Karlsruher Institut für Technologie

Vorlesung 16:

Teilchenphysik I (Particle Physics I)

Precision Physics at the Z Pole

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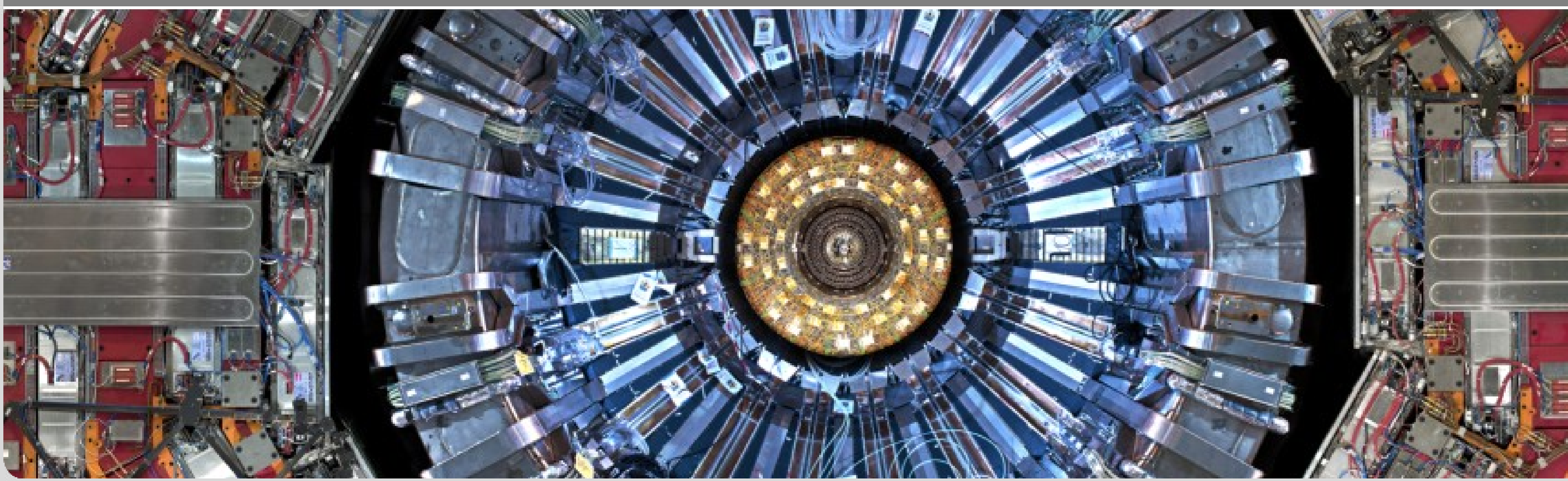
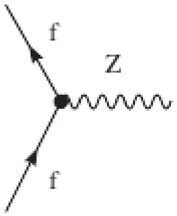


Short summary

- The observation of neutral current reactions in neutrino scattering gave evidence for the existence of a neutral, weak gauge boson, as expected in the $SU(2)_L$ x U(1) model
- The W and Z bosons were discovered and their properties studied in proton-antiproton collisions at the $Sp\bar{p}S$ collider by the experiments UA1 and UA2
- Detailed studies of (anti-)neutrino electron scattering during the 80ies gave first constraints on the values of the vector and axial vector couplings of the Z boson to the electron

Tree-level couplings of the Z boson to fermions in the SM:



$g_a^{\mathrm{f}} = I_3^{\mathrm{f}}$ axial vector coupling (factor of $\gamma_5\gamma_\mu$ term in $\mathcal{L}$)

$g_v^{\mathrm{f}} = I_3^{\mathrm{f}} - 2\, Q^{\mathrm{f}} \sin^2\Theta_{\mathrm{W}}$ vector coupling (factor of γ_μ term in $\mathcal{L}$)

- The Z boson factories SLC as Slac and LEP at CERN were designed to produce large numbers of Z bosons. Precision detectors were able to precisely measure all visible decay channels of Z bosons

Reminder

the differential cross section for fermion production
with photon and Z exchange at lowest Order:

$$\frac{2s}{\pi}\frac{1}{N_c^{\mathrm{f}}}\frac{d\sigma_{\mathrm{ew}}}{d\cos\theta}(\mathrm{e}^+\mathrm{e}^-\to \mathrm{f\bar{f}}) \;=$$

$$\underbrace{|\alpha|^2\,(1+\cos^2\theta)}_{\gamma}$$

$$\underbrace{-8\alpha\Re\{\chi(s)\}\left[g_{\mathrm{Ve}}g_{\mathrm{Vf}}(1+\cos^2\theta)+2g_{\mathrm{Ae}}g_{\mathrm{Af}}\cos\theta\right]}_{\gamma-\mathrm{Z\ interference}}$$

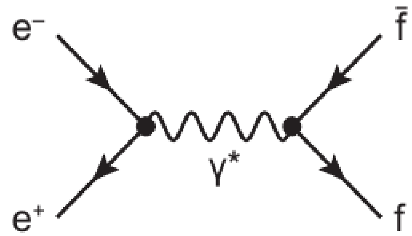
$$\underbrace{\begin{array}{c}+16|\chi(s)|^2\left[\,(g_{\mathrm{Ve}}^2+g_{\mathrm{Ae}}^2)(g_{\mathrm{Vf}}^2+g_{\mathrm{Af}}^2)(1+\cos^2\theta)\right.\\ \left.+8g_{\mathrm{Ve}}g_{\mathrm{Ae}}g_{\mathrm{Vf}}g_{\mathrm{Af}}\;\cos\theta\;\right]\end{array}}_{\mathrm{Z}}$$

with $\quad \chi(s)=\frac{m_{\mathrm{Z}}^2}{8\pi\sqrt{2}}\frac{s}{s-m_{\mathrm{Z}}^2+i\Gamma_{\mathrm{Z}}m_{\mathrm{Z}}}$

Lecture 13

The cross section in detail: Photon Exchange

$e^+e^- \rightarrow f\bar{f}$ for $\sqrt{s} \ll m_Z$: pure **QED process**



Total cross section: decreasing with 1/(center-of-mass energy)^2

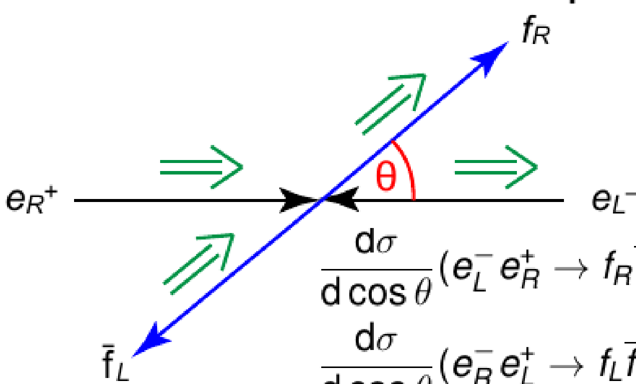
$$\sigma_\gamma = N_{C,f}\, Q_f^2\, \frac{4\pi\alpha^2}{3s}$$

- Assumption: all fermion masses can be neglected
- $N_{C,f}$: number of **color degrees of freedom** (3 for quarks, 1 for leptons)
- Q_f: fermion charge in units of elementary charge e
- α: **fine structure constant**

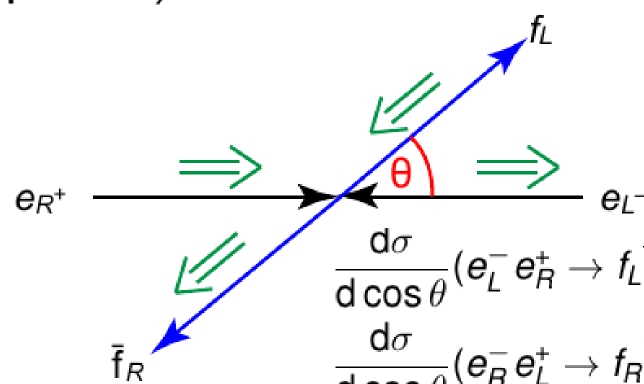
$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

Differential cross section as a function of scattering angle θ:

- Angular dependence from **particle spins**: spin-1 photon exchanged between fermion/antifermion pairs (spin 1/2)

