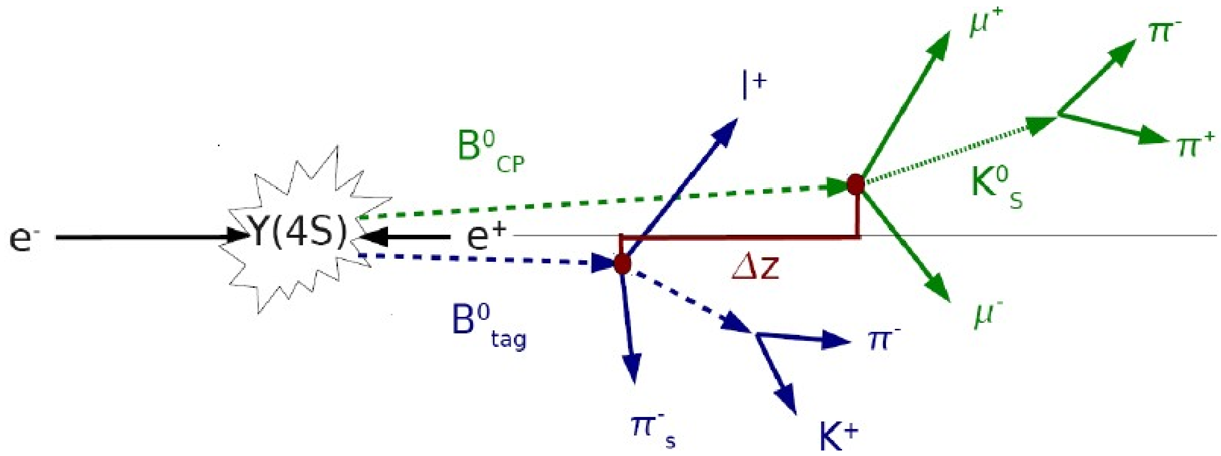
B-Factories

Electron-positron collider with **asymmetric** beam energies:

- $\Upsilon(4S)$: B$\bar{\text{B}}$-pair produced approximately **at rest** in e^+e^- center-of-mass frame
- B$\bar{\text{B}}$ rest frame **moving** relative to laboratory frame→ all decay lengths Lorentz boosted → better measurement

B$\bar{\text{B}}$ pair = **entangled** quantum state (cf. EPR paradoxon)

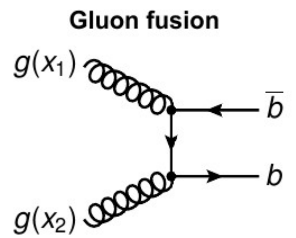
- Oscillations in $B^0\bar{B}^0$ system
  → first decay of a B^0 or $\bar{B}^0$ determines flavor of other $\bar{B}^0$ or B^0
- Observable: **decay length difference** Δz = O(200 μm)

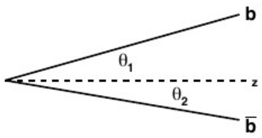


The LHC as a B Factory

Heavy quarks are **copiously produced in hadron colliders**

- Dominant process: **gluon fusion**
- Process kinematics: $m_b \ll \sqrt{s}$
  $\rightarrow$ momentum fractions of gluons $x_1 \gg x_2$
  $\rightarrow$ b and $\bar{b}$: both emitted either into forward or backward direction





