Deep-inelastic scattering (DIS)

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Karlsruhe Institute of Technology

See the CMS cavern in a live visit!

Instead of the traditional CERN excursion, not possible because of COVID19, we propose for Thursday, 28.01.2021, from 14:00 – 15:00h a virtual visit of the CMS experiment to all students of the Particle & Detector Physics courses.

The event can be reached from this link:
https://indico.cern.ch/event/971070/

(I will send this also via Ilias after the lecture today.)
Under connection you'll find the ZOOM link to the webinar.
It is possible to ask questions via a Q&A box.

Local Karlsruhe host: Günter Quast
CMS surface host: KR
Underground guide: Erik Butz
Technical support: Noemi Beni & Zoltan Szillasi

Summary Z pole

- $e^+e^- \to f\bar{f}$ annihilation via s-channel photon exchange: $\sigma_\gamma = N_C Q_f^2 \frac{4\pi\alpha^2}{3s}$
- $e^+e^- \to e^+e^-$ special case, also t-channel dominating for small angles
- Approaching M_Z also Z exchange, depends on left-/righthanded chiralities,
- Identical final states → γ/Z interference → forward-backward asymmetries
- At M_Z resonance peak of cross section depends on decay widths
- Lineshape depends on Z decay width including invisible decays (neutrinos)
- Determine no. of neutrino generations: $N_\nu = 2.9840 \pm 0.0082$
- Higher-order diagrams of QED important for correct predictions at Z pole
- Radiative corrections mandatory
- Virtual corrections also contain information of heavy particles in loops → predict top quark mass

Recall QCD & Jets

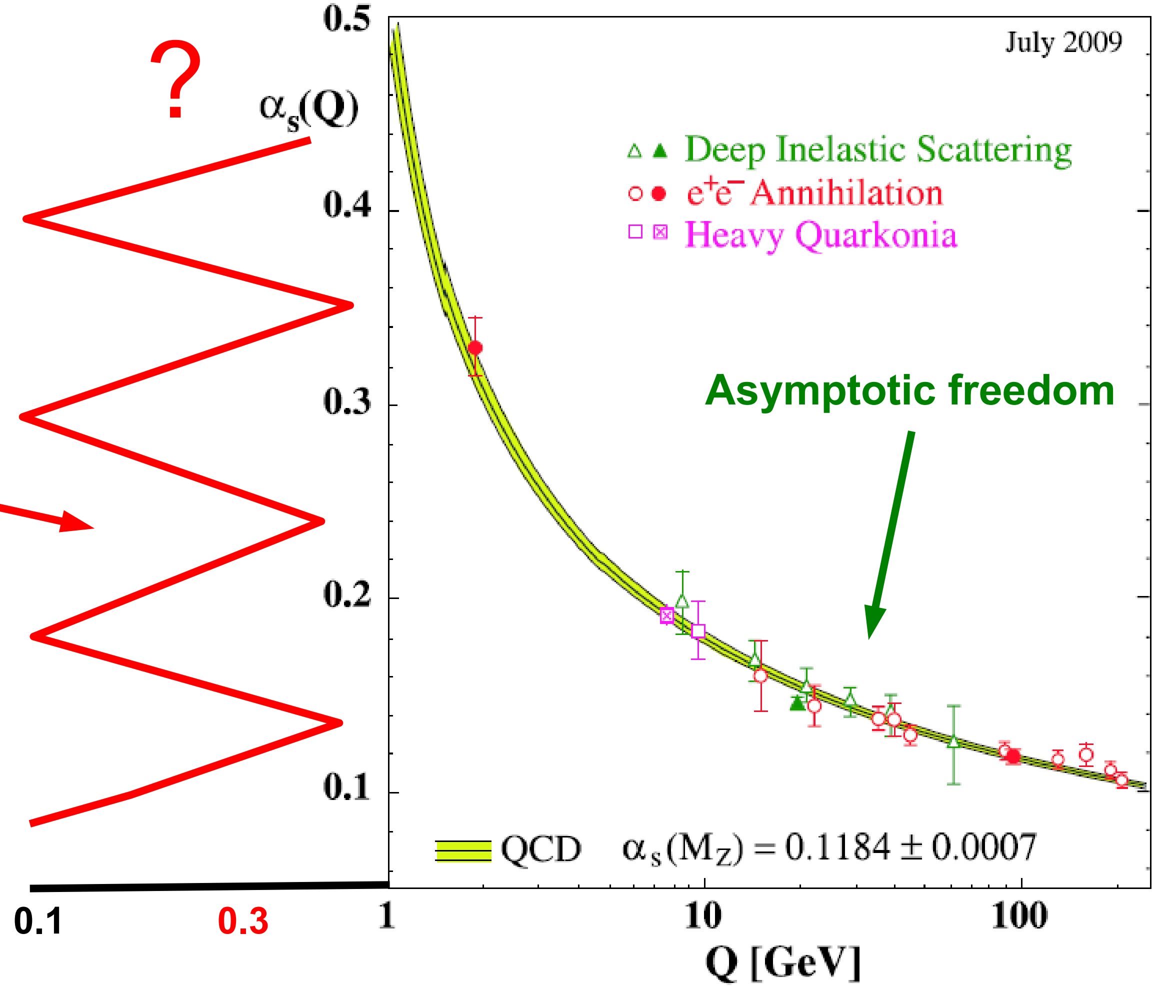
- Quantum corrections lead to energy (distance) dependent couplings
- QCD coupling decreases with energy → asymptotic freedom
- QCD coupling increases with distance (confinement) → string potential ~ r
- Free (anti-)quarks or gluons are not measurable
- But final state pattern (foot print) of hard interaction measurable if energy scale of interaction >> hadronisation scale
  - Energy flow pattern in events, “event shapes”
  - Streams of collimated particles, “jets”
- Jet definition (event shape, too) requires algorithm that is infrared- and collinear-safe to compare with perturbative predictions of theory

Running coupling constant

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)}$$

with Λ typically ≈ 200 – 300 MeV

Non-perturbative regime



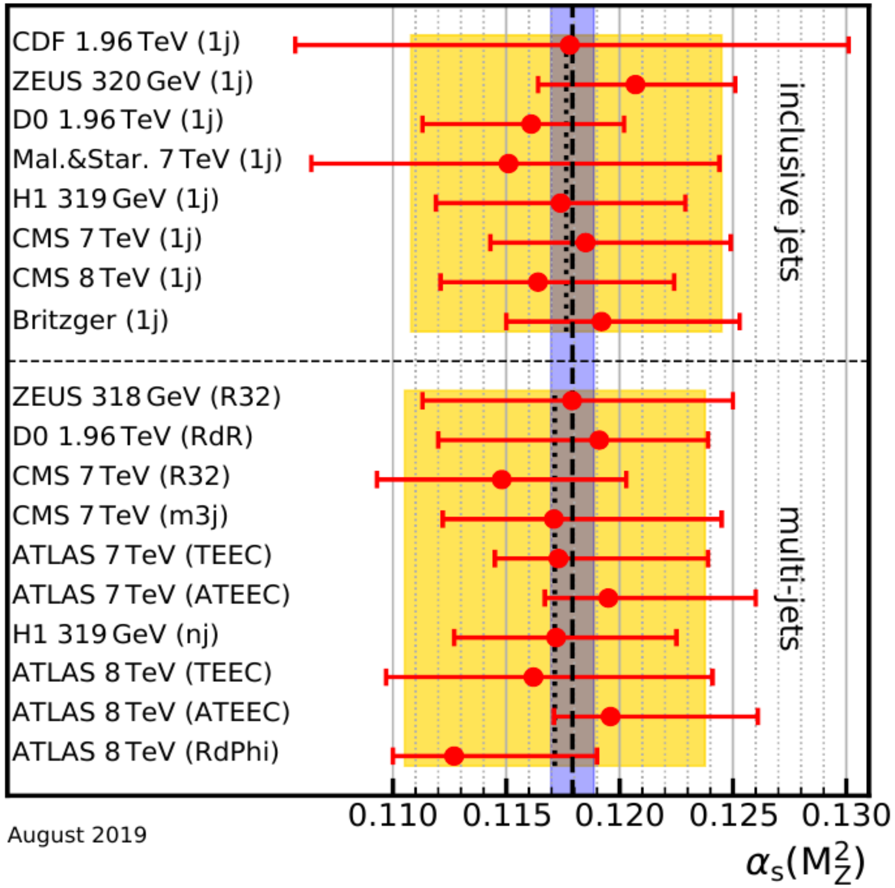
S. Bethke, EPJC 64 (2009).

QCD potential grows linearly with larger distances:

$$V = \sigma \cdot r \approx 1\text{GeV}/\text{fm} \cdot r$$

→ No free quarks (or gluons)
→ Confinement

Running coupling from PDG

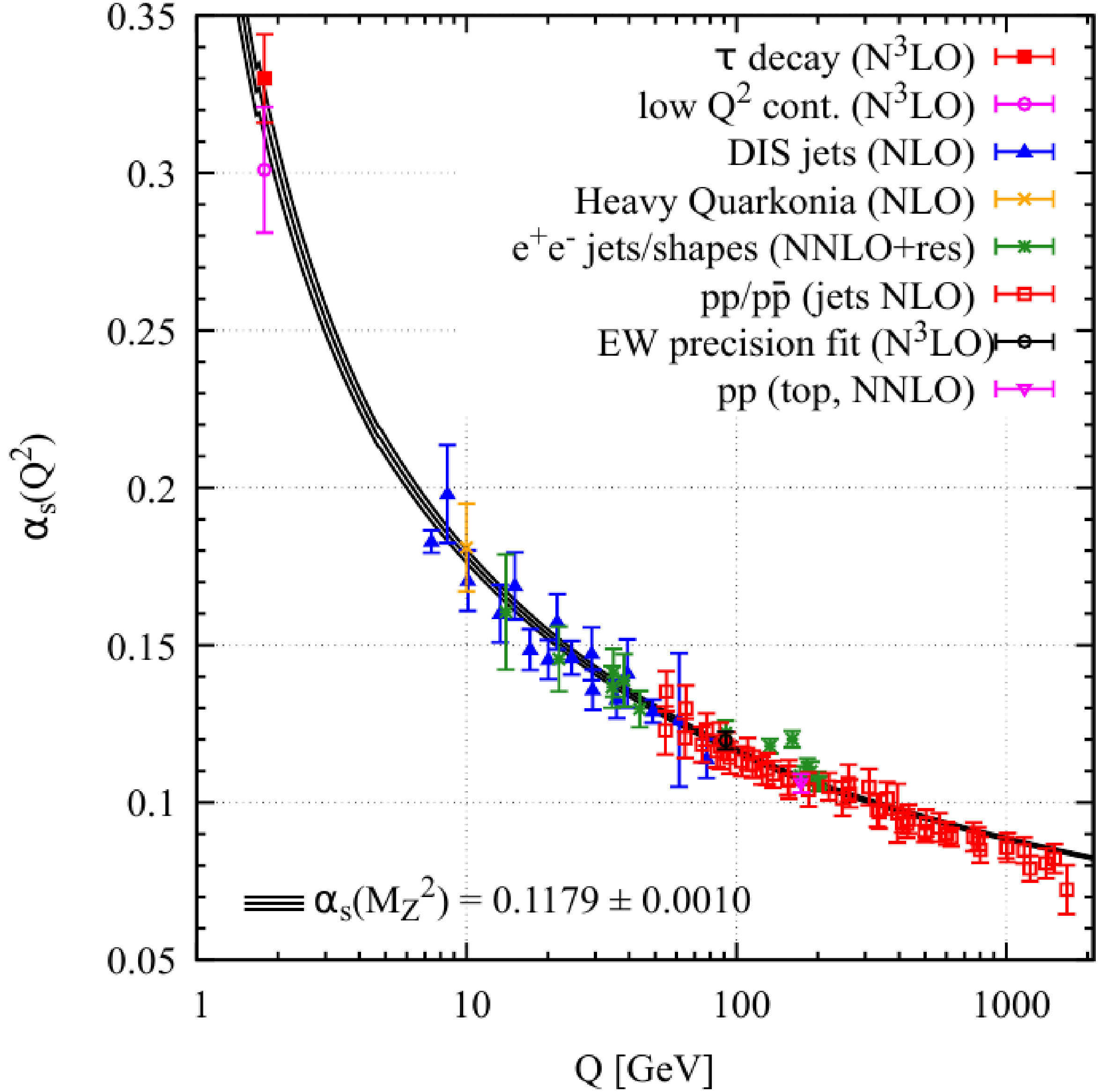


Not yet in world average – need NNLO

$$\alpha_s(M_Z) = 0.1179 \pm 0.0010$$

Most precise "single" result from lattice gauge theory.