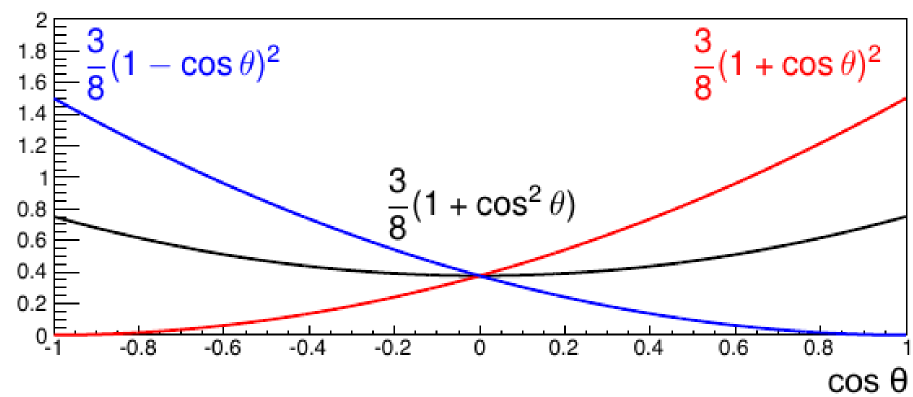


$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to f_R \bar{f}_L),$$
$$\frac{d\sigma}{d\cos\theta}(e_R^- e_L^+ \to f_L \bar{f}_R):$$
$$\sim (1-\cos\theta)^2$$



$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \to f_L \bar{f}_R),$$
$$\frac{d\sigma}{d\cos\theta}(e_R^- e_L^+ \to f_R \bar{f}_L):$$
$$\sim (1+\cos\theta)^2$$

Can be derived using Wigner's d-matrices $d^1{}_{1,-1}$ and $d^1{}_{1,1}$



- Superposition of all combinations:

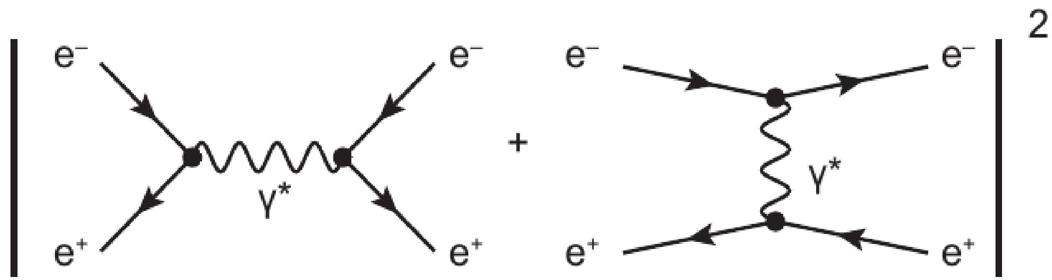
$$\frac{d\sigma_\gamma}{d\cos\theta} = N_{C,f} Q_f^2 \frac{\pi\alpha^2}{2s}(1+\cos^2\theta)$$

The cross section in detail: Photon Exchange (3)

Special case: $e^+e^- \to e^+e^-$

- **Identical** particles in initial and final state → t-channel process in addition
- Matrix element:

$|M|^2 =$



- *t*-channel process: **Bhabha scattering** → differential cross section

$$\frac{d\sigma}{d\cos\theta} \sim \frac{1}{\sin^4(\theta/2)}$$

→ dominates for **small scattering angles** (cf. Rutherford scattering)

Application at LEP: measurement of **luminosity** (precision: 10^{-3})

- Theory: **pure QED process** → can be calculated very precisely
- Experiment: number of Bhabha events = **3× to 4×** number of Z events

Particle chiralities in Z-boson exchange

consider left and right-handed couplings instead of vector and axial vector couplings:

$g_L = (g_V + g_a)/2 \qquad g_R = (g_V - g_a)/2$

- Left-right: $\dfrac{d\sigma(e^-_L e^+_R \to f_R \bar{f}_L)}{d\cos\theta} \sim (g_L{}^\ell)^2 (g_R{}^f)^2 \, (1 - \cos\theta)^2$
- Right-left: $\dfrac{d\sigma(e^-_R e^+_L \to f_L \bar{f}_R)}{d\cos\theta} \sim (g_R{}^\ell)^2 (g_L{}^f)^2 \, (1 - \cos\theta)^2$
- Left-left: $\dfrac{d\sigma(e^-_L e^+_R \to f_L \bar{f}_R)}{d\cos\theta} \sim (g_L{}^\ell)^2 (g_L{}^f)^2 \, (1 + \cos\theta)^2$
- Right-right: $\dfrac{d\sigma(e^-_R e^+_L \to f_R \bar{f}_L)}{d\cos\theta} \sim (g_R{}^\ell)^2 (g_R{}^f)^2 \, (1 + \cos\theta)^2$

→ Differential cross section for pure Z boson exchange:

$$\frac{d\sigma_f}{d\cos\Theta} \propto \sigma_f^0 \left[(1 + \cos^2\Theta) + 2\mathcal{A}_e \mathcal{A}_f \cos\Theta \right]$$

with $\mathcal{A}_f = \dfrac{g_{Lf}^2 - g_{Rf}^2}{g_{Lf}^2 + g_{Rf}^2} = \dfrac{2 g_{Vf} g_{Af}}{g_{Vf}^2 + g_{Af}^2} = 2 \dfrac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2}$

Asymmetry Parameter

The cross section in detail: γ + Z
Forward-Backward asymmetry

Z boson and γ exchange and **γ/Z inteference** lead to a forward-backward asymmetry,

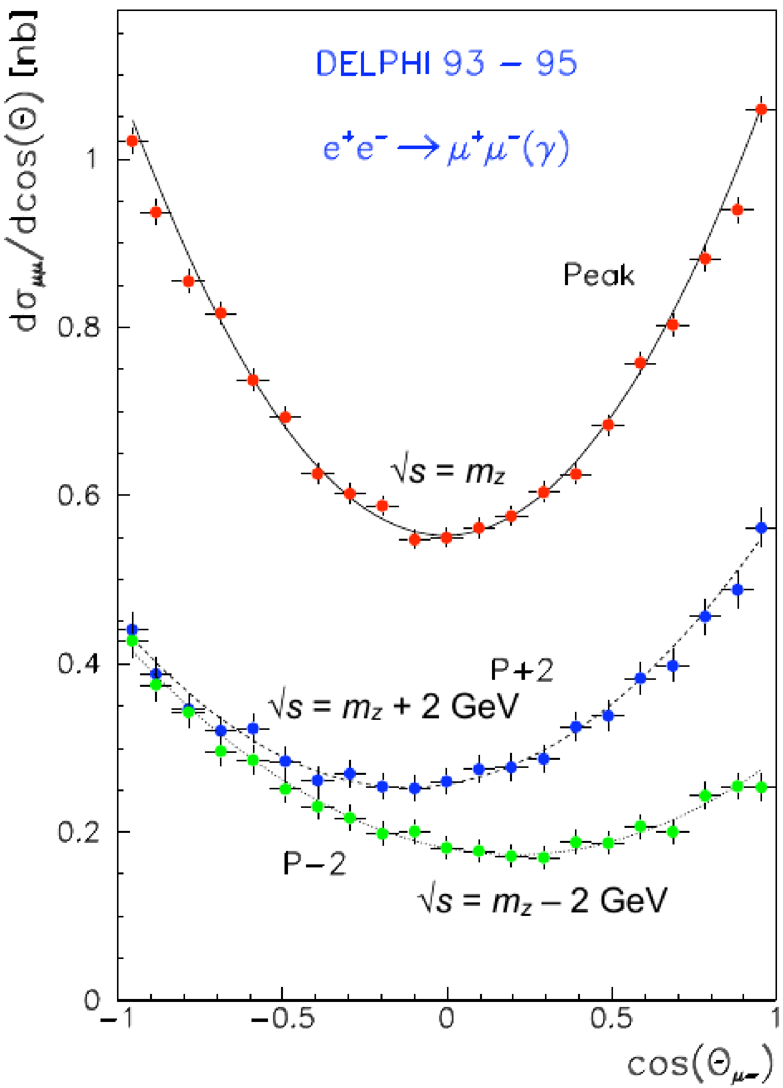
which depends on:
- charges and weak isospin, i.e. the couplings g_V and g_A
- centre-of-mass energy

- interference contribution vanishes at the Z pole

→ **Peak Asymmetry A_{FB}^0**

$$A^0_{\mathrm{FB,f}} = \frac{3}{4}\frac{2g_{\mathrm{Ve}}g_{\mathrm{Ae}}}{g_{\mathrm{Ve}}{}^2+g_{\mathrm{Ve}}{}^2}\frac{2g_{\mathrm{Ve}}g_{\mathrm{Af}}}{g_{\mathrm{Vf}}{}^2+g_{\mathrm{Vf}}{}^2} =: \frac{3}{4}\mathcal{A}_{\mathrm{e}}\mathcal{A}_{\mathrm{f}}$$

