Exercise sheet 1. Theoretical Nanooptics. WS 2020/2021

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In order for the exercise sheet to count towards obtaining the course credits, you must send your solutions to me by email (ivan.fernandez-corbaton@kit.edu) by the 10th of December. Make sure that the contents of all the scanned/photographed pages are clearly visible.

1 The $\sqrt{2}\mathbf{G}_\pm = \mathbf{E} \pm i\mathbf{H}$ fields

1.1 True or false?

State whether the following statements are true or false and write a short reasoning for your answer.

a) There is no difference in the physical content between Maxwell equations written with $\mathbf{E}$ and $\mathbf{H}$, and Maxwell equations written with $\mathbf{G}_\pm$.

b) There is no obvious advantage of writing Maxwell equations using $\mathbf{G}_\pm$.

c) In the definition $\mathbf{G}_\pm(\mathbf{r},t) = \frac{\mathbf{E}(\mathbf{r},t)\pm i\mathbf{H}(\mathbf{r},t)}{\sqrt{2}}$, it does not matter whether $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{H}(\mathbf{r},t)$ are real or complex fields. In both cases the $\mathbf{G}_\pm(\mathbf{r},t)$ splits the two handedness of the electromagnetic field.

2 One-sided vs. two-sided Fourier transforms

Consider a **real** field $\boldsymbol{\mathcal{X}}(\mathbf{r},t) \in \mathbb{R}^3\ \forall(\mathbf{r},t)$:

a) Write down the expression for its Fourier transform $\mathbf{X}(\mathbf{r},\omega)$ using a range of $\omega \in [-\infty,\infty]$.

b) Does $\mathbf{X}(\mathbf{r},\omega)$ contain all the information from $\boldsymbol{\mathcal{X}}(\mathbf{r},t)$? Why?

c) Show that, with such definition of the Fourier transform, the following identity is met $\mathbf{X}(\mathbf{r},\omega) = [\mathbf{X}(\mathbf{r},-\omega)]^*$, where * means complex conjugation. Discuss what the identity means regarding the information contained in $\mathbf{X}(\mathbf{r},\omega)$ for $\omega \in [-\infty,\infty]$.

d) Write down an inverse Fourier transform that uses $\mathbf{X}(\mathbf{r},\omega) \in (0,\infty]$ to produce a $(\mathbf{r},t)$-dependent field $\mathbf{X}(\mathbf{r},t)$.

e) Can $\mathbf{X}(\mathbf{r},t)$ be real? Why?

f) Discuss the possible differences between the information contained in $\boldsymbol{\mathcal{X}}(\mathbf{r},t)$ and $\mathbf{X}(\mathbf{r},t)$

g) Give at least two reasons why discarding the $\omega = 0$ point is appropriate

h) In the lectures, we use the one-sided Fourier transform with $\omega \in (0,\infty]$. Can such Fourier transform be used to represent a general $(\mathbf{r},t)$-dependent complex field? Why?

3 The helicity of a plane-wave

Consider the expression for a plane-wave of well-defined helicity $\lambda = \pm 1$ and wavevector $\mathbf{k}$:

$$|\mathbf{k}\ \lambda = \pm 1\rangle \equiv \hat{\mathbf{e}}_\pm(\hat{\mathbf{k}}) \exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r} - \mathrm{i}\omega t) = \begin{bmatrix} \mathrm{i}\sin\phi - \lambda\cos\phi\cos\theta \\ -\mathrm{i}\cos\phi - \lambda\sin\phi\cos\theta \\ \lambda\sin\theta \end{bmatrix} \exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r} - \mathrm{i}kt), \quad (1)$$

where $k = \omega = +\sqrt{\mathbf{k}\cdot\mathbf{k}}$, $\phi = \arctan(k_y, k_x)$, $\theta = \arccos(k_z/k)$, $\hat{\mathbf{k}}$ is the unit vector in the direction of $\mathbf{k}$, and the $\hat{\mathbf{e}}_\pm(\hat{\mathbf{k}})$ are polarization vectors corresponding to the two handedness.

a) Write the xyz coordinates of the wavevector as a function of ω, θ, and ϕ.

b) Show that the plane-wave is transverse, that is, that its polarization is perpendicular to its wavevector.

c) Show that the plane-wave is an eigenstate of the $\frac{\nabla\times}{\omega}$ operator with eigenvalue λ.