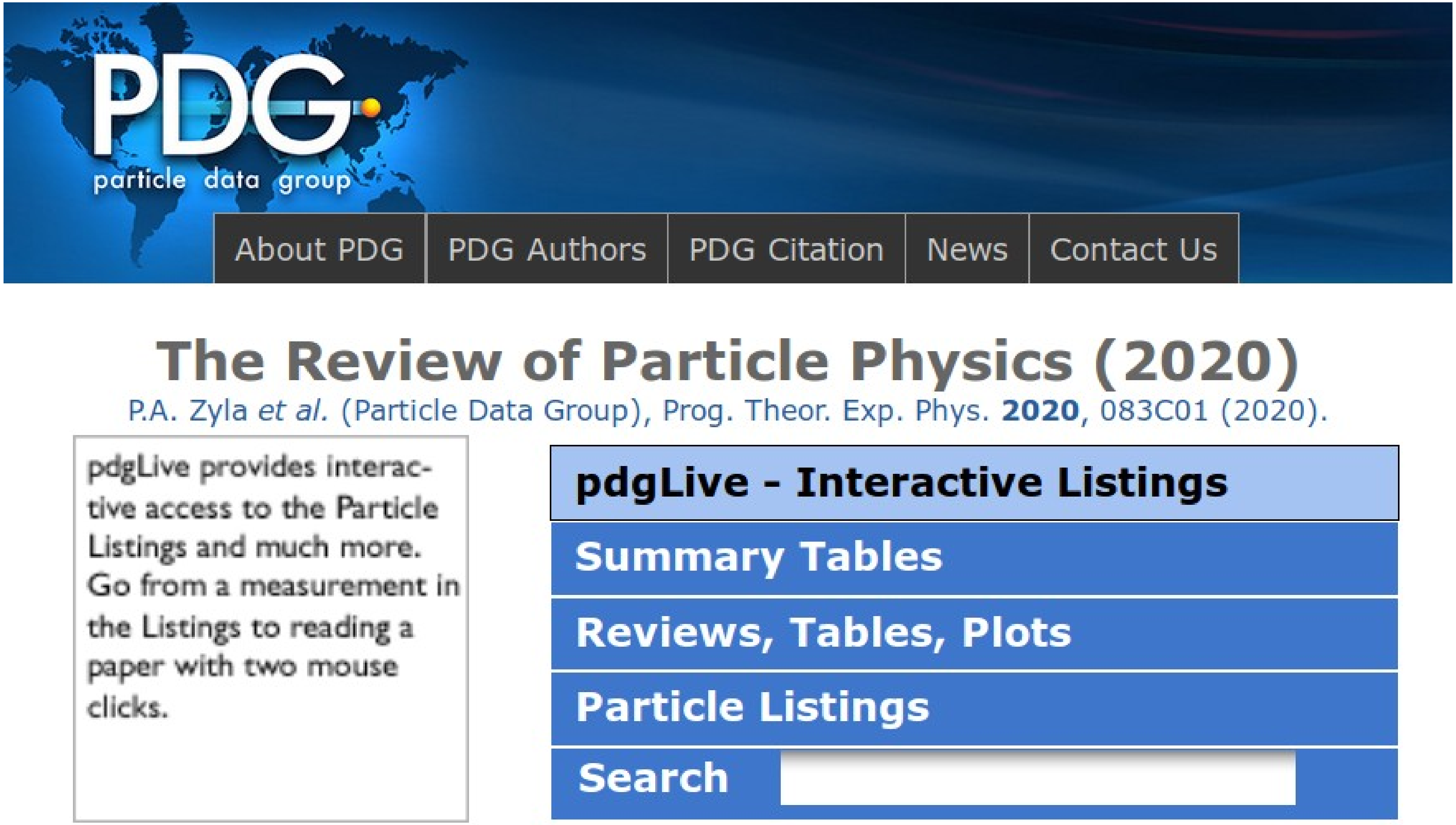
2019



Particle Data Group, Review of Particle Physics 2020, QCD ch., J. Huston, KR, G. Zanderighi

Online reference

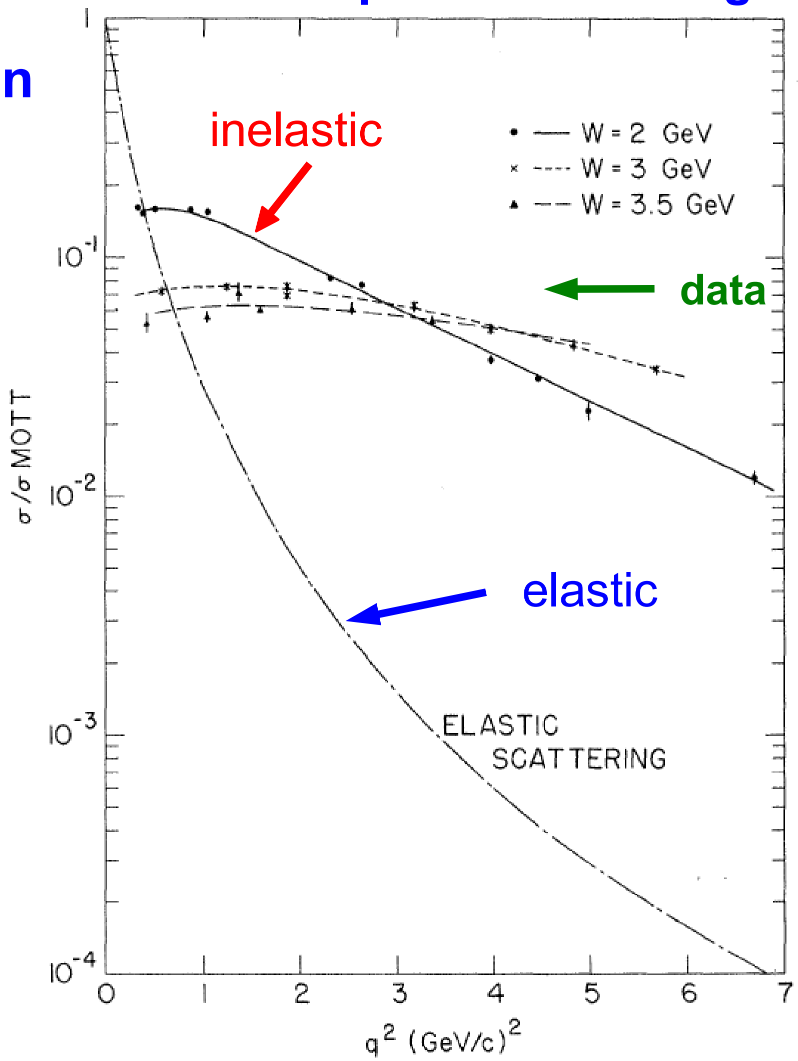
- **Literature: Review of Particle Physics (PDG)**
  - **Online version of “Bible” of particle physics: http://pdg.lbl.gov/**



Scale invariance

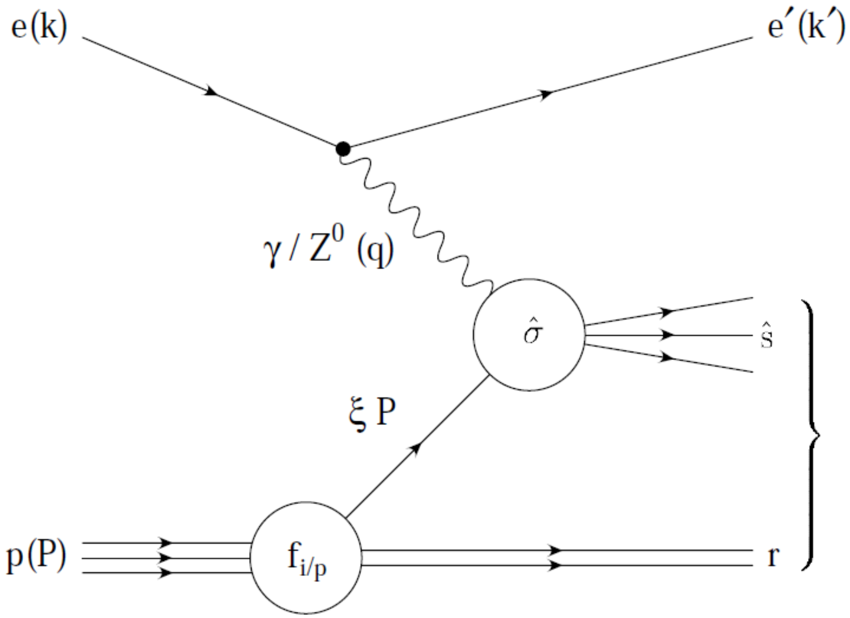
- **Inelastic >> elastic cross section**
- **Inelastic cross section ~ const. * Mott x section**
  - **approximately independent of resolution ~ q^2**
  - **scale invariant, i.e. no natural length scale**
  - **like scattering at point-like objects**

electron-proton scattering



PRL 23 (1969) 935.

Deep-inelastic scattering (DIS)



Scattered electron

Many Hadrons

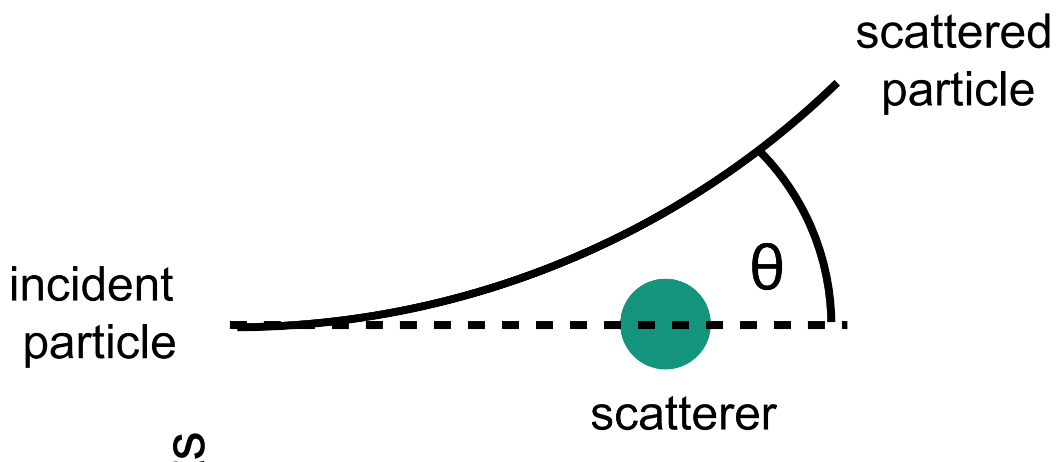
X(h)

Scattering off nuclei

Rutherford cross section:

- **Non-relativistic electron with kinetic energy E_{kin} on static Coulomb potential**
- **Only interacting charge (electric)**
- **No purely spin-related interaction (magnetic)**