LEP: $\sqrt{s} = m_Z \pm 2$ GeV



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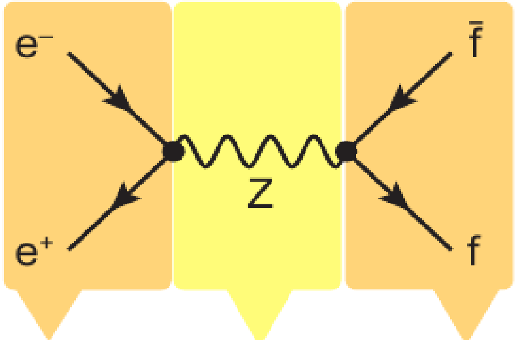
The cross section in detail: Z pole

For $\sqrt{s} \approx m_Z$: Z boson exchange dominates



$$\mathcal{M} \sim \frac{g^2}{(2\cos\theta_W)^2} J^e_{\mathrm{NC},\mu} \frac{i g^{\mu\nu}}{s - m_Z^2 + is\frac{\Gamma_Z}{m_Z}} J^f_{\mathrm{NC},\nu}$$

Propagator: Z boson **unstable** → **resonance** in scattering amplitude

- Wave function for stable particle at rest: $\psi \sim \exp[-imt]$
- Unstable particle: $\psi^*\psi \sim \exp\left[-\frac{t}{\tau}\right] = \exp[-\Gamma t] \quad \rightarrow \quad \psi \sim \exp[-imt]\exp\left[-\frac{\Gamma t}{2}\right]$
- **Decay width** = inverse of lifetime: $\Gamma = 1/\tau$
- **Breit-Wigner** prescription: in the propagator, **replace *m* by *m* – iΓ/2**
  (quantum field theory: scattering amplitude contains **pole** in complex plane)

The cross section in detail: Decay Width

Total width Γ_Z of the Z resonance:

- Sum of **partial (decay) width**
- Consider all possible Z-boson decays in the standard model: 5 quarks (top quark too heavy), 3 charged leptons, 3 neutrinos

$$\Gamma_Z = \sum_f \Gamma_f = \sum_{q=u,d,s,c,b} \Gamma_q + \sum_{\ell=e,\mu,\tau} \Gamma_\ell + \sum_{\nu=\nu_e,\nu_\mu,\nu_\tau} \Gamma_\nu$$

Partial widths Γ_f (standard model in leading order):

$$\Gamma_f = \Gamma(Z \to f\bar{f}) = N_{C,f} \frac{G_F m_Z^3}{6\sqrt{2}\pi}\left[(g_V^f)^2 + (g_A^f)^2\right] \text{ with } g_V^f = I_{3,f} - 2Q_f \sin^2\theta_W,\ g_A^f = I_{3,f}$$

$\rightarrow$ Measure **quadratic sum** of vector couplings and axial vector couplings

Lepton universality:

- Same decay width for all charged leptons: $\Gamma_e = \Gamma_\mu = \Gamma_\tau \equiv \Gamma_\ell$
- Same decay width for all neutrinos: $\Gamma_{\nu_e} = \Gamma_{\nu_\mu} = \Gamma_{\nu_\tau} \equiv \Gamma_\nu$

!!! Γ_τ recieves a mass correction !

The cross section in detail: Z Resonance

Parametrerization of the cross section around the Z-pole

(neglecting γ contributions)

$$\sigma_f = \underbrace{\frac{12\pi}{m_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}}_{\sigma_f^0} \cdot \underbrace{\frac{s\Gamma_Z^2}{(s-m_Z^2)^2 + s^2\frac{\Gamma_Z^2}{m_Z^2}}}_{\text{Breit-Wigner}}$$

Note: s-dependent width is a special choice at LEP !

Resonance peak **height**: $\sigma_f^0 \sim \Gamma_e\Gamma_f$

- Measure of **product of partial decay widths**, e.g. $\sigma_{had}^0 \sim \Gamma_e\Gamma_{had}$
- Determine **single** partial widths: combine certain **ratios** of cross sections

Resonance shape depends on number of neutrino generations:

- Partial decay with to neutrinos is invisible in the detector, but contribues to Γ_Z
- with three neutrino generations: $\Gamma_{inv} = \Gamma_{\nu_e} + \Gamma_{\nu_\mu} + \Gamma_{\nu_\tau} = 3\Gamma_\nu$
- assume there are more (or fewer) neutrino generations:

  $\rightarrow$ height and width of the reconance would change

Measurement of the Z Resonance Shape

Final measurements of the Z resonance shape at LEP

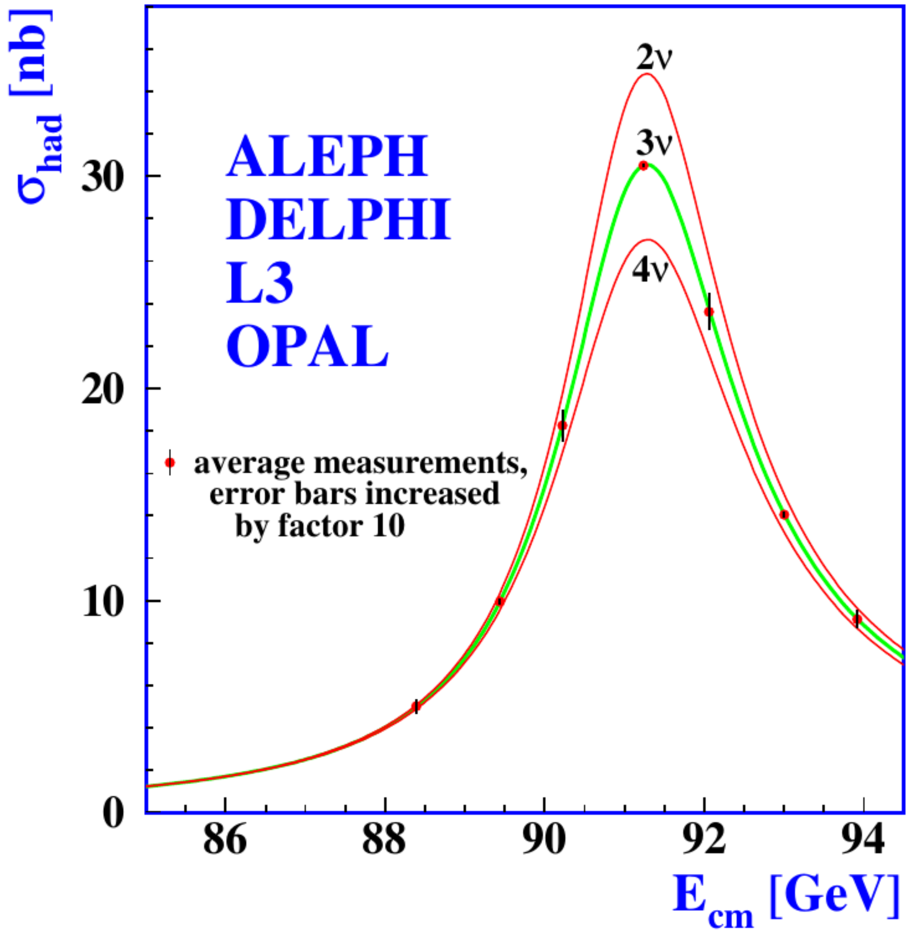
curves assuming 2, 3 or 4 neutrino flavours are also shown

clearly confirms that there are three light neutrino generations

Measurement of the number of light neutrino generations was the first exciting results from LEP and SLC.

very robust observable:

$$R^0_{\rm inv} \equiv \frac{\Gamma_{\rm inv}}{\Gamma_{\ell\ell}} = N_\nu \left(\frac{\Gamma_{\nu\overline{\nu}}}{\Gamma_{\ell\ell}}\right)_{\rm SM}$$



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best value today:

$$N_\nu = 2.9840 \pm 0.0082$$

Z Boson decay channels

Particle	Branching Fraction (PDG 2017)	Detection at Colliders
Left-handed neutrinos	20.00(06)% in total	No direct detection
Left-handed and right-handed charged leptons	3.3658(23)% each	e, μ “simple” τ: depends on decay
Left-handed and right-handed up-type quarks (u,c) in three colors	11.6(6)% each	Jets = collimated bundles of hadrons
Left-handed and right-handed down-type quarks (d,s,b) in three colors	15.6(4)% each	Jets = collimated bundles of hadrons

