Theoretical Optics

Exercise 2

C. Rockstuhl, M. Paszkiewicz, N. Perdana, M. Vavilin

Institute of Theoretical Solid State Physics

Karlsruhe Institute of Technology

___/24 points

Drop point: Your tutorial group in ILIAS

Due Date: May 19th 2022, 16:00

Problem 1. (13 points) Pulse propagation in dispersive media

A Gaussian pulse with spectrum $U(z=0,\omega) = \frac{Tu_0}{\sqrt{2\pi}} e^{-\frac{T^2(\omega-\omega_0)^2}{2}}$, where u_0 is the pulse amplitude, T is the pulse duration and ω_0 the carrier frequency, propagates through a piece of glass with a thickness of $\Delta z = d$ and a dielectric function $\varepsilon(\omega)$.

(a) Expand the dispersion relation $k(\omega)$ in the neighborhood of $\omega = \omega_0$ up to the quadratic term. (2 points)

(b) Discuss the physical meaning of the three constant coefficients that appear in the expansion. (3 points)

(c) Write down an expression of the spectrum at the output, $U(z=d,\omega)$. (2 points)

(d) Compute the pulse wave form at the output, $u(z=d,t)$. (4 points)

(e) Write down an expression of the reformed pulse duration at the output. (1 point)

(f) Discuss the dependency of the term $\frac{\mathrm{d}^2k}{\mathrm{d}\omega^2}\big|_{\omega=\omega_0}$ and the distance d for the reformed pulse duration. (1 point)

(*Hint* 1: Use the answer for question 1(a) in the other questions.)
(*Hint* 2: Use the formula $\int_{-\infty}^{\infty} e^{-AX^2+BX}\mathrm{d}X = \sqrt{\frac{\pi}{A}} e^{\frac{B^2}{4A}}$ for $\mathrm{Re}[A] > 0$.)

Problem 2. (11 points) Poynting Vector and Normal Mode

Consider a monochromatic plane wave of frequency ω_0, propagating in a homogeneous isotropic weakly lossy dielectric medium of relative permittivity $\varepsilon = \varepsilon' + i\varepsilon''$ (where $\varepsilon', \varepsilon'' > 0$ and $\varepsilon' >> \varepsilon''$). Its electric field has the form $\mathbf{E}_r(\mathbf{r},t) = E_0\, \mathbf{e}_x\, e^{-\alpha z}\, \cos(\beta z - \omega_0 t + \phi)$, where the subscript r is used for the real valued fields.

(a) Even though the field is given here as a real valued quantity, in some of the following tasks, it might be better to use the complex representation as a mean to simplify your calculations. Therefore, write at first the same field as above but in complex notation. (1 point)

(b) By starting from the dispersion relation of the plane wave in the medium, show that

$$\beta \approx \frac{\omega_0}{c}\sqrt{\epsilon'} \quad \text{and} \quad \alpha \approx \frac{\omega_0}{c}\frac{\varepsilon''}{2\sqrt{\varepsilon'}}$$

Useful formula: $\sqrt{1+z} \approx 1 + \frac{1}{2}z, \quad z \in \mathbb{C}, \quad |z| \ll 1.$ (2 points)

(c) Start from Maxwell's equations to find the real valued magnetic field $\mathbf{H}_r(\mathbf{r},t)$. (4 points)

(d) Continue to use the real valued representation of the field to write down the formula for the instantaneous Poynting vector, $\mathbf{S}_r(\mathbf{r},t)$. (1 point)

(e) Find the time averaged Poynting vector using the formula $\langle \mathbf{S}_r(\mathbf{r},t)\rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{+T} \mathbf{S}_r(\mathbf{r},t)\mathrm{d}t$.
Useful formula: $\cos(a)\cos(b) = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$, $\sin(a)\cos(b) = \frac{1}{2}[\sin(a-b) + \sin(a+b)]$. (3 points)