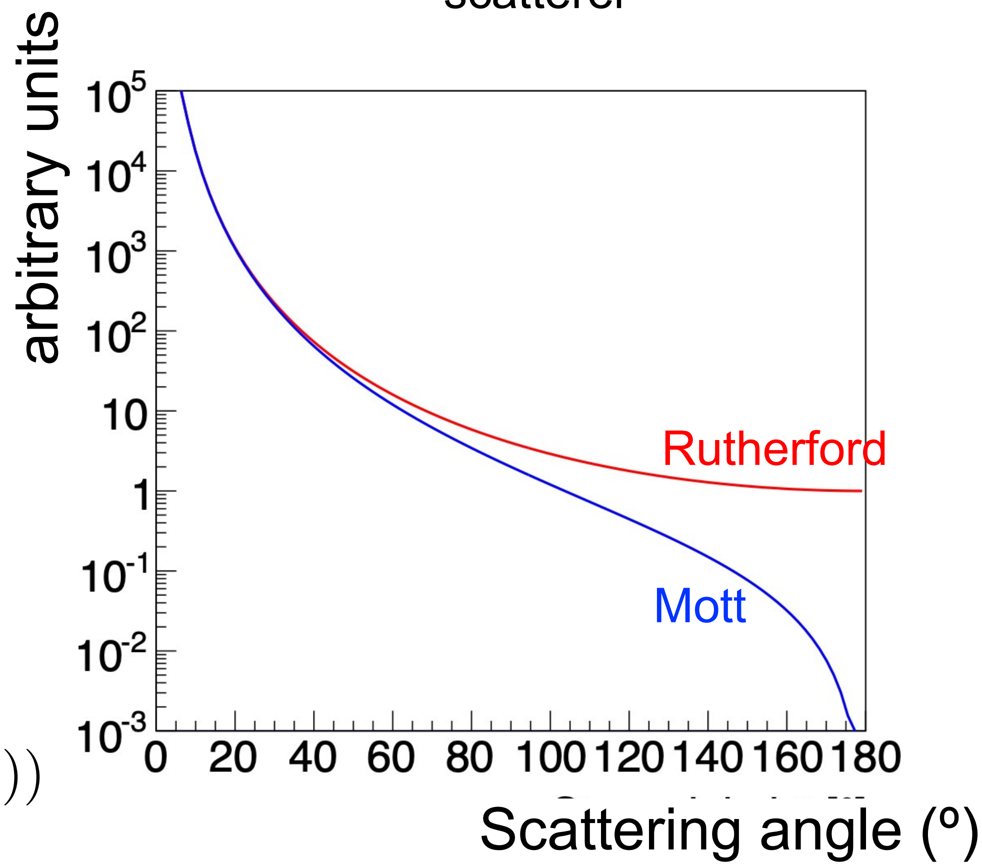
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} = \frac{Z^2\alpha^2}{16E_{\text{kin}}^2 \sin^4(\theta/2)}$$



Mott cross section:

- **Relativistic spin-1/2 electron with energy E on pointlike spinless particle**
- **Considers nuclear recoil E = E'**
- **Still no spin-spin interaction; spin flip impossible → backscatter suppressed**

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} \cdot \frac{E'}{E}\left(1-\beta^2\sin^2(\theta/2)\right)$$



Scattering off nucleon

Rosenbluth cross section:

- **Elastic electron-nucleon scattering**
- **Considers finite size of nucleon**
- **Introduces electric and magnetic form factors G_E and G_M**

$$Q^2 = -q^2 = \vec{q}^2 - (E - E')^2$$

$$\tau = \frac{Q^2}{4M_p^2}$$

$$Q^2(1+\tau) = \vec{q}^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rosenbluth}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2(\theta/2)\right]$$

- **For $Q^2 << M_p^2$, that is $\tau \to 0$:**
  - **G_E and G_M become the Fourier transforms of the electric charge and magnetic moment distributions**
  - **Electric interaction dominant**

$$\left(\frac{d\sigma}{d\Omega}\right) \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \approx G_E^2$$

- **At high Q^2, $\tau >> 1$:**
  - **Magnetic interaction dominant**

$$\left(\frac{d\sigma}{d\Omega}\right) \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \approx \left(1 + 2\tau\tan^2(\theta/2)\right) G_M^2$$

Form factors

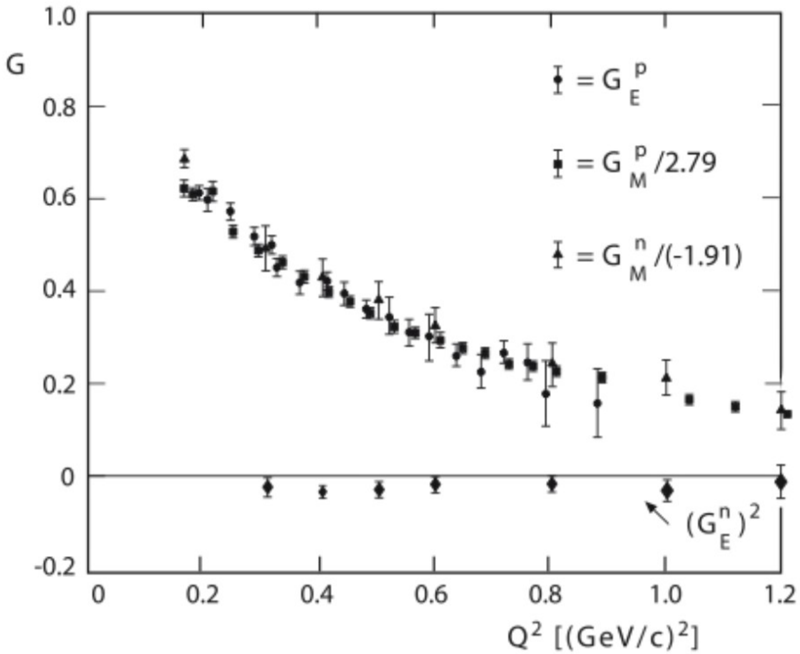
- **Electric form factor:**
  - **Point-like particle → flat**
  - **Proton measurements → "dipole" like (exponential charge distribution)**
  - **Other shapes possible, e.g. Gaussian (^{6}Li nucleus)**

$$G_E(Q^2) \propto \frac{1}{(1 + f \cdot Q^2)^2}$$

Nobel prize 1943

Otto Stern

- **Magnetic form factor:**
  - **Measurement from 1933:** **Anomalous magnetic moment of 2.79 μ_N**
    - **Not a point-like particle ...**
  - **No reason to assume shape G_M / 2.79 ~ G_E**
    - **But measurements from the 1950's show that it's the case ...**

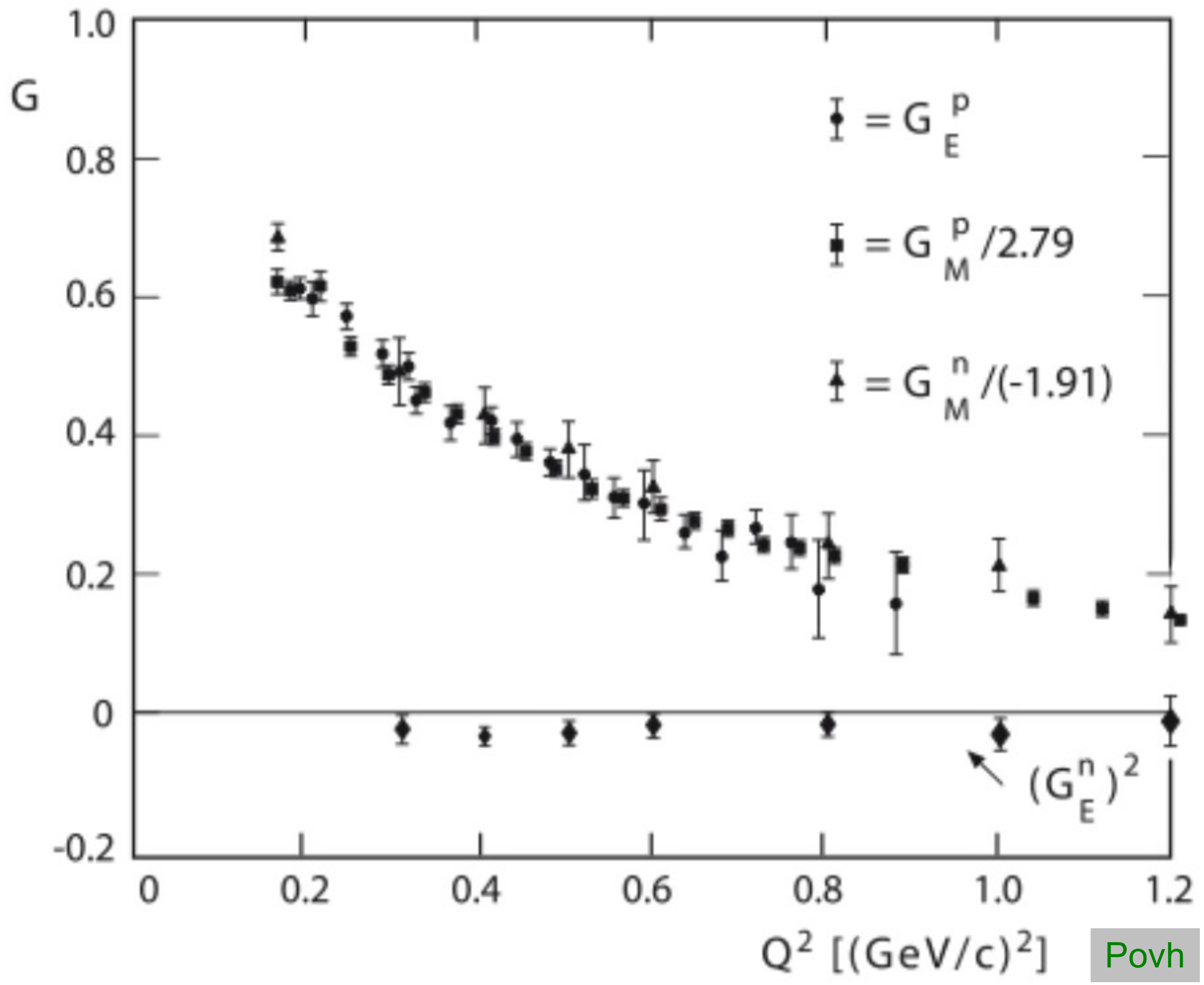


Povh

Form factors

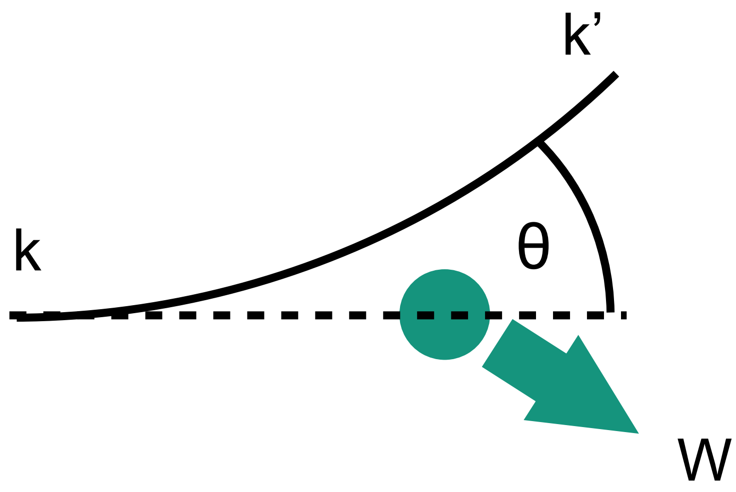
"Dipole"-like G_E of proton ~ G_M of proton and neutron if normalised to anomalous magnetic moments $\mu_p = 2.79\ \mu_N$ and $\mu_n = -1.91\ \mu_N$