1. History
2. Basics principles
3. Detectors and Accelerators
4. Theoretical Foundations
   1. Relativistic Quantum Mechanics
   2. Quantum Field Theory and symmetries
   3. Elctroweak Symmetry and Higgs Mechanism
5. QCD and Jets
6. Analysis Chain
7. Flavour Physics
   1. Quark mixing and CKM Matrix
   2. Meson-Antimeson Mixing
   3. CP Violation

8. Tests of Electroweak Theory
   1. Discoveries
   2. Precision Physics with Z bosons
      1. Higher order corrections

Radiative corrections are large around the Z pole and must be considered to correctly interpret the measurements

Loop contributions are sizeable compared to the ultimate precision obtained at Z factories

Photon Radiation

Photon radiation from initial or final states modifies differential cross section

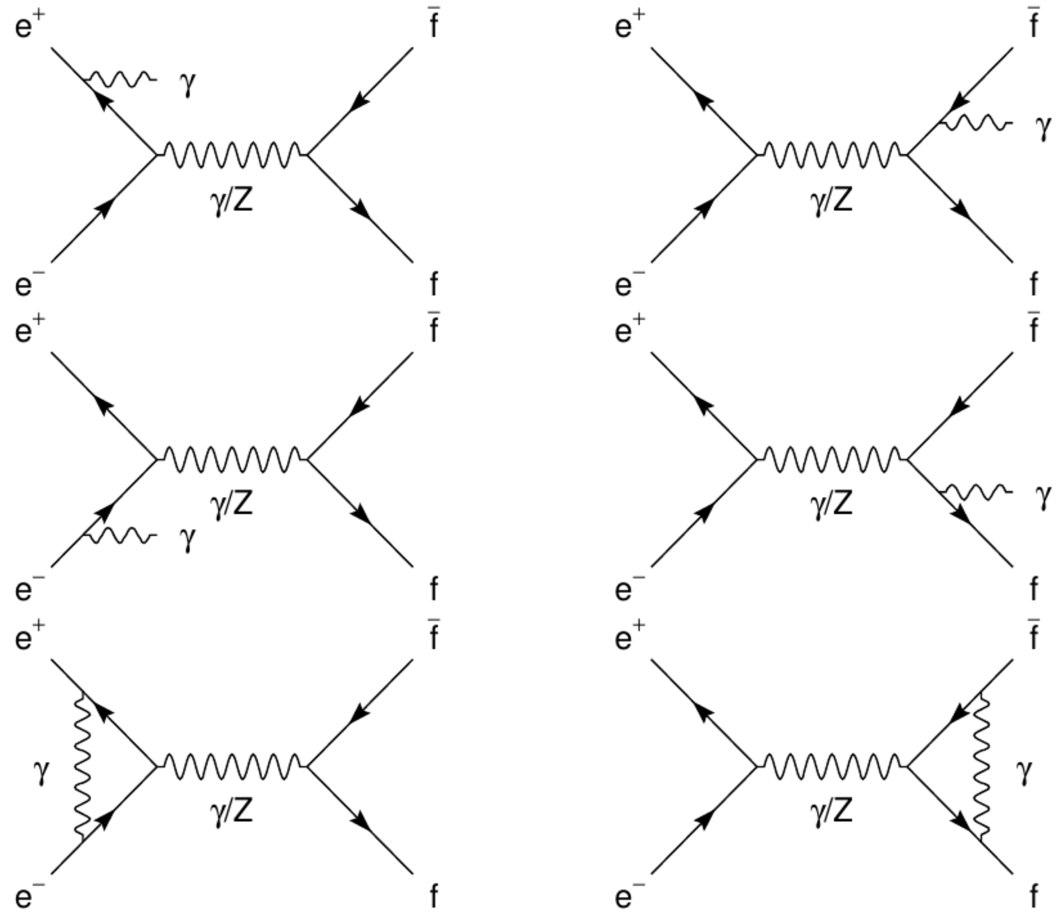
- radiation from final state

  increases observed cross section

- radiation from initial state:

  reduction of initial centre-of mass energy distort shape of the resonance

  → events radiate down
  - lower cross section at pole
  - increased cross section at energies above the pole



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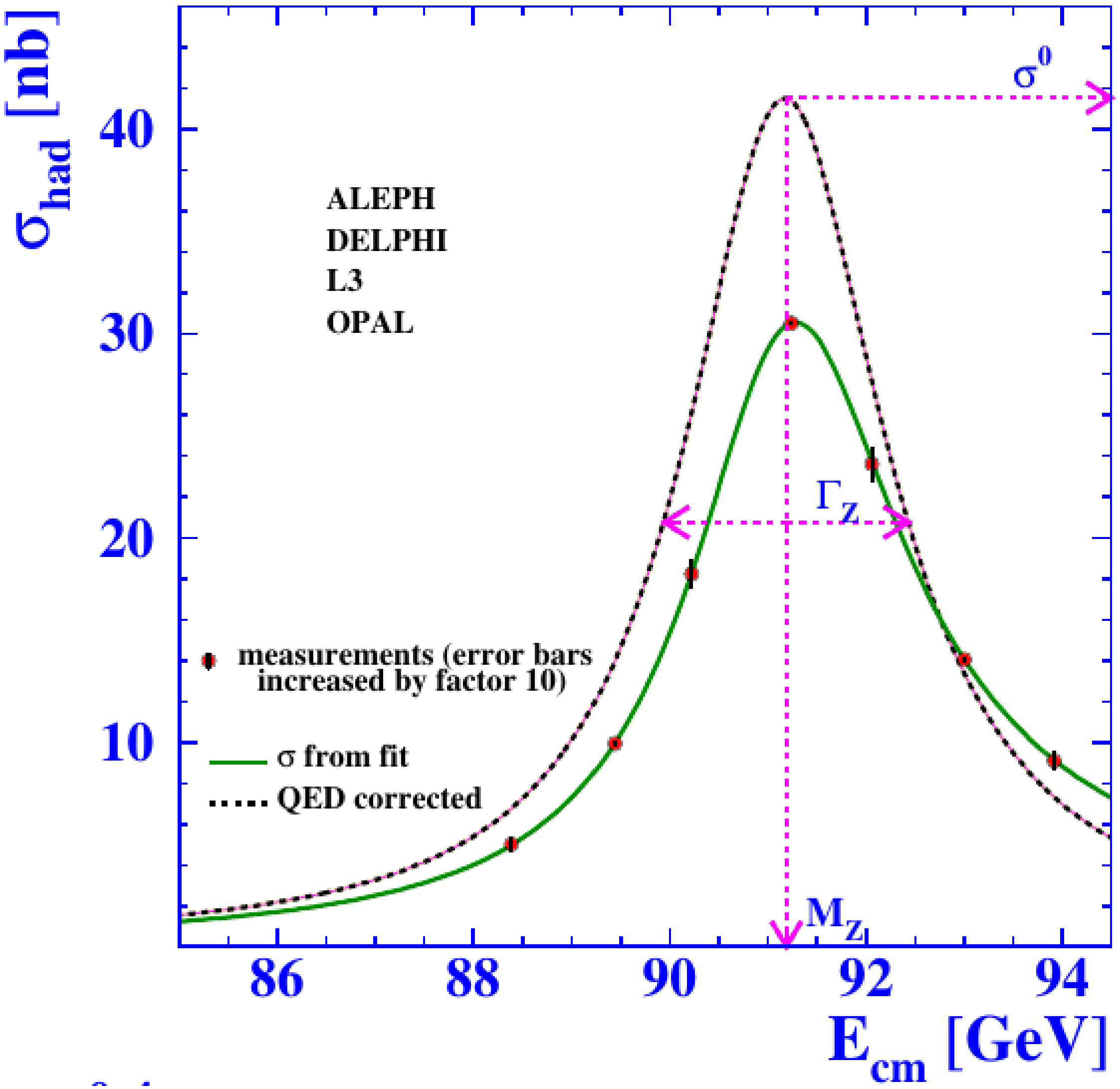
QED-corrected Z Cross Section



