Theoretical Optics
recorded for the summer term 2021

Prof. Carsten Rockstuhl

Institute of Theoretical Solid State Physics and Institute of Nanotechnology

carsten.rockstuhl@kit.edu

Phone: +49-721-608-46054

Office: 30.23/10-23

course information available at ilias.studium.kit.edu

Structure of the course

1. Peculiarities of Maxwell's equations in the context of optics

2. Diffraction theory

3. Optics in anisotropic media (a.k.a. Crystal optics)

4. Coherence optics

Suggested literature:

“Fundamentals of Photonics” by B. E. A. Saleh and M. C. Teich (excellent overview over the basics)

“Principles of Optics” by E. Wolf (referential but sometimes too complete)
"Theoretical Optics: An Introduction" by H. Römer

"Classical Electrodynamics" by J. D. Jackson

"Introduction to Fourier Optics" by J. W. Goodman (chapter on diffraction)
"Introduction to the Theory of Coherence and Polarization of Light" by E. Wolf (chapter on coherence properties of light)

Basics of Maxwell's equations

Prof. Carsten Rockstuhl

Karlsruhe Institute of Technology

1. Review of Electromagnetism

1.1 Maxwell's equations

1.1.1 Maxwell's equations in optics

Maxwell:

$$\mathbf{rot}\,\mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \qquad \mathbf{div}\,\mathbf{D}(\mathbf{r},t) = \rho_{\mathrm{ext}}(\mathbf{r},t)$$

$$\mathbf{rot}\,\mathbf{H}(\mathbf{r},t) = \mathbf{j}_{\mathrm{makr}}(\mathbf{r},t) + \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} \qquad \mathbf{div}\,\mathbf{B}(\mathbf{r},t) = 0$$

Constitutive relations:

$$\mathbf{D}(\mathbf{r},t) = \varepsilon_0\,\mathbf{E}(\mathbf{r},t) + \mathbf{P}(\mathbf{r},t)$$

$$\mathbf{H}(\mathbf{r},t) = \frac{1}{\mu_0}\left[\,\mathbf{B}(\mathbf{r},t) - \mathbf{M}(\mathbf{r},t)\right]$$

Impact of matter:

$\mathbf{P}(\mathbf{r},t) = f[\mathbf{E}(\mathbf{r},t)]$ and $\mathbf{M}(\mathbf{r},t) = f[\mathbf{B}(\mathbf{r},t)]$

Sources of electromagnetic radiation:

- free charge density $\rho_{\text{ext}}(\mathbf{r},t)$
- macroscopic current density: $\mathbf{j}_{\text{makr}}(\mathbf{r},t) = \mathbf{j}_{\text{cond}}(\mathbf{r},t) + \mathbf{j}_{\text{conv}}(\mathbf{r},t)$
- conductive current density: $\mathbf{j}_{\text{cond}}(\mathbf{r},t) = f[\mathbf{E}(\mathbf{r},t)]$
- convective current density: $\mathbf{j}_{\text{conv}}(\mathbf{r},t) = \rho_{\text{ext}}(\mathbf{r},t)\mathbf{v}$

Assumption: $\mathbf{M}(\mathbf{r},t) = 0$ $\quad \rho_{\text{ext}}(\mathbf{r},t) = 0 \rightarrow \mathbf{j}_{\text{conv}}(\mathbf{r},t) = 0 \rightarrow \mathbf{j}_{\text{cond}}(\mathbf{r},t) \equiv \mathbf{j}(\mathbf{r},t)$

Maxwell in optics

$$\mathbf{rot}\,\mathbf{E}(\mathbf{r},t) = -\mu_0 \frac{\partial \mathbf{H}(\mathbf{r},t)}{\partial t} \qquad \varepsilon_0 \mathbf{div}\,\mathbf{E}(\mathbf{r},t) = -\mathbf{div}\,\mathbf{P}(\mathbf{r},t)$$

$$\mathbf{rot}\,\mathbf{H}(\mathbf{r},t) = \mathbf{j}(\mathbf{r},t) + \frac{\partial \mathbf{P}(\mathbf{r},t)}{\partial t} + \varepsilon_0 \frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t} \qquad \mathbf{div}\,\mathbf{H}(\mathbf{r},t) = 0$$

task in optics: given $\mathbf{P}(\mathbf{E})$ and $\mathbf{j}(\mathbf{E})$ $\rightarrow$ solve Maxwell consistent

1.1.2 Temporal dependencies of the fields

A) monochromatic, stationary fields

- $\mathrm{e}^{-i\omega_0 t}$

  → fixed phase relation

  → perfect coherence

  → infinite wave train

$$\mathbf{E}(\mathbf{r},t) = \bar{\mathbf{E}}(\mathbf{r},\omega_0)\mathrm{e}^{-i\omega_0 t}$$