G_E of neutron ~ 0

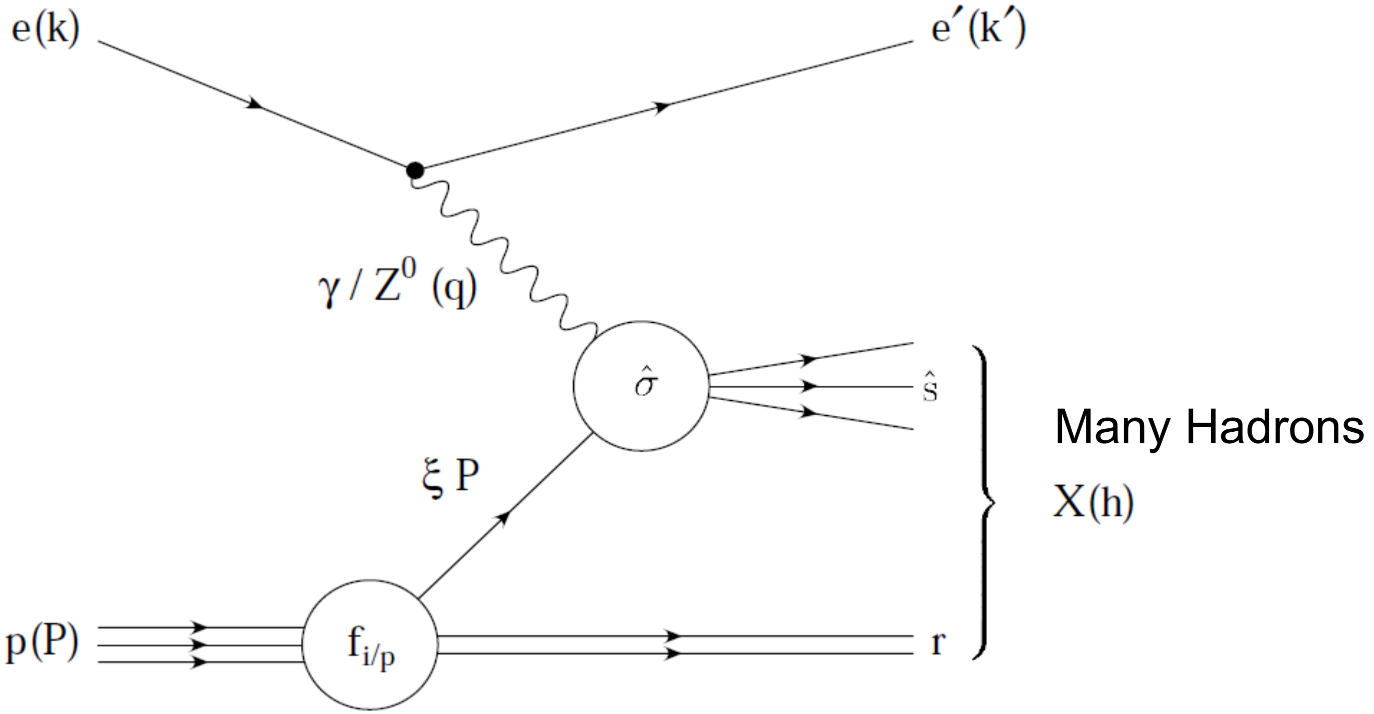


Povh

- **Inelastic scattering:**
  - **Energy transfer to target nucleon**
  - **Target nucleon broken up**
  - **Recoil absorbed by hadronic system with mass W > M_p**



Deep-inelastic scattering (DIS)



DIS kinematics

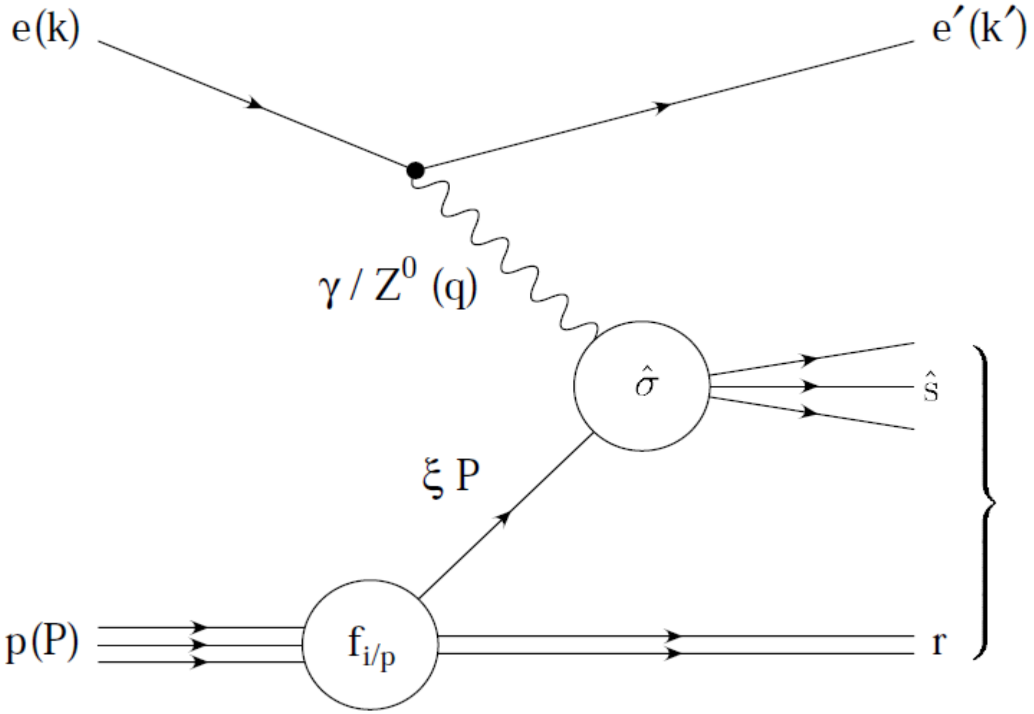
Neglect electron and proton masses: $M_p = M_e = 0$

Center-of-mass energy: $s = (k+P)^2 = 2k \cdot P = 4E_e E_p$

Elastic → one independent variable:

$$Q^2 = -q^2 = (k-k')^2 = 2k \cdot k' = 2E_e E_{e'} (1 + \cos(\theta_{e'}))$$

Deep-inelastic scattering (DIS)



Inelastic → 2nd variable:

$$W^2 = (q+P)^2 = 2P \cdot q - q^2$$

Alternative:

Bjorken scaling variable: $x = \dfrac{Q^2}{2P \cdot q}$

Inelasticity: $y = \dfrac{P \cdot q}{P \cdot k}$

DIS kinematics

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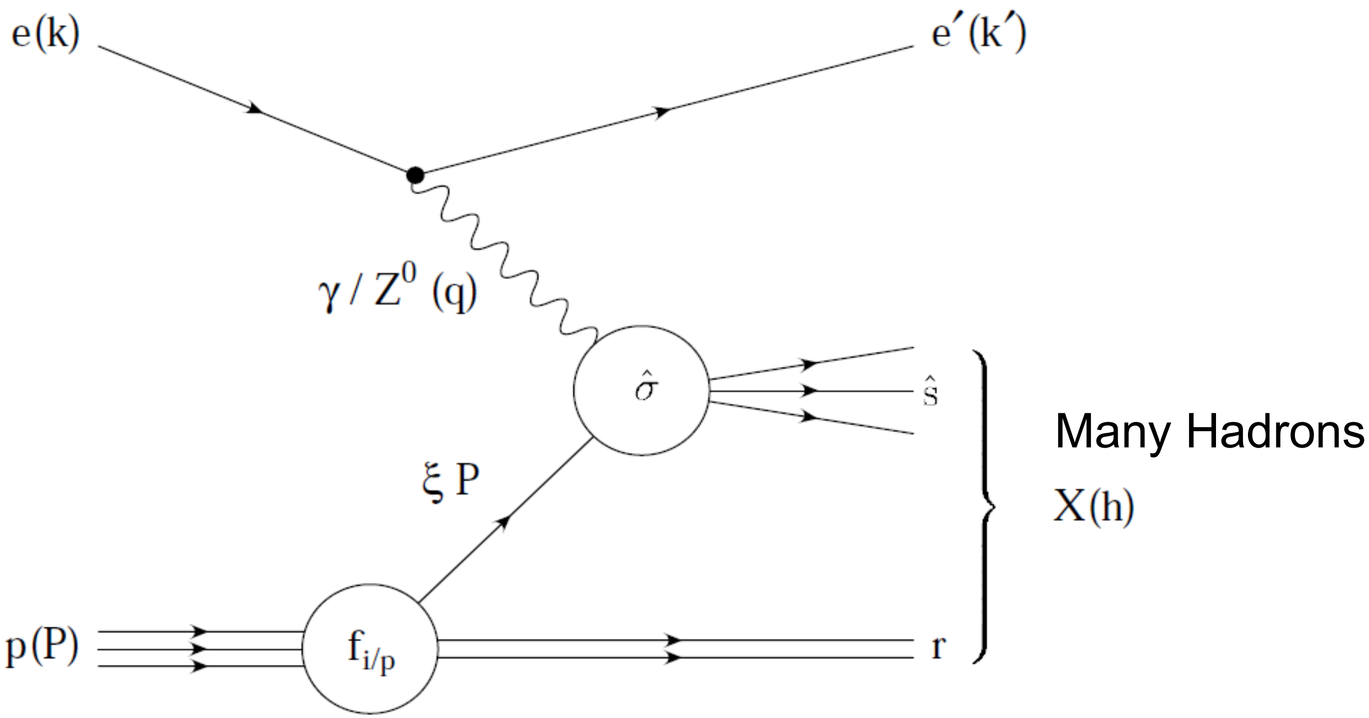
Deep-inelastic scattering (DIS)



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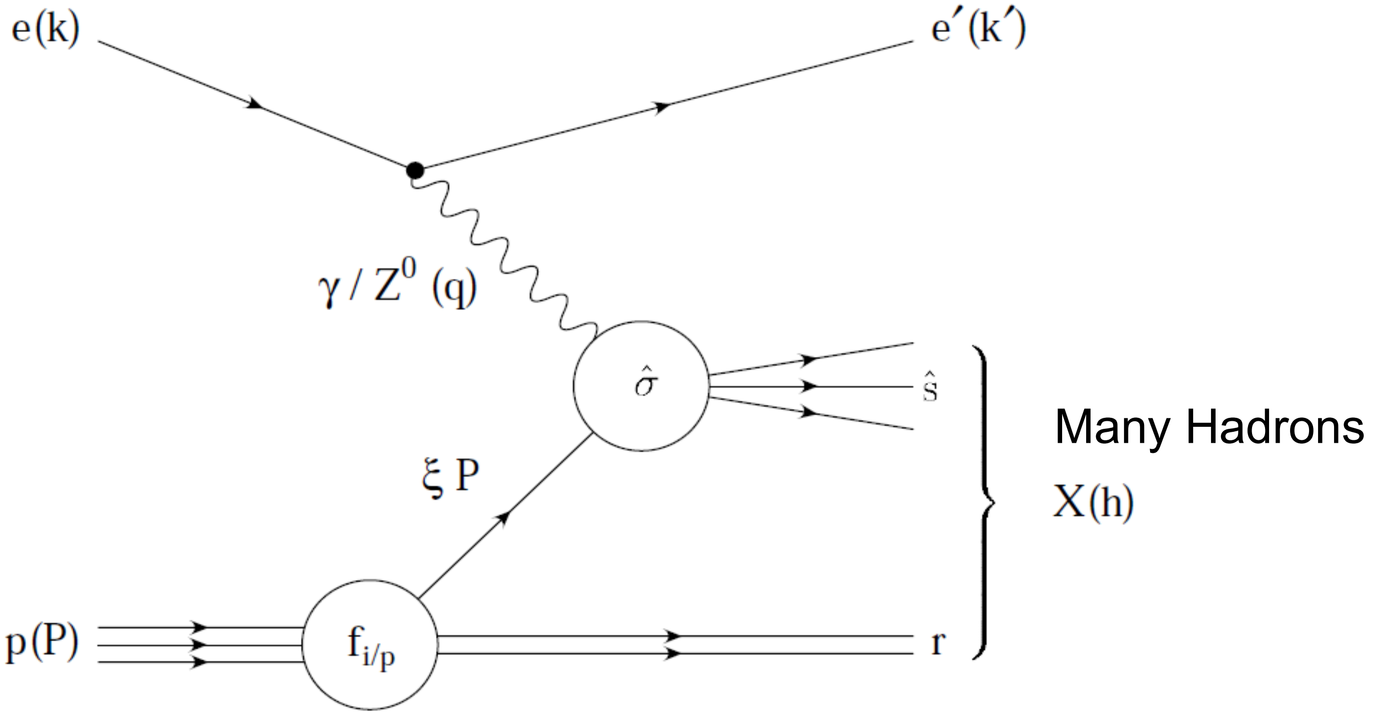
$$0 \leq (x, y) \leq 1$$