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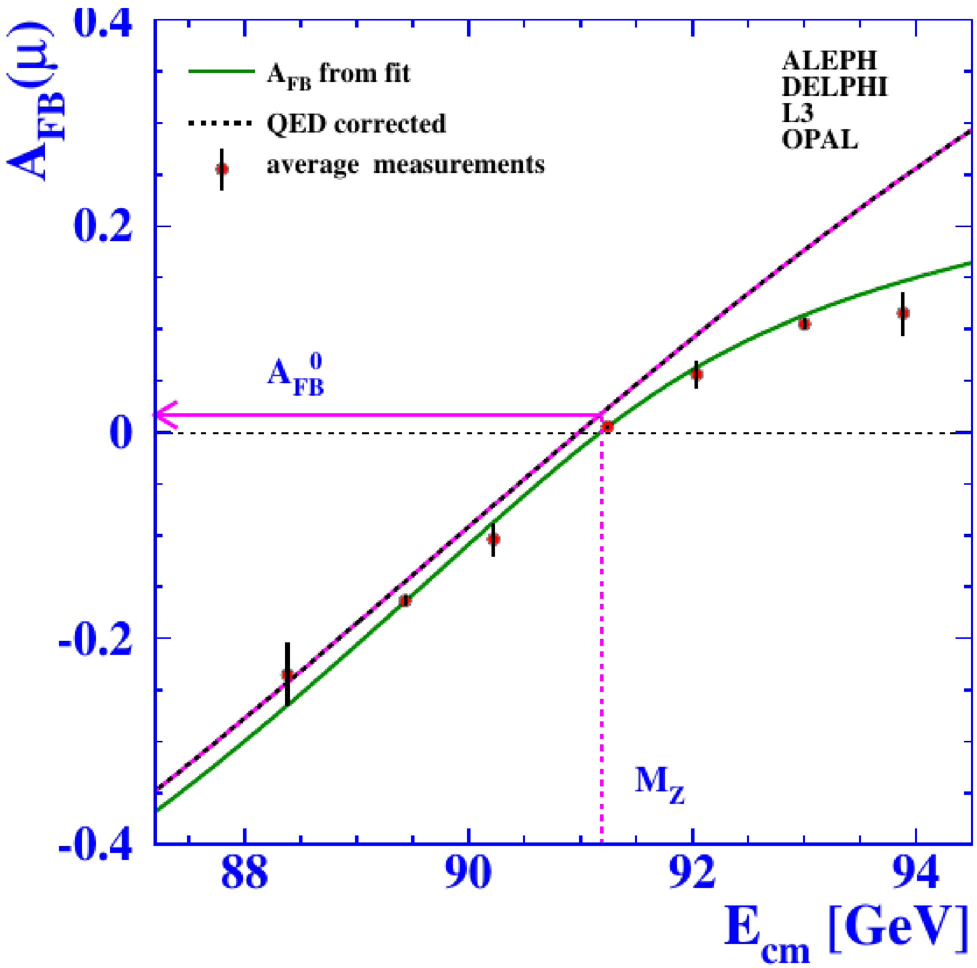
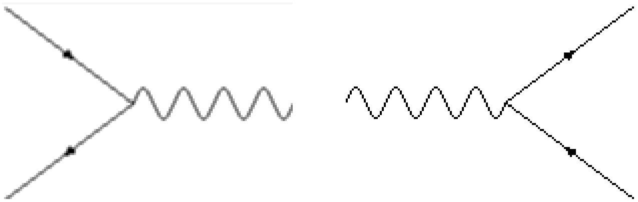
QED-corrected Forward-Backward Asymmetry

γ - Z interference and Z couplings lead to a forward-backward asymmetry

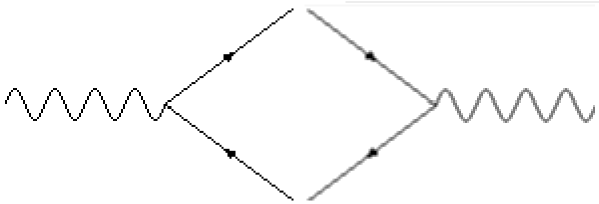
Forward-Backward asymmetry around the Z pole

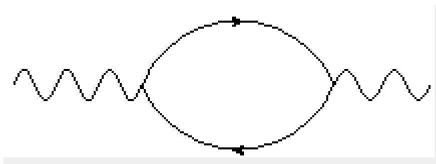


Feynman diagrams from building blocks

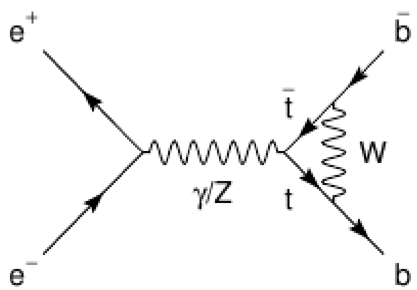


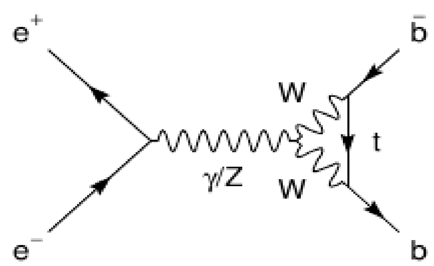
$$f\bar{f} \to f'\bar{f}'$$



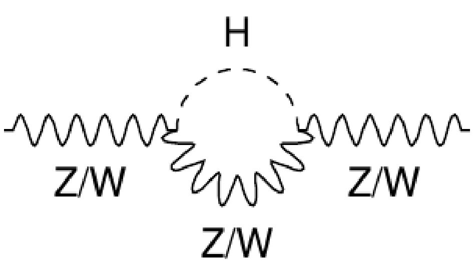


“fermion loop”
(modifies photon propatator)





“vertex corrections”
involving top quark



“Higgs loop”
(modifies Z/W propatator)

Very many combinations possible, up to an infinite number of loops
→ enormous number of diagrams contributing to a given process

Radiative corrections

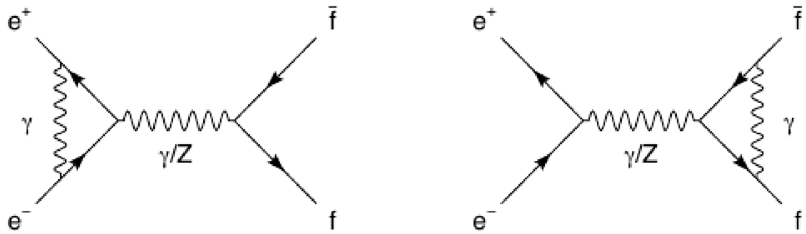
Precision of LEP and SLC data: sensitive to **higher-order corrections**

- **Real emission** of photons and **loop corrections**
- Consequence: “running” QED coupling constant:

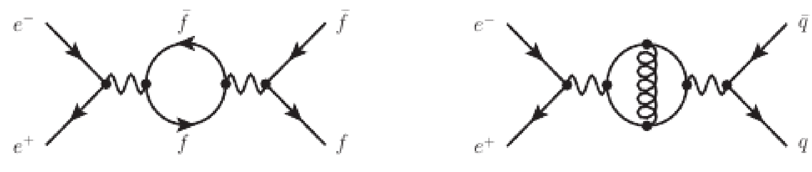
$$\alpha(m_Z^2) \approx \frac{1}{128} > \alpha \approx \frac{1}{137}$$

- **Pseudo-observables**: redefine experimental observables to compare to theory predictions, e.g. effective weak mixing angle

QED Vertex Correction



Vacuum Polarization



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Via higher-order corrections, precision measrurements become sensitive to all parameters of the theory, including new, so far unobserved particles !

Radiative Corrections (2)

The resulting corrections can be absorbed in a **redefinition of the couplings**, retaining the tree-level structure of the cross section formula :

- $g_a^{\mathrm{f}} = I_3 \rightarrow g_a^{\mathrm{eff,f}} = \sqrt{\rho^{\mathrm{eff,f}}} I_3$ with $\rho^{\mathrm{eff}} = 1 + \Delta\rho$
- $g_v^{\mathrm{f}} = I_3 - 2Q_f \sin^2\Theta_W \rightarrow g_v^{\mathrm{eff,f}} = \sqrt{\rho^{\mathrm{eff,f}}}(I_3 - 2Q_f \sin^2\Theta_W^{\mathrm{eff,f}})$
- relation between boson masses and weak electromagnetic and weak couplings

  also receives a correction, *Δr* : $m_{\mathrm{W}}^2\left(1 - \frac{m_{\mathrm{W}}^2}{m_{\mathrm{Z}}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_{\mathrm{F}}}\left(\frac{1}{1-\Delta r}\right)$
- keep $\sin^2\Theta_W := 1 - \frac{{M_W}^2}{M_Z^2}$
- running of the electromagnetic coupling constant: $\alpha(M_Z) = \frac{\alpha^{(0)}}{1 - \Delta\alpha}$

$\Delta r_w,\ \Delta\rho$ and $\sin^2\Theta_W^{\mathrm{eff}}$ **absorb higher orders to very good approximation**