- $B^{\pm}$ mesons: 40%
- B^0_d mesons: 40%
- B^0_s mesons: 10%
- B baryons: 10%

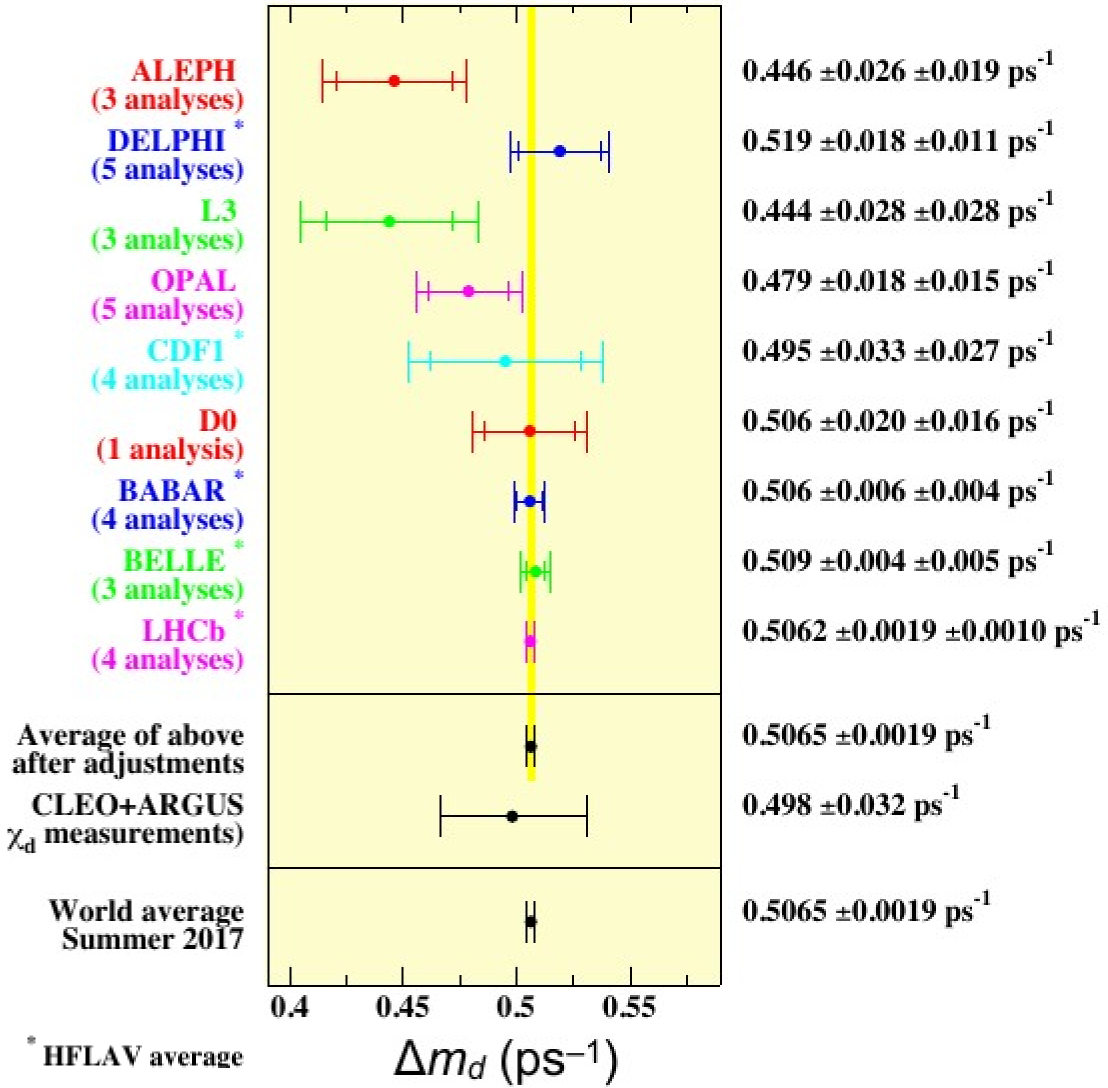
Note: Contrary to e^+e^- b factories, QCD effects lead to rapid distruction of quantum-mecanical entanglement of the two b quarks.

Appropriate detector setup: forward spectrometer (LHCb)

Measurements of B_d Mixing

- ARGUS/CLEO:
  - Symmetric beam energies → **time-integrated** mixing probability measurement
  - Idea: mixing parameters from **number of lepton pairs with same charge sign** in semileptonic B and $\bar{B}$ decays (observable: asymmetry χ_d)

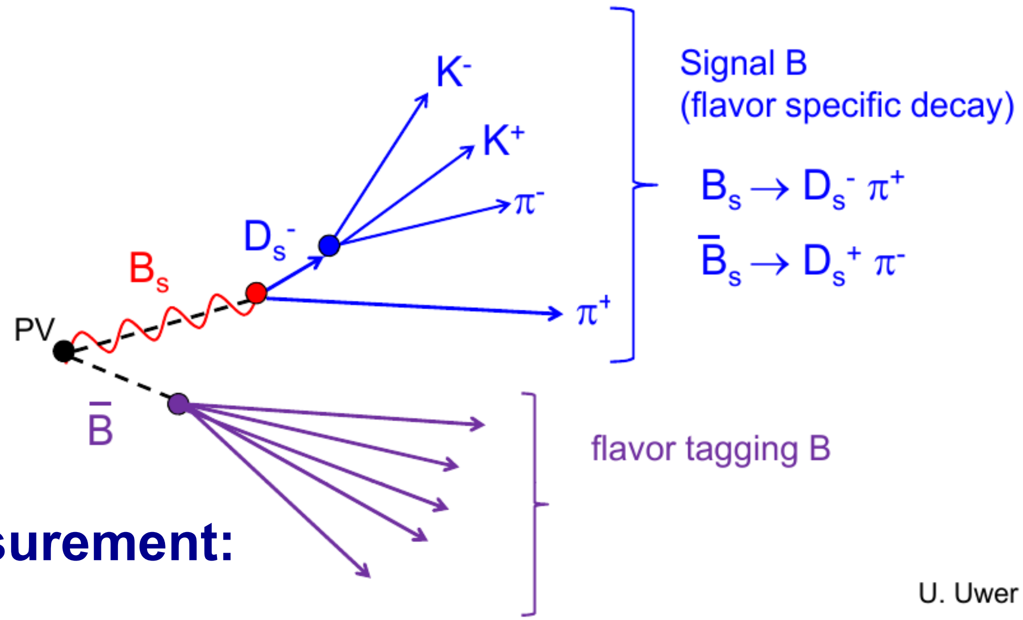
- LEP/B factories/Tevatron/LHCb:
  - **Time-resolved** measurement of oscillation frequency Δm_d
  - Rough idea: count number of B and $\bar{B}$ decays as a function of Δz (various methods)



