Theoretical Optics

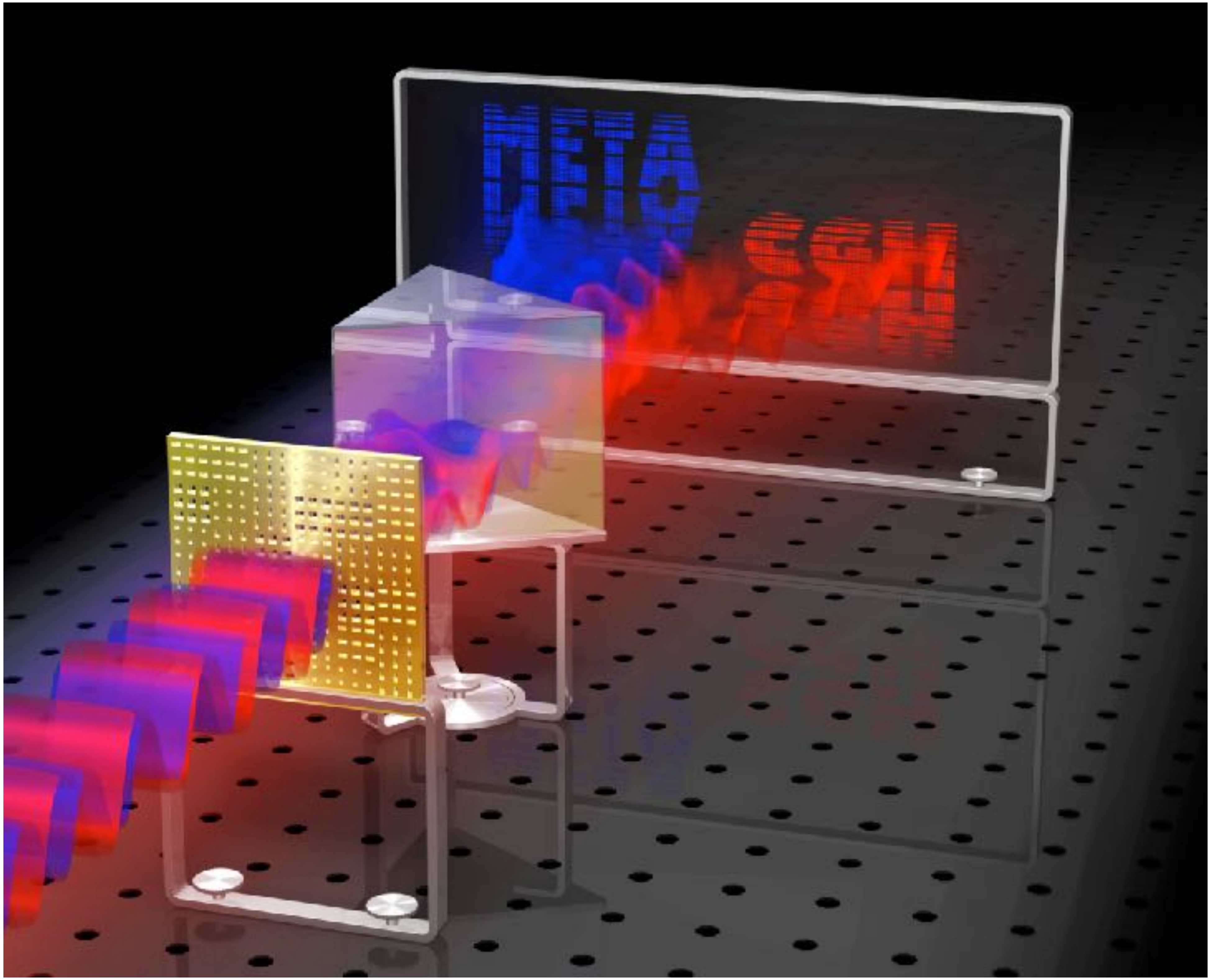
Basics of holography

Prof. Carsten Rockstuhl

Karlsruhe Institute of Technology