Note: couplings also receive (small) complex contributions

Higher Orders (3)

furthermore, partial decay widths $\Gamma_{\mathrm{f}}^{(ew)} = N_c^{\mathrm{f}} \frac{G_F {m_Z}^3}{6\sqrt{2}\pi} \left({g_V^{\mathrm{eff}}}^2 + {g_A^{\mathrm{eff}}}^2 \right)$

receive radiative corrections for real photon and gluon emmission:

$$\Gamma_{\mathrm{f}} = \Gamma_{\mathrm{f}}^{(ew)} \cdot \left(1 + \frac{3}{4\pi} Q_{\mathrm{f}}^2 \alpha(m_Z) + \ldots \right) \cdot \left(1 + \frac{\alpha_s(m_Z)}{\pi} + \ldots \right)$$

for quarks only

Loop corrections depend

- quadratically on top-quark mass
- logarithmically on Higgs-boson mass

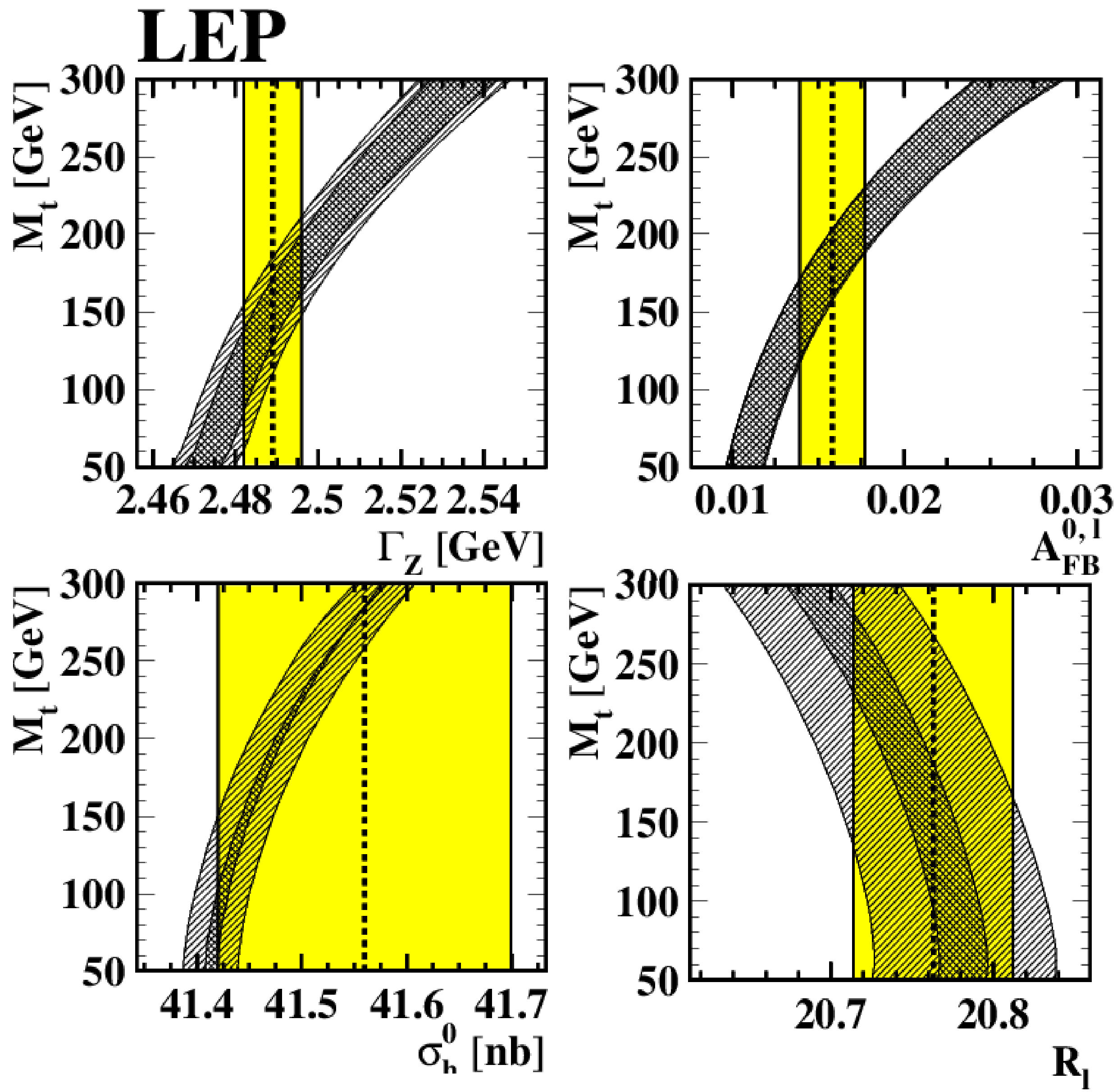
and can be calculated from quantum field theory

$$\text{e. g. } \Delta r_W(M_t, M_H) = \frac{3\alpha \cos^2 \Theta_W}{16\pi \sin^4 \Theta_W} \frac{{M_t}^2}{{M_W}^2} + \frac{11\alpha}{48\pi \sin^2 \Theta_W} \log \frac{{M_H}^2}{{M_W}^2} + \ldots$$

Software implementations of higher-order calculations exist to calculate Observables relevant observables $\mathcal{O}(\alpha, G_F, M_Z, M_{\mathrm{top}}, M_{\mathrm{H}}, \ldots)$

→ masses of particels out of reach at LEP (top quark, Higgs boson) could be predicted from precision measurements

Example: dependence of ovservables on m_t



LEP EWWG Report 1993

- single hatched area: variation of α_s by ±0.006
- cross-hatched area: 60 < M_H(GeV) < 1000

Cross section formula with effective couplings

$$\frac{2s}{\pi}\frac{1}{N_c^{\mathrm{f}}}\frac{d\sigma_{\mathrm{ew}}}{d\cos\theta}(\mathrm{e^+e^-}\to\mathrm{f\bar{f}}) =$$

Form of tree-level formula retained !

$$\underbrace{|\alpha(s)|^2\,(1+\cos^2\theta)}_{\gamma}$$

$$\underbrace{-8\Re\left\{\alpha^*(s)\chi(s)\left[\mathcal{G}_{\mathrm{V}e}\mathcal{G}_{\mathrm{V}f}(1+\cos^2\theta)+2\mathcal{G}_{\mathrm{A}e}\mathcal{G}_{\mathrm{A}f}\cos\theta\right]\right\}}_{\gamma-\mathrm{Z\ interference}}$$

$$\underbrace{\begin{array}{c}+16|\chi(s)|^2\left[(|\mathcal{G}_{\mathrm{V}e}|^2+|\mathcal{G}_{\mathrm{A}e}|^2)(|\mathcal{G}_{\mathrm{V}f}|^2+|\mathcal{G}_{\mathrm{A}f}|^2)(1+\cos^2\theta)\right.\\ \left.+8\Re\left\{\mathcal{G}_{\mathrm{V}e}\mathcal{G}_{\mathrm{A}e}{}^*\right\}\Re\left\{\mathcal{G}_{\mathrm{V}f}\mathcal{G}_{\mathrm{A}f}{}^*\right\}\cos\theta\right]\end{array}}_{\mathrm{Z}}$$

