Theoretical Optics

Exercise 5

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___/24 points

Drop point: Your tutorial group in ILIAS

Due Date: July 7$^{\text{th}}$ 2022, 16:00

Problem 1. (7 points) Holography

Holography involves the recording and reconstruction of optical waves. Consider a monochromatic optical wave, with complex amplitude in some plane at $z = 0$ is $U(x, y)$. We record a wave with a transparency (hologram) with the complex transmittance $t(x, y)$ (see Figure 1a). Optical detectors used to make transparencies (e.g. photographic emulsion) are responsive to the optical intensity $|U(x, y)|^2$ and therefore do not store the information about the phase $\arg\{U(x, y)\}$. A solution to this problem is mixing of the original wave (object wave) U with a known reference wave U_0 and recording their interference pattern in the $z = 0$ plane.

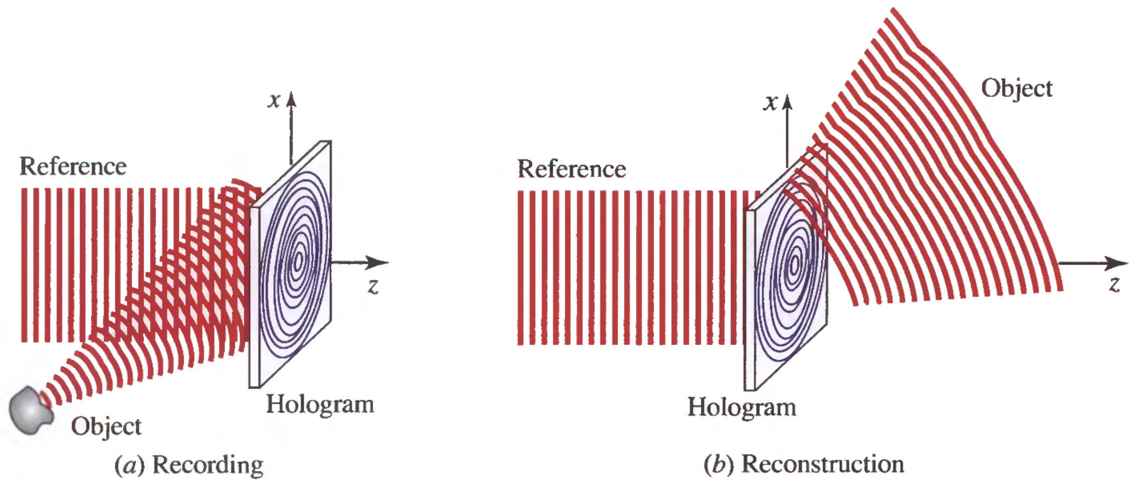

Figure 1: (a) Recording the interference pattern between the object wave and reference wave. (b) Reconstruction of the object wave by illuminating the hologram with a reference wave. Taken from Saleh, B.E. and Teich, M.C., 2007. Fundamentals of photonics. john Wiley & sons.

(a) Show how the information about the amplitude and phase differences between the reference wave and the object wave can be recorded in the transparency. *Hint: The transmittance t is proportional to the intensity* $|U + U_0|^2$. (2 points)

(b) To reconstruct the object wave, we illuminate the hologram with the reference wave U (Figure 1b). Write the amplitude of resulting wave with a complex amplitude in the hologram plane $z = 0$ in terms of the amplitudes and intensities of the reference and object wave. Explain the meaning of the particular terms in the obtained sum. (2 points)

(c) Assuming that the reference wave is a spherical wave centered about the point $(0, 0, -d)$ and the object wave is a plane wave travelling at an angle θ_x, determine the hologram pattern and the reconstructed wave at $z = 0$. *Hint: Use Fresnel approximation for the spherical wave.* (3 points)

Problem 2. (17 points) Principal axis of anisotropic crystals

Consider an anisotropic crystal characterized in the laboratory coordinate system by the following relative permittivity tensor

$$\hat{\varepsilon} = \begin{pmatrix} a & 0 & 0 \\ 0 & 1.25a & \alpha a \\ 0 & \alpha a & 1.75a \end{pmatrix},$$

where a is some non-zero number that eventually defines the material properties.
Hint: The dielectric functions are the eigenvalues of the permittivity tensor and the principal axes are defined by the corresponding eigenvectors.

(a) Find all possible solutions for α such that the crystal is uniaxial. (5 points)

(b) For α's obtained from the previous question, rewrite the crystal permittivity tensor in the respective principal axes. (3 points)

(c) For the real positive solution of α from the previous question, write the principal axes of the crystal in terms of the laboratory coordinate basis. (7 points)

(d) How much does the crystal need to be rotated with respect to laboratory coordinate such that we have a diagonalized permittivity tensor in the new coordinate system? (2 points)