Infinite momentum frame

Proton equals collinear stream of fast moving "partons", masses negligible
→ factorised incoherent partonic scatter

$$\hat{s} = (q + \xi P)^2 = 2\xi q \cdot P - Q^2 = \left(\frac{\xi}{x} - 1\right) Q^2$$

Deep-inelastic scattering (DIS)



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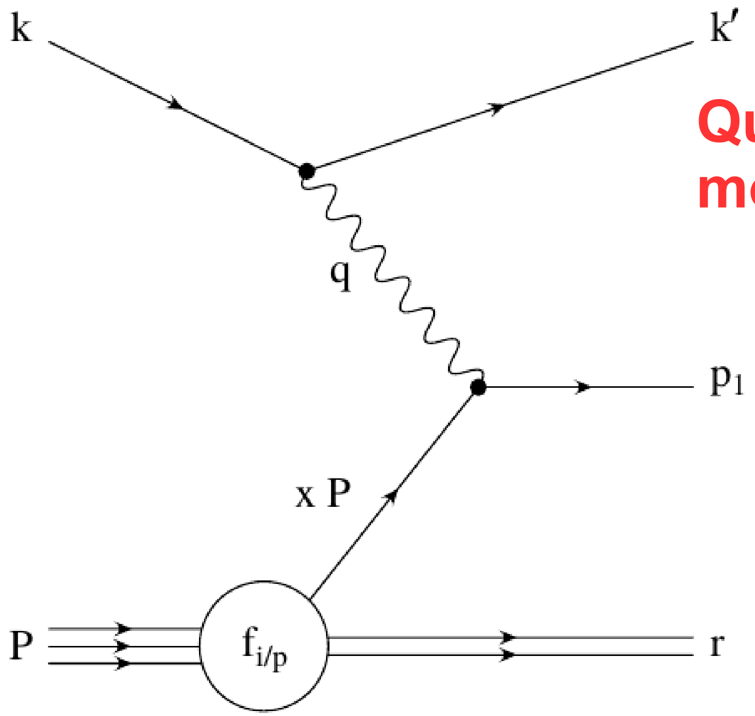
$$\hat{s} = (q + \xi P)^2 = 2\xi q \cdot P - Q^2 = \left(\frac{\xi}{x} - 1\right) Q^2$$

Deep-inelastic scattering (DIS) at LO $\hat{s} = 0 \rightarrow \xi = x$ $\xi = \left(1 + \frac{\hat{s}}{Q^2}\right) x$



One parton

Quark parton model (QPM) → scaling variable x: momentum fraction of struck parton in proton

$$x = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

$$0 \leq (x, y) \leq 1$$

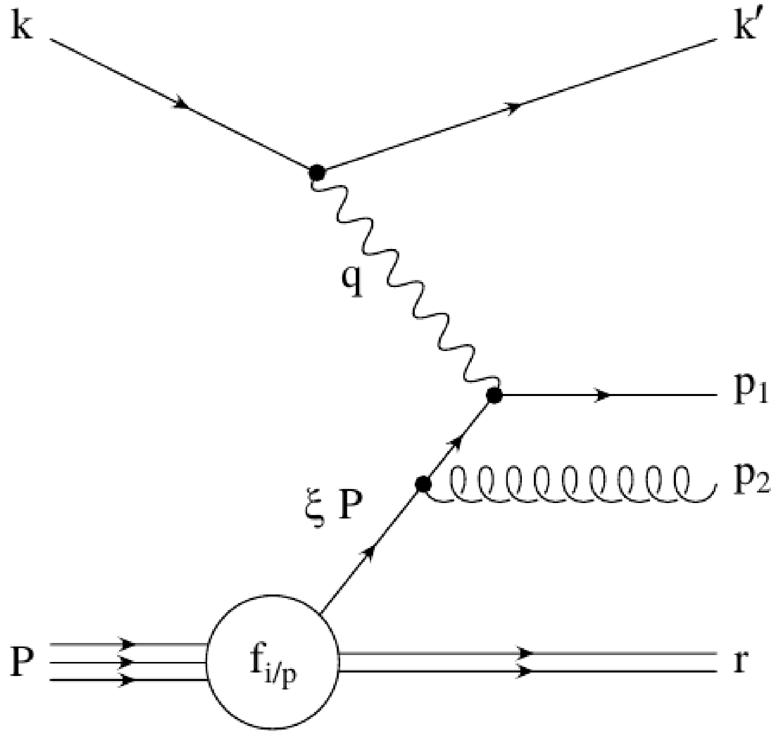
Infinite momentum frame

Proton equals collinear stream of fast moving “partons”, masses negligible
→ factorised incoherent partonic scatter

$$\hat{s} = (q + \xi P)^2 = 2\xi q \cdot P - Q^2 = \left(\frac{\xi}{x} - 1\right) Q^2$$

Deep-inelastic scattering (DIS) at NLO

$$x \leq \xi \leq 1 \qquad \xi = \left(1 + \frac{\hat{s}}{Q^2}\right) x$$



More partons

$$x = \frac{Q^2}{2P \cdot q}$$

$$y = \frac{P \cdot q}{P \cdot k}$$

$$0 \leq (x, y) \leq 1$$

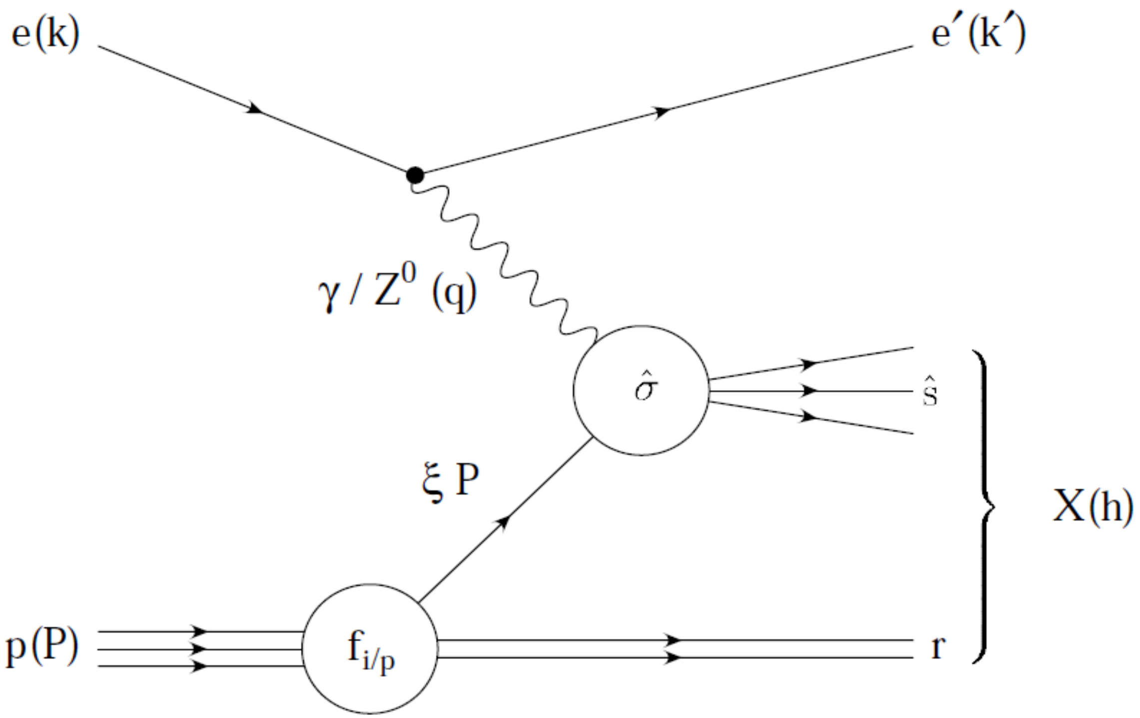
Target rest frame

Inelasticity y is relative energy loss of electron in target rest frame:

$$y = \frac{E_e - E_{e'}}{E_e}$$

Only two of the four invariant variables are independent!

Deep-inelastic scattering (DIS)