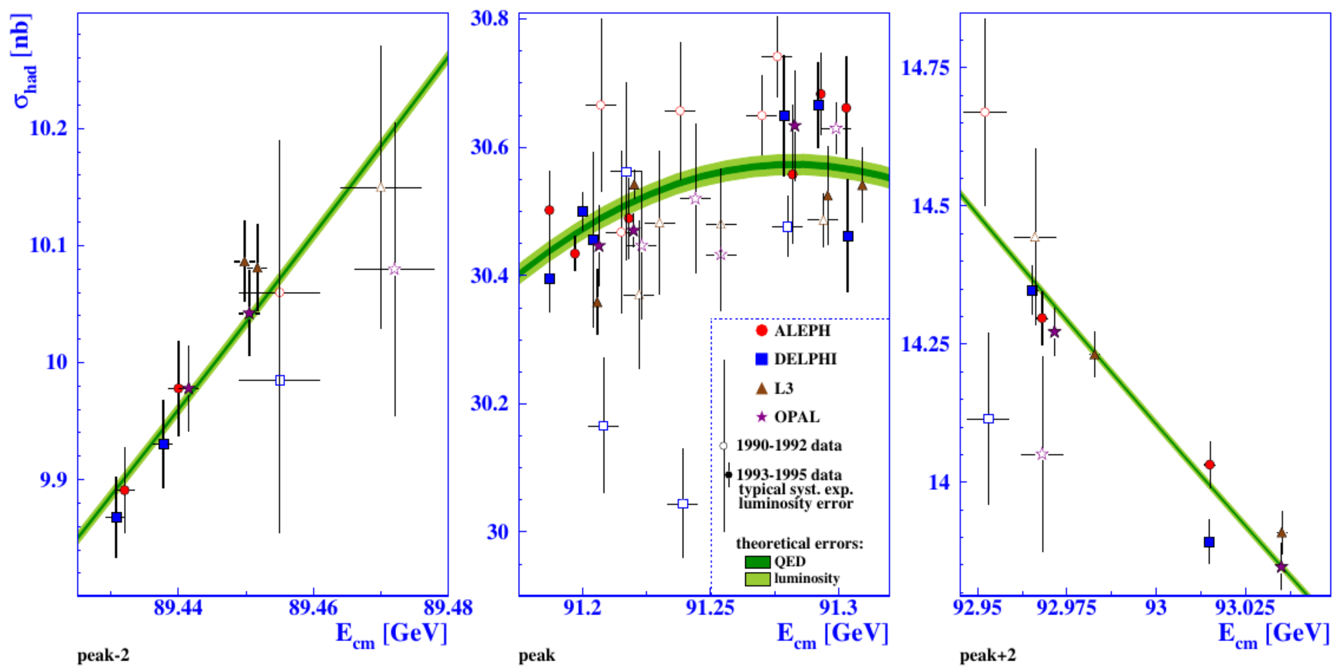
$$\text{with } \chi(s)=\frac{m_{\mathrm{Z}}^2}{8\pi\sqrt{2}}\frac{s}{s-m_{\mathrm{Z}}^2+is\Gamma_{\mathrm{Z}}/m_{\mathrm{Z}}}$$

Breit-Wigner with s-dependent width

Measurements at the Z resonance

- Data taking determined by “fills” of the LEP machine, each typically lasting ~8 h
- for maximum statisctics, mostly ran at the peak energy
- during energy scans, beam energy altered between peak, peak – 2 GeV and peak + 2 GeV

raw cross-section measruements $e^+e^- \rightarrow$ hadrons at LEP



Parameterization of Cross Section Measurements

Total cross sections …

$$e^+e^- \rightarrow \mathrm{Hadrons}$$
$$e^+e^- \rightarrow e^+e^-$$
$$e^+e^- \rightarrow \mu^+\mu^-$$
$$e^+e^- \rightarrow \tau^+\tau^-$$

… as functions of E_{CM}

~30 per channel and experiment, at seven energy points

… parameterizes by 6 "Pseudo Observables"

- m_Z
- Γ_Z
- $\sigma^o_{had} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{had}}{\Gamma_Z^2}$
- $R_e = \Gamma_{had} / \Gamma_{ee}$
- $R_\mu = \Gamma_{had} / \Gamma_{\mu\mu}$
- $R_\tau = \Gamma_{had} / \Gamma_{\tau\tau}$

$\Gamma_{ff} \propto \left({g_{Vf}}^2 + {g_{Af}}^2\right)$ for f=e, μ, τ

advantages of this parameter set:

- based on "simple" measurements
- minimal correlated uncertainties

Parameterization of Cross Section Measurements

Forward Backwards asymmetries …

$$e^+e^- \rightarrow e^+e^-$$
$$e^+e^- \rightarrow \mu^+\mu^-$$
$$e^+e^- \rightarrow \tau^+\tau^-$$

… as functions of E_{CM}
~30 per channel and experiment,
at seven energy points

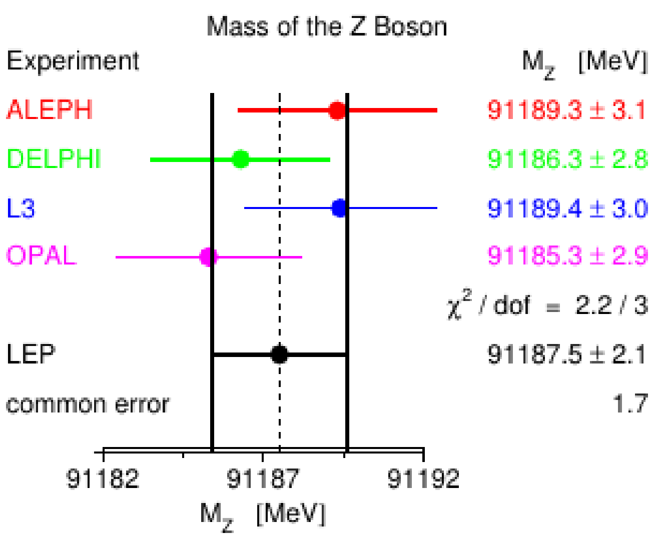
… parameterizes by 3 "Pseudo Observables"

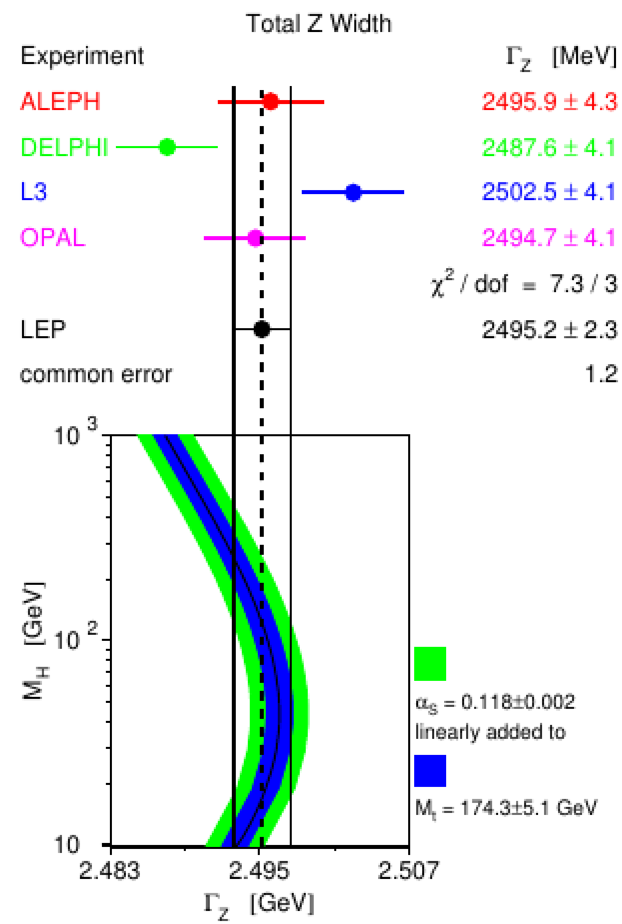
- $A_{FB}^{0,e}$
- $A_{FB}^{0,\mu}$
- $A_{FB}^{0,\tau}$

Parameter sets of the four LEP experiments were combined, taking into account common correlated uncertainties arising dominantly from:
- LEP energy scale
- theory uncertainties

Results of Combination

based on 9 POs, χ^2/DoF = 33/27 (22 % prob.)





$$m_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

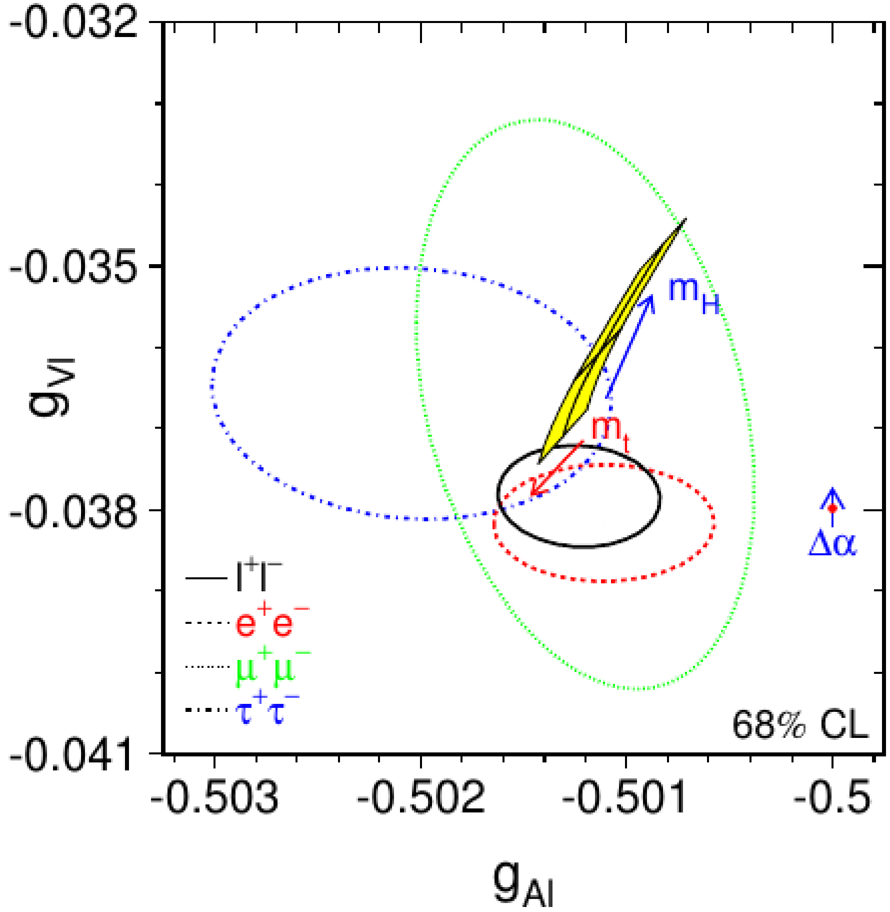
$$\Gamma_Z = \quad 2.4952 \pm 0.0023 \text{ GeV}$$