Y. Amhis et al., "Averages of b-hadron, c-hadron, and tau-lepton properties as of summer 2016," arXiv:1612.07233 and online update at http://www.slac.stanford.edu/xorg/hflav

Measurements of B_s Mixing

- First measurement of B_s oscillation frequency by the CDF experiment at Tevatron in 2006
- most precise measurements by the LCHb experiment at the LHC



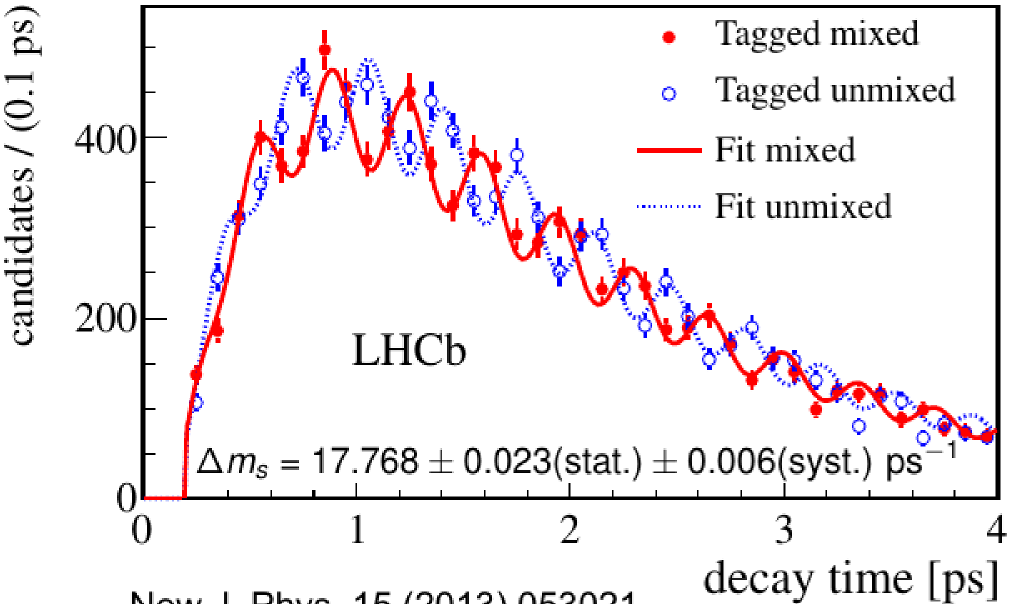
Principle of measurement:

- B_s **flavor** at **decay time**: charge of **decay products**
- B_s **flavor** at **production time**: flavor of **other B hadron** ("opposite sign tag") or search for partner of $\bar{s}$ quark in B_s ("same sign tag")
- B_s **lifetime**: **decay length** $L \to$ **decay time** $t = Lm/p$

LHCb: B_s Mixing results

- Theoretical transition probability:

$$|g_{\pm}(t)|^2 = \frac{\exp[-\Gamma_s t]}{2}\left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right) \pm \cos(\Delta m_s t)\right]$$



New J. Phys. 15 (2013) 053021

- Probability density function to describe observed oscillation signal:

$$P(t|\sigma_t) \sim \Gamma_s \frac{\exp[-\Gamma_s t]}{2}\left[\cosh\left(\frac{\Delta\Gamma_s t}{2}\right)\Theta(t) \pm D\cos(\Delta m_s t)\right] \otimes R(t, \sigma_t)$$

Dilution factor (due to mistags)

Time resolution and acceptance