Basics of holography

information of an object encoded in amplitude and phase of scattered light

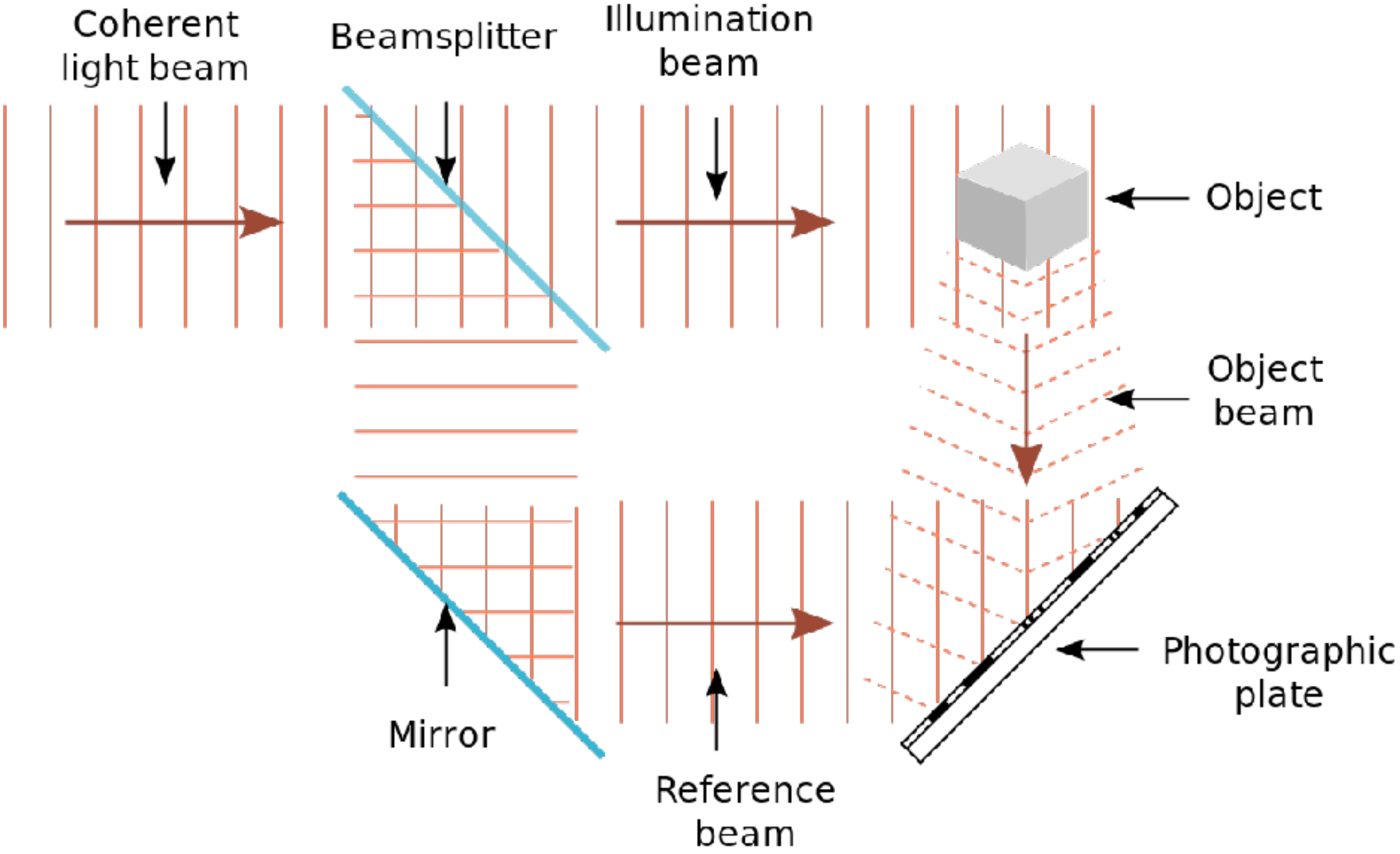