B) polychromatic, non-stationary fields

- finite or an infinite number of different frequencies
- fully coherent
- Fourier decomposition

$$\mathbf{E}(\mathbf{r},t) = \int_{-\infty}^{\infty} \bar{\mathbf{E}}(\mathbf{r},\omega)\mathrm{e}^{-i\omega t}\, d\omega$$

$$\bar{\mathbf{E}}(\mathbf{r},\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{E}(\mathbf{r},t)\mathrm{e}^{i\omega t}\, dt$$

C) partially coherent or incoherent light

- only statements on statistical properties possible
- expressing their evolution
- later part of the lecture

1.1.3 Maxwell's equations in frequency space

Fourier transforming the equations and herewith all quantities

Integration by parts

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dt \left[\frac{\partial}{\partial t}\mathbf{E}(\mathbf{r},t)\right] \mathrm{e}^{i\omega t} = -i\omega \, \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\mathbf{E}(\mathbf{r},t)\mathrm{e}^{i\omega t} = -i\omega\bar{\mathbf{E}}(\mathbf{r},\omega)$$

$$\frac{\partial}{\partial t} \xrightarrow{FT} -i\omega$$

$$\mathbf{rot}\,\bar{\mathbf{E}}(\mathbf{r},\omega) = i\omega\mu_0\bar{\mathbf{H}}(\mathbf{r},\omega) \qquad \varepsilon_0\mathbf{div}\,\bar{\mathbf{E}}(\mathbf{r},\omega) = -\mathbf{div}\,\bar{\mathbf{P}}(\mathbf{r},\omega)$$

$$\mathbf{rot}\,\bar{\mathbf{H}}(\mathbf{r},\omega) = \bar{\mathbf{j}}(\mathbf{r},\omega) - i\omega\bar{\mathbf{P}}(\mathbf{r},\omega) - i\omega\varepsilon_0\bar{\mathbf{E}}(\mathbf{r},\omega) \qquad \mathbf{div}\,\bar{\mathbf{H}}(\mathbf{r},\omega) = 0$$

1.1.4 Wave equation

A) Time domain

$$\mathbf{rot}\,\mathbf{rot}\,\mathbf{E}(\mathbf{r},t) = -\mu_0\mathbf{rot}\frac{\partial\mathbf{H}(\mathbf{r},t)}{\partial t} = -\mu_0\frac{\partial}{\partial t}\left[\mathbf{j}(\mathbf{r},t) + \frac{\partial\mathbf{P}(\mathbf{r},t)}{\partial t} + \varepsilon_0\frac{\partial\mathbf{E}(\mathbf{r},t)}{\partial t}\right]$$

$$\mathbf{rot}\,\mathbf{rot}\,\mathbf{E}(\mathbf{r},t) + \frac{1}{c_0^2}\frac{\partial^2\mathbf{E}(\mathbf{r},t)}{\partial t^2} = -\mu_0\frac{\partial\mathbf{j}(\mathbf{r},t)}{\partial t} - \mu_0\frac{\partial^2\mathbf{P}(\mathbf{r},t)}{\partial t^2}$$

$$\mathbf{div}[\ \varepsilon_0 \mathbf{E}(\mathbf{r},t) + \mathbf{P}(\mathbf{r},t)] = 0$$

magnetic field: $$\frac{\partial \mathbf{H}(\mathbf{r},t)}{\partial t} = -\frac{\mathbf{1}}{\mu_0}\mathbf{rot}\,\mathbf{E}(\mathbf{r},t)$$

B) Frequency domain

$$\mathbf{rot}\,\mathbf{rot}\,\bar{\mathbf{E}}(\mathbf{r},\omega) - \frac{\omega^2}{c_0^2}\ \bar{\mathbf{E}}(\mathbf{r},\omega) = i\omega\mu_0\bar{\mathbf{j}}(\mathbf{r},\omega) + \omega^2\mu_0\bar{\mathbf{P}}(\mathbf{r},\omega)$$

$$\mathbf{div}[\ \varepsilon_0 \bar{\mathbf{E}}(\mathbf{r},\omega) + \bar{\mathbf{P}}(\mathbf{r},\omega)] = 0$$

magnetic field: $$\bar{\mathbf{H}}(\mathbf{r},\omega) = -\frac{\boldsymbol{i}}{\mu_0\omega}\mathbf{rot}\,\bar{\mathbf{E}}(\mathbf{r},\omega)$$

Non-stationary fields and linear media: $\mathbf{E}(\mathbf{r},t) = \int_{-\infty}^{\infty} \bar{\mathbf{E}}(\mathbf{r},\omega)\mathrm{e}^{-i\omega t}\,d\omega$