Lepton Couplings

Fit includes measurements with polarized electron beam by SLD, which allows to measure A_e from cross section asymmetry:

$$A_{LR} = \frac{N_L - N_R}{N_L + N_R} \frac{1}{\langle \mathcal{P}_e \rangle}$$

ratio of cross-sections with left- and right-handed electrons

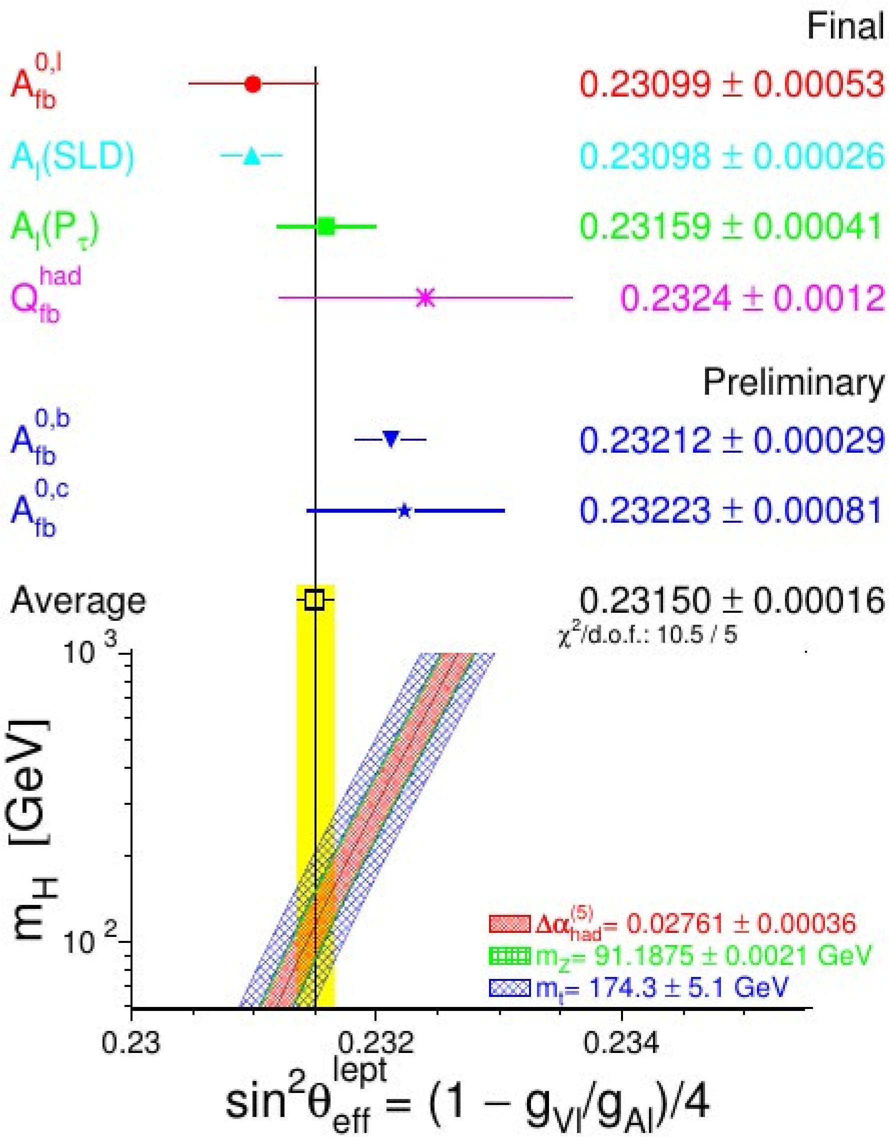


Consistent with Standard Model expectation, preferring small Higgs boson mass

Standard Model Prediction includes measurements of the W boson and Top quark masses

see later !

Effecitve weak mixing angle



χ^2 of average: 10.5/5

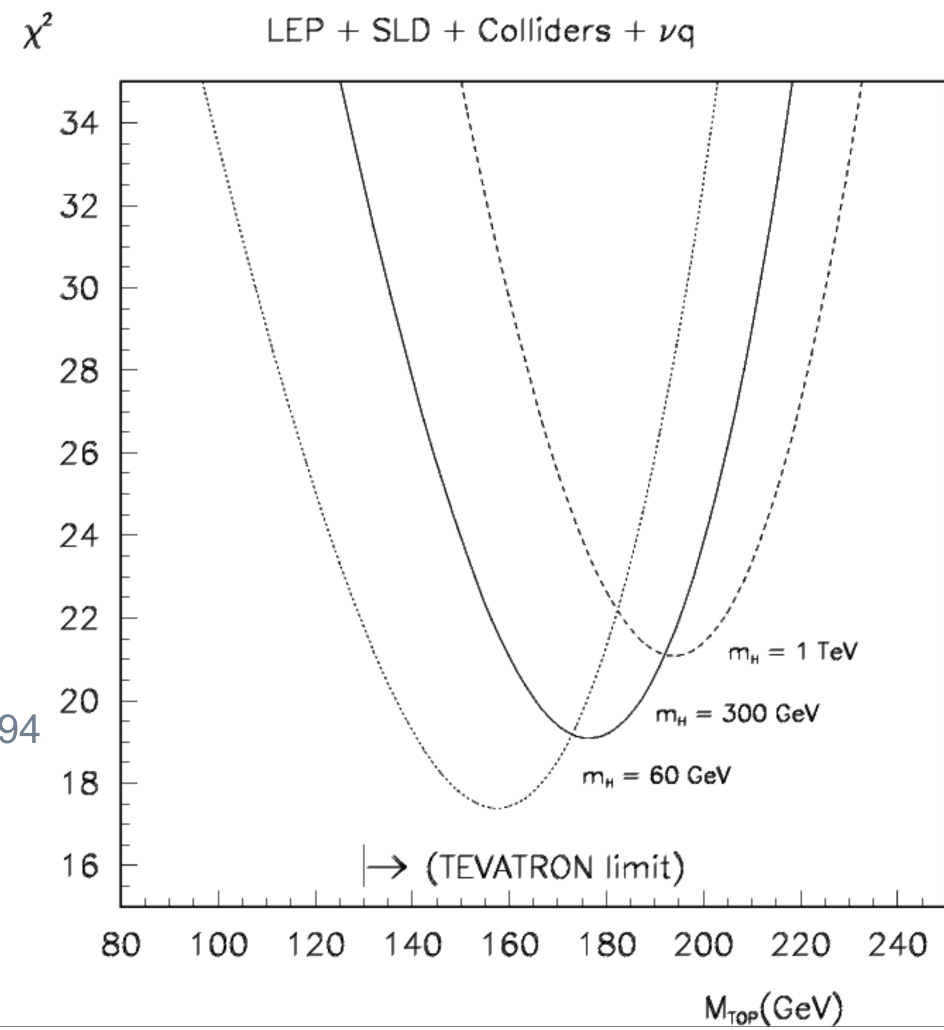
χ^2 – probability: 6.2 %

1994: Prediction of Top Quark mass

LEP + Collider and ν data + A_{LR} from SLC
$177^{+11}_{-11}\,{}^{+18}_{-19}$

Top quark mass [GeV] from electroweak fit.
The second uncertainty stems from the - at that time – unknown value of the Higgs-boson mass

LEP EWWG, Moriond Conference, March 1994



First hints for top quark production at the Tevatron at a mass around 175 GeV reported in April 1994, combined published mass value by CDF and D0 in 1995:

m_t = 176±18 GeV