Basics of holography

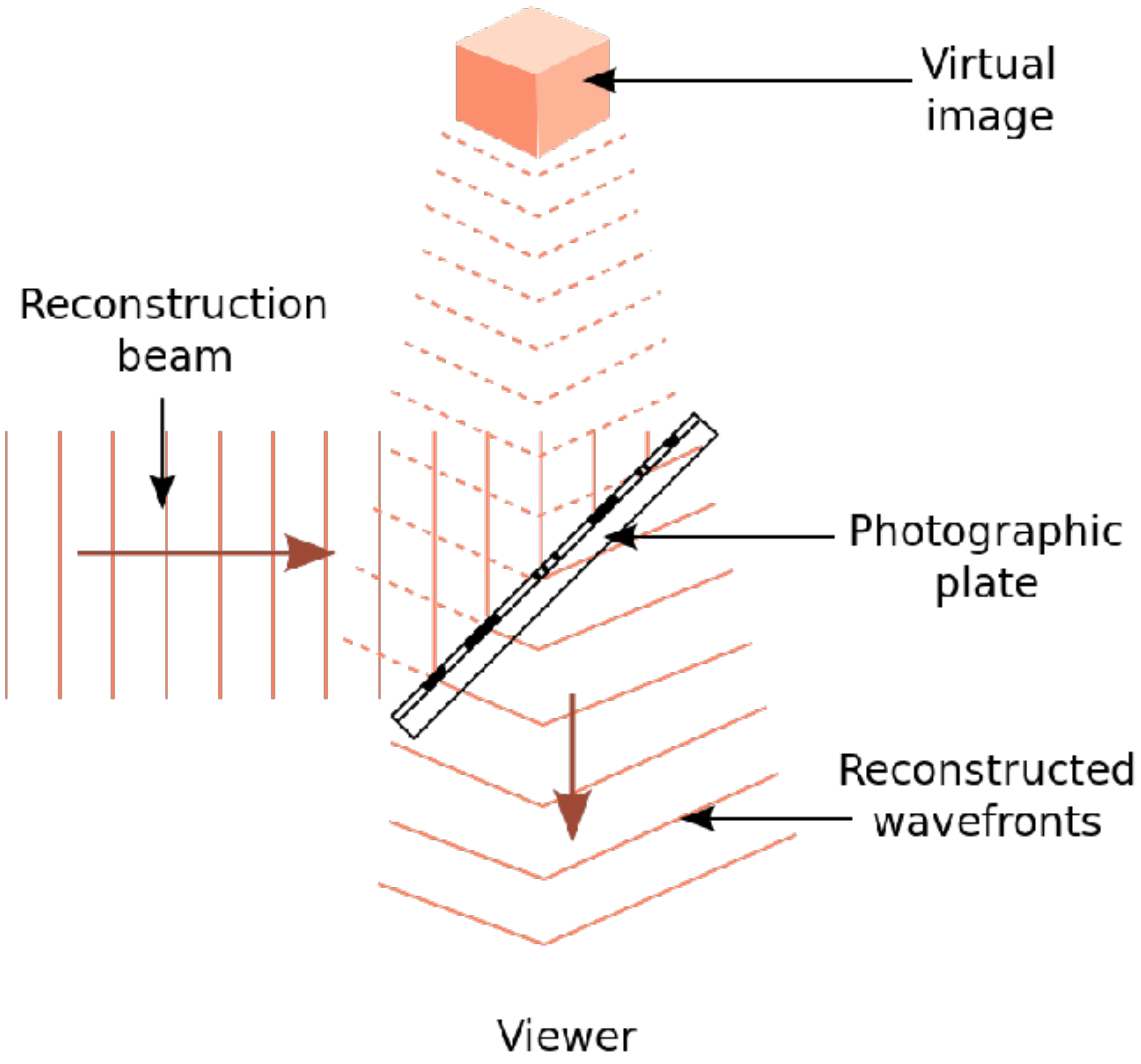
information of an object encoded in amplitude and phase of scattered light