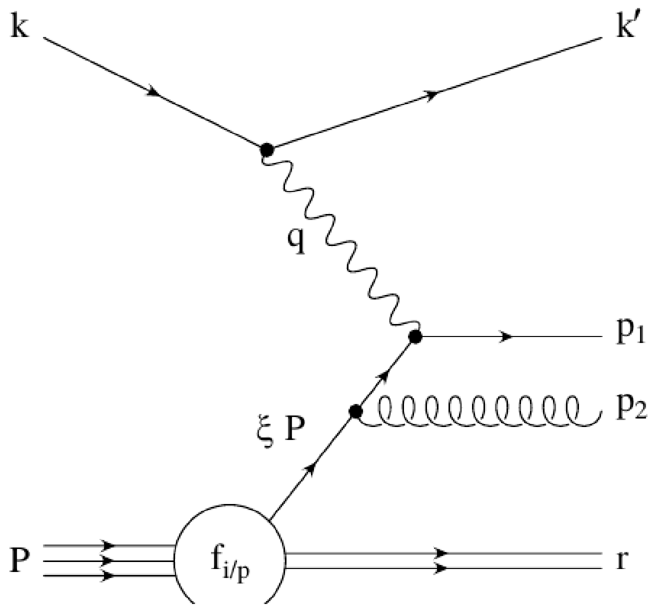


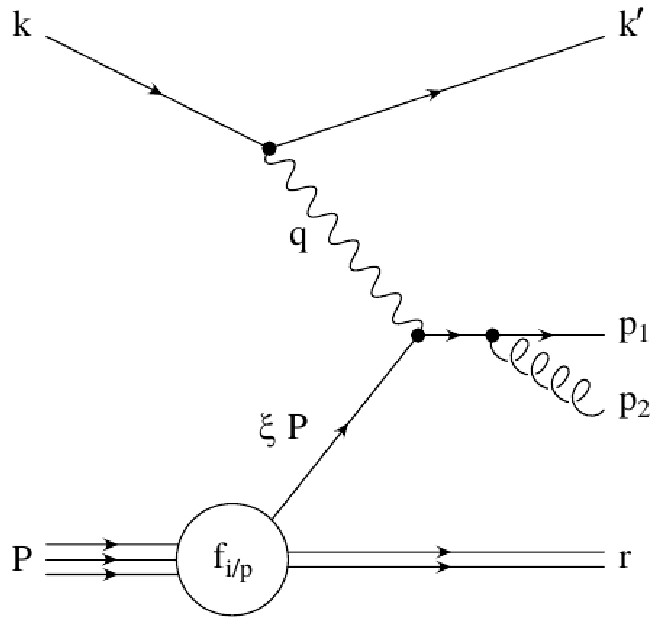
Conversion formulae

$$x = \frac{Q^2}{Q^2 + W^2} \qquad y = \frac{Q^2 + W^2}{s}$$

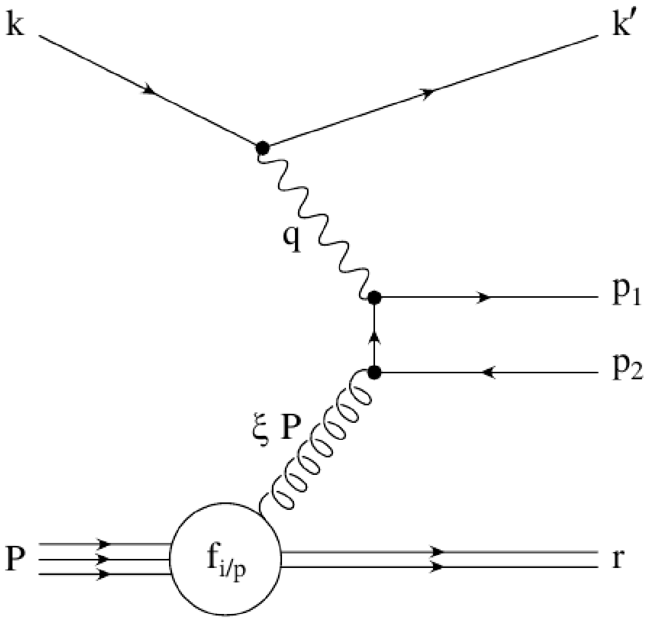
$$Q^2 = sxy \qquad W^2 = s(1-x)y$$

QCD-Compton

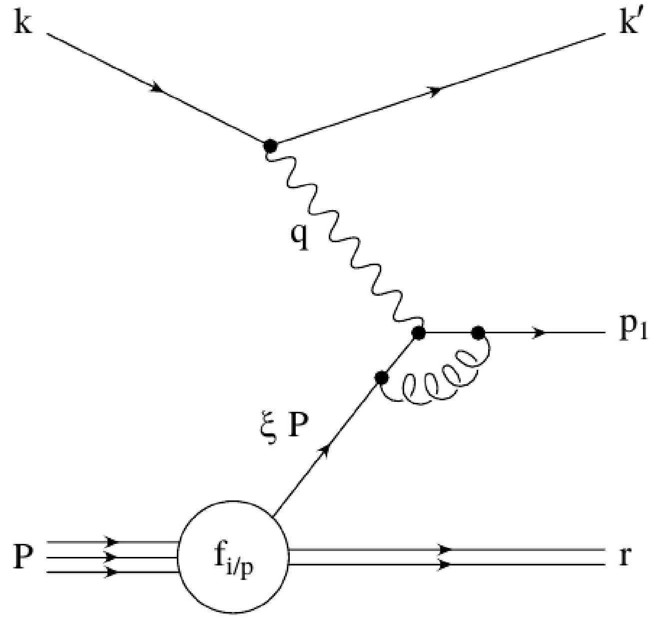




Boson-gluon fusion

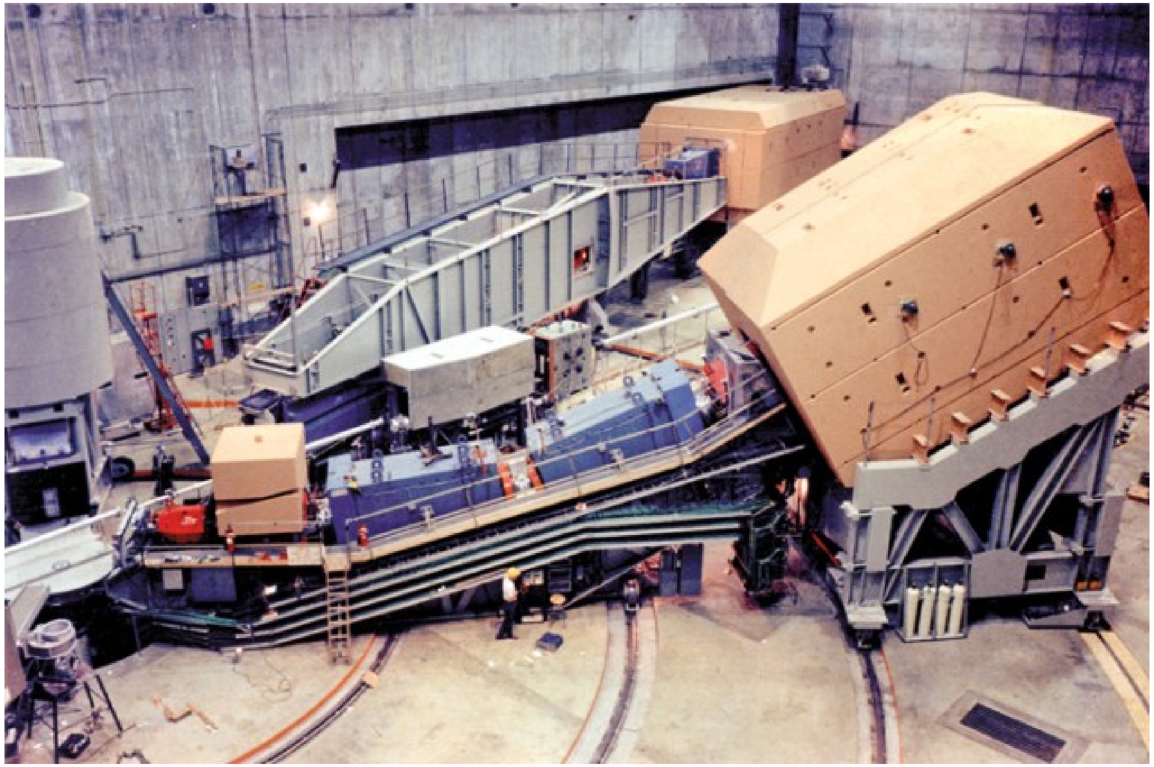


Virtual correction



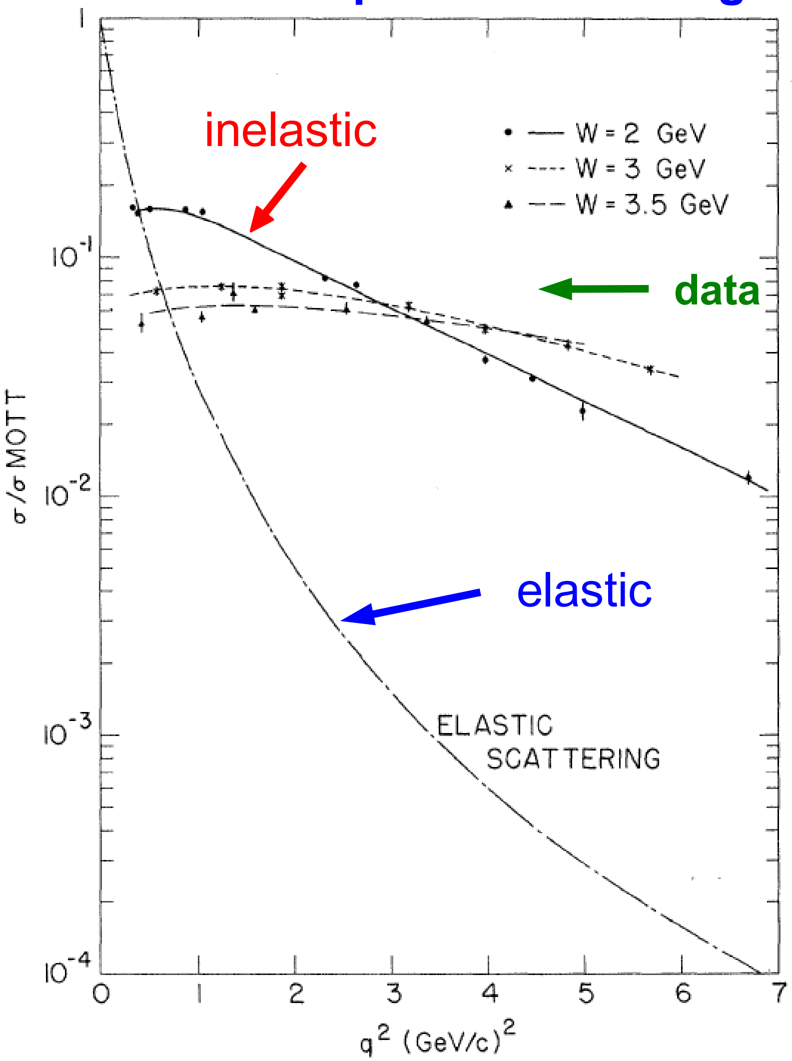
Scale invariance

- Inelastic >> elastic cross section ~ $G^2_{E,M}$ * Mott ~ $1/Q^8$ * Mott
- Inelastic cross section ~ const. * Mott
  - approximately independent of resolution ~ q^2
  - scale invariant, i.e. no natural length scale
  - like scattering at point-like objects
  - proton broken up → $W > M_p$



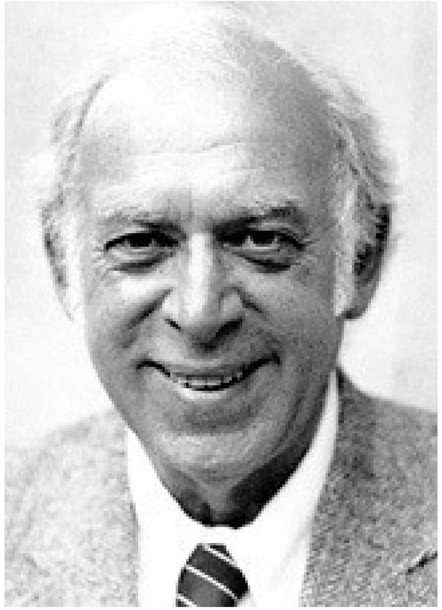
SLAC.

electron-proton scattering

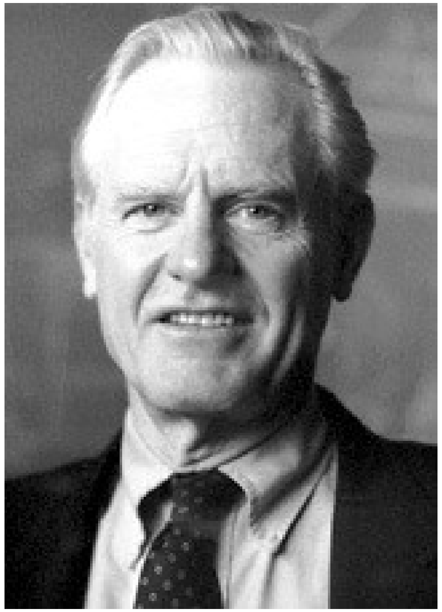


PRL 23 (1969) 935.

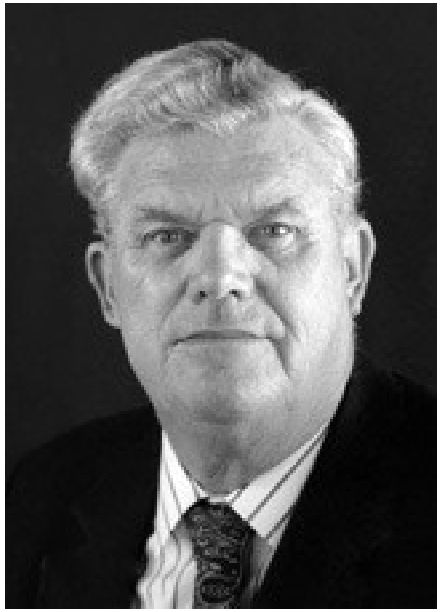
Nobel prize 1990: DIS



J.I. Friedman



H.W. Kendall



R.E. Taylor

nobelprize.org

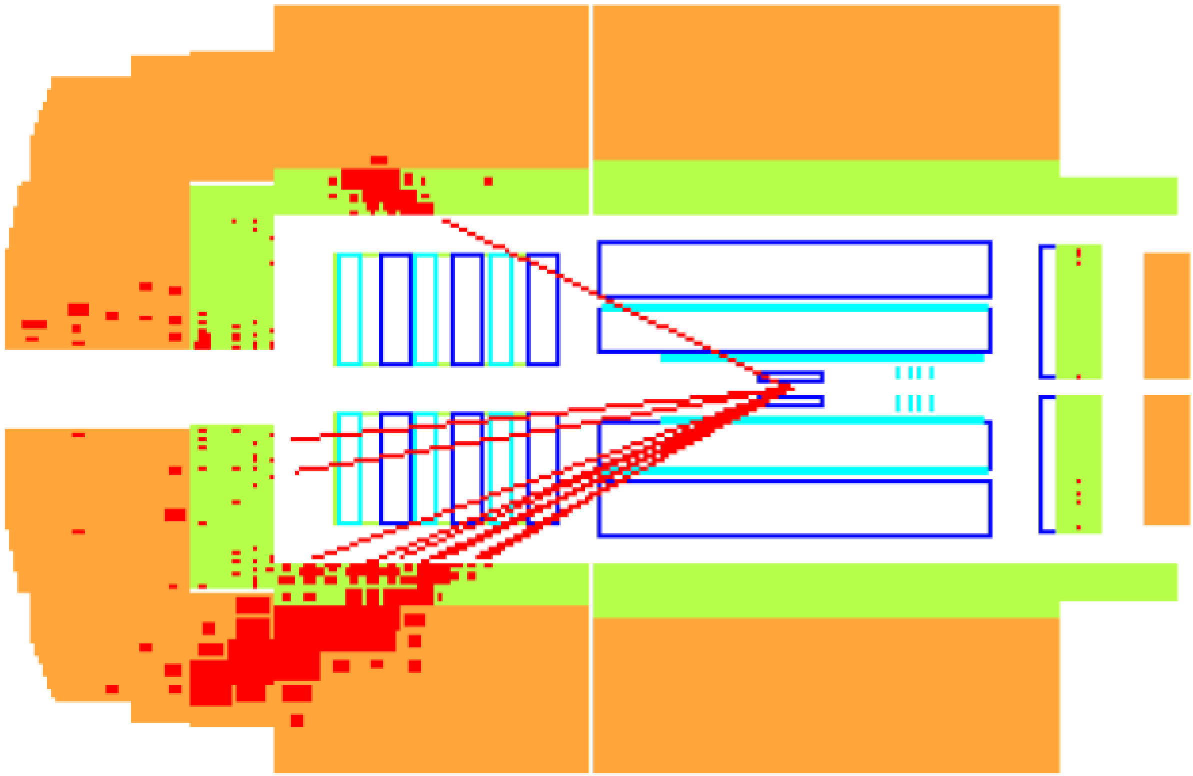
HERA Collider at DESY

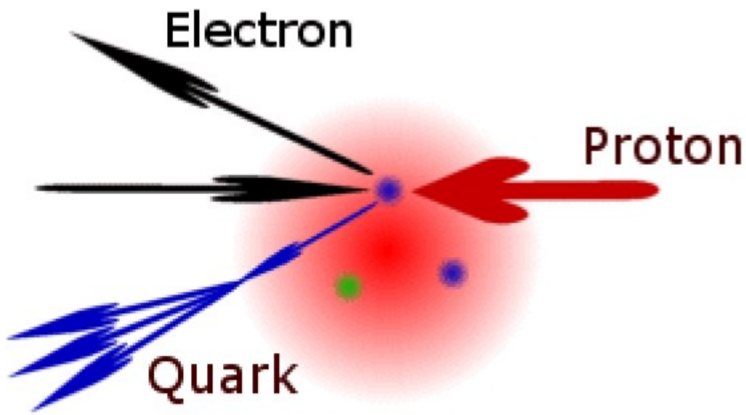
HERA: 1992 – 2007
Collisions of e^+-p, e^--p
HERA II: E_{cms} = 319 GeV

Electromagnetic reaction:

Backscattering of electron off charged proton constituent

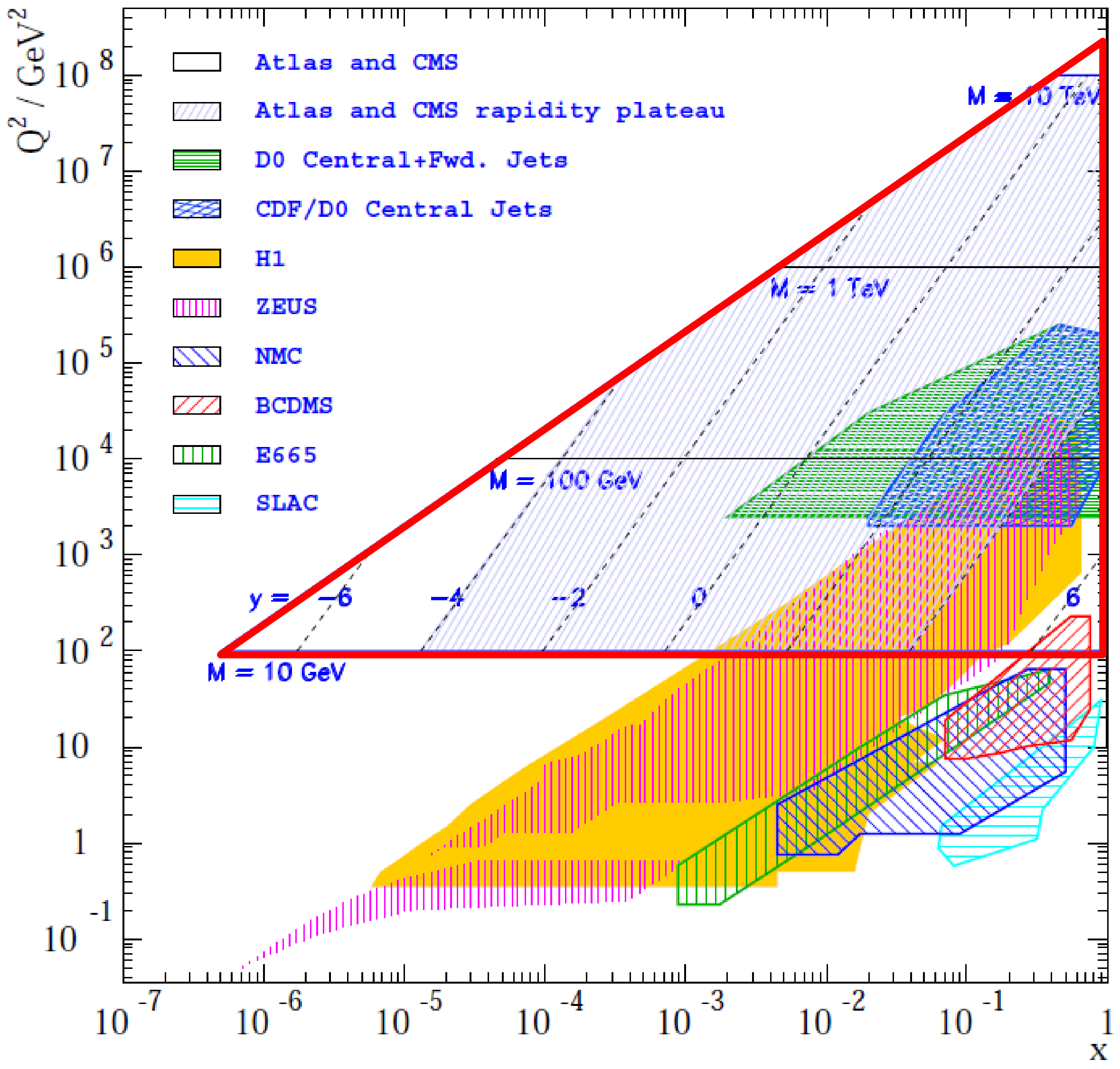
H1 Detector



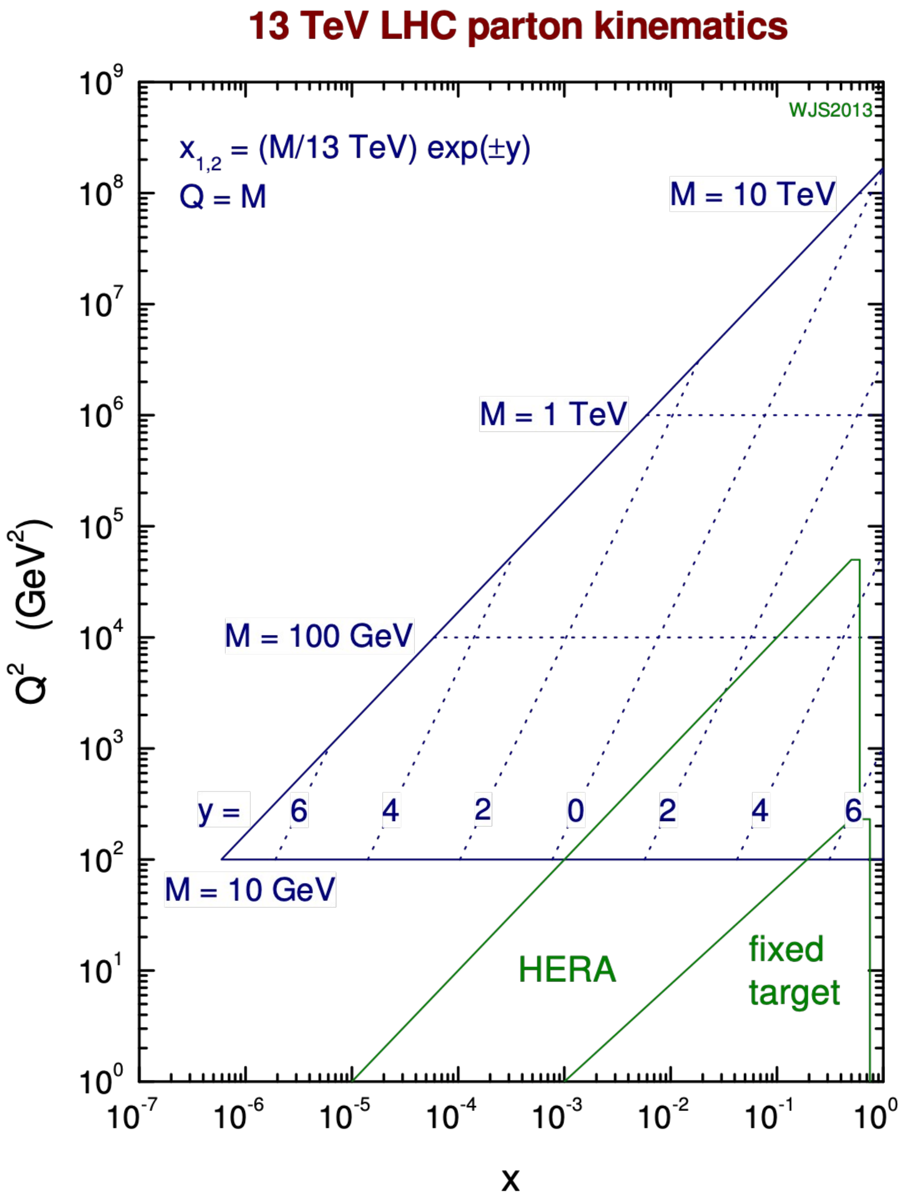


H1 Event Tutorial, J Meyer, DESY (2005)

Phase space in x and Q^2



S. Glazov, Braz.J.Ph. 37 (2007) 793.



W.J. Stirling

DIS cross section

Rosenbluth formula can be rewritten to include inelastic scattering.

Most general Lorentz-invariant and parity conserving expression:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right] \qquad Q^2 \gg M_p^2 y^2$$

$F_1(x,Q^2)$ and $F_2(x,Q^2)$ are structure functions incorporating the form factors (and kinematic ones, τ), but cannot be related to Fourier transforms any more since dependent on x.

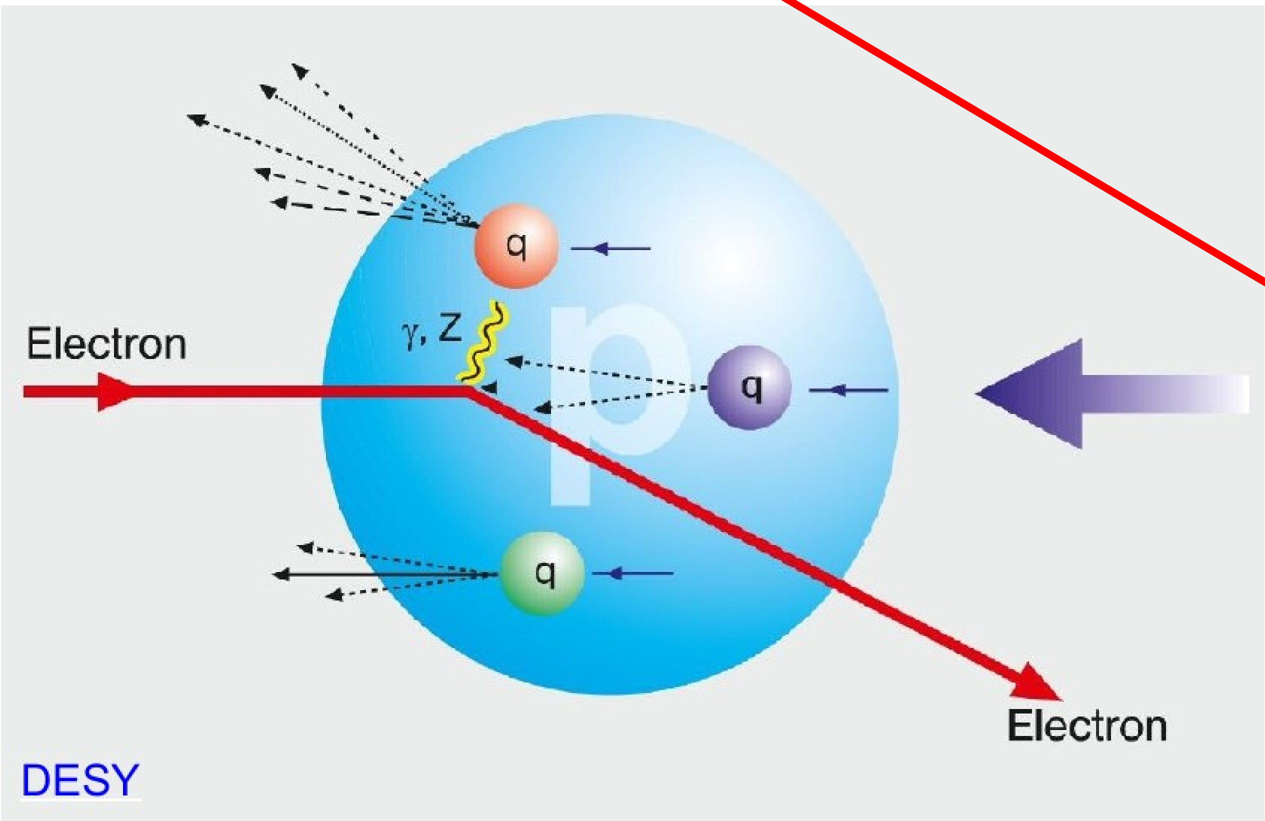
Still, $F_1(x,Q^2)$ is of purely magnetic origin, while $F_2(x,Q^2)$ originates from both, electric and magnetic effects.

What do they mean?

Quark-Parton-Model

J.D. Bjorken, R.P. Feynman 1969:

- **Infinite momentum frame**
  - incoherent superposition of elastic scatterings with point-like “partons”
  - scale invariant, i.e. independent of resolution ~ q^2, no natural length scale
  - partons have spin 1/2



Bjorken scaling:

$$F_1(x, Q^2) \rightarrow F_1(x)$$
$$F_2(x, Q^2) \rightarrow F_2(x)$$

Callan-Gross relation:

$$F_1(x) = \frac{F_2(x)}{2x}$$

Spin 0 would give:

$$F_1(x) = 0$$

Quark-Parton-Model

Modern writing:

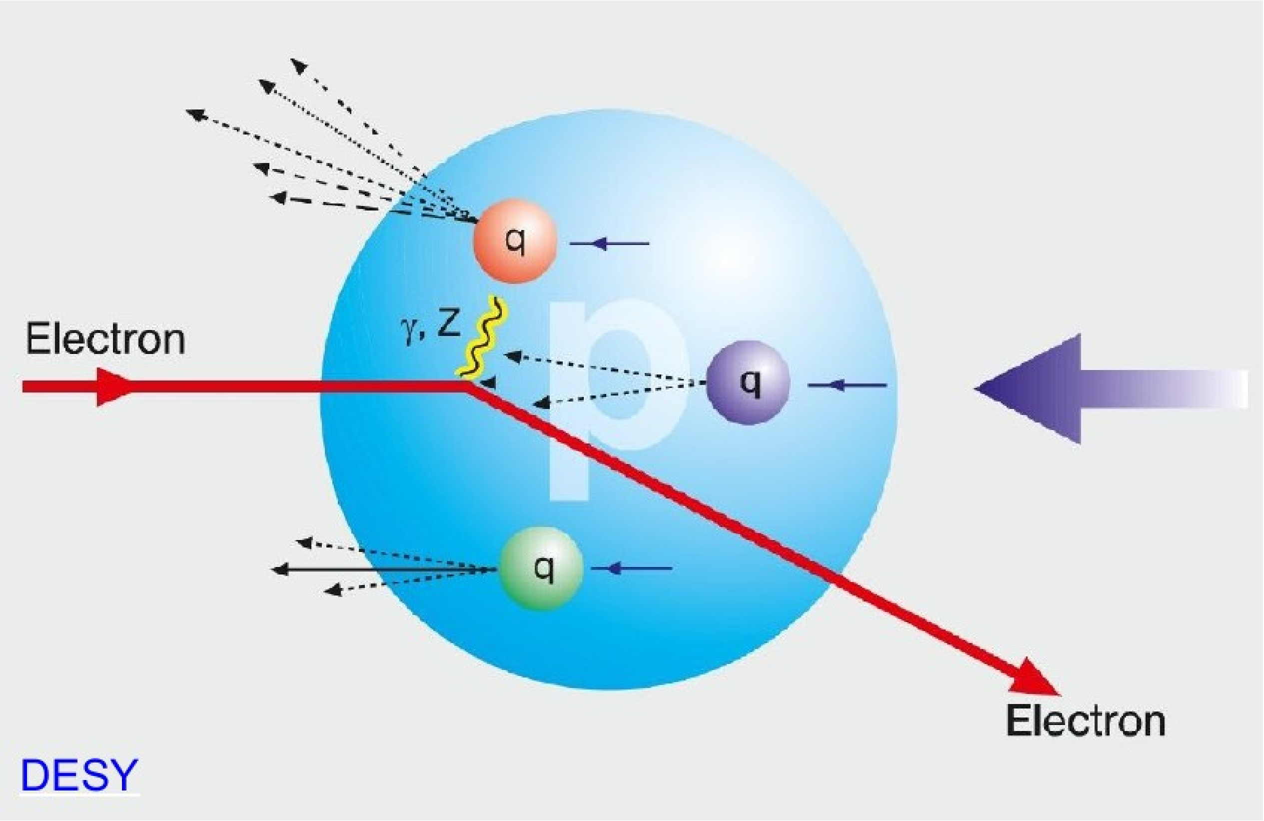
$$F_2(x) = x \sum_i e_i^2 \left[q_i(x) + \bar{q}(x)\right]$$

quark charges

anti-quark momentum distribution

quark momentum distribution

q_i : parton distribution functions (PDFs)



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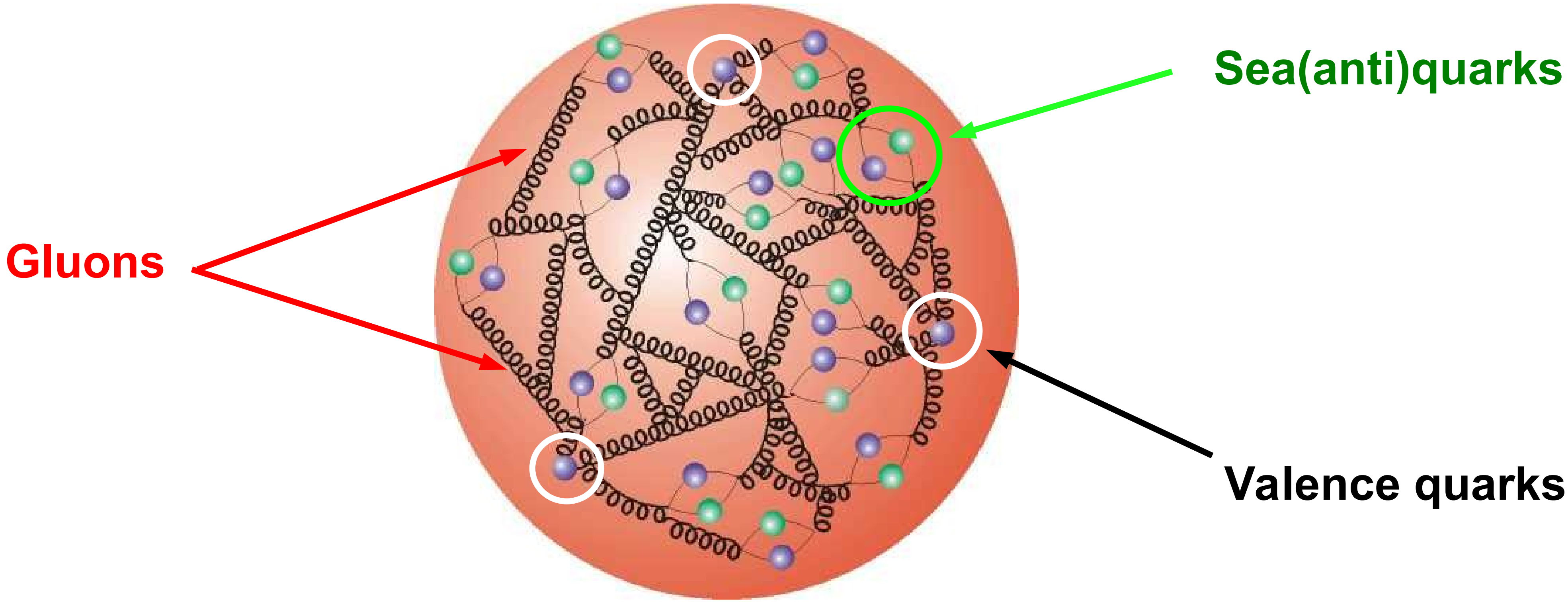
Proton structure



- Example of generic functional form of proton structure:

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$$

Normalisation — Behaviour for x → 0 — Behaviour for x → 1 — Middle region largest variability

Summary

- The new world average from PDG on $\alpha_s(M^2_Z)$ is 0.1179 ± 0.0010
- Elastic scattering of electrons on protons:
  - "Simplest" elastic approximation: Rutherford scattering
    - Has one independent variable, e.g. θ or Q^2
  - Mott scattering includes energy transfer from electron to nucleon and spin-1/2
  - Rosenbluth formula introduces internal structure of the proton; electric and magnetic form factors
- Inelastic scattering requires two independent variables, e.g. scaling variable x and momentum transfer squared Q^2
- Can be converted to other combination with inelasticity y and hadron final state mass squared W^2
- Lorentz-invariant description involves structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$

$$F_1(x) = \frac{F_2(x)}{2x}$$

- Depend mostly on x, not Q^2: Spin-1/2 partons lead to Callan-Gross relation