1948 Dennis Gabor
(Nobel prize 1971)

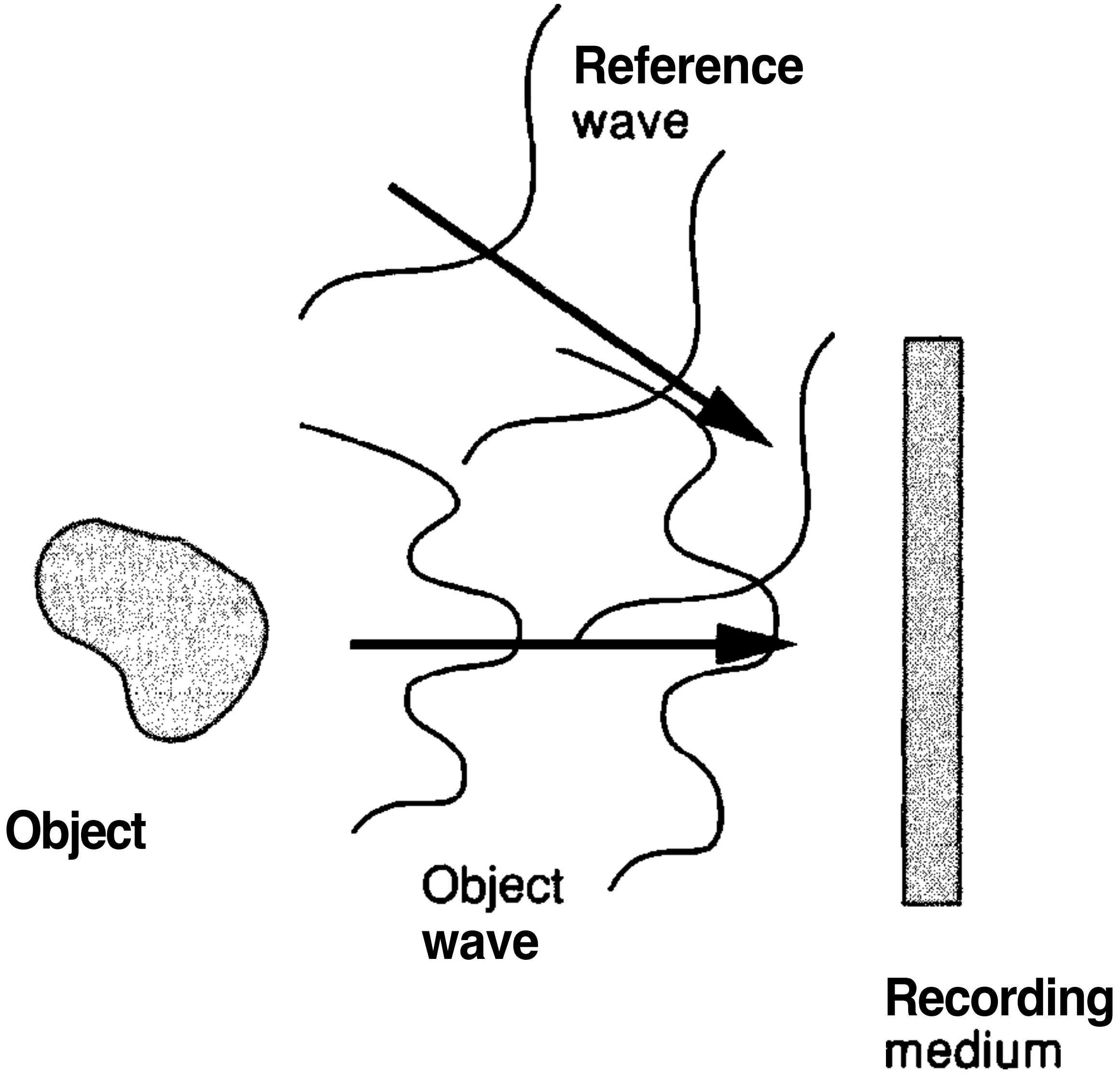
recording


Coherent light beam
Beamsplitter
Illumination beam
Object
Object beam
Photographic plate
Mirror
Reference beam

reconstruction



Recording and reconstruction



object wave $u(\mathbf{r})$

reference wave $u_0(\mathbf{r}) = a_0 e^{i\mathbf{k}_0\cdot\mathbf{r}}$

coordinate of recording medium ξ

intensity in recording plane

$$\boxed{I(\xi) = |u(\xi) + u_0(\xi)|^2 = |a_0|^2 + a_0^* u(\xi) e^{-i\mathbf{k}_0\cdot\xi} + a_0 u^*(\xi) e^{i\mathbf{k}_0\cdot\xi} + |u(\xi)|^2}$$

intensity of reference much larger, last term drops

blackening proportional to local intensity

filter has local transmission function → $t(\xi) = 1 - \gamma I(\xi)$

contrast function

illumination the filter with reconstruction wave: $u_1(\mathbf{r}) = a_1 e^{i\mathbf{k}_1\cdot\mathbf{r}}$

field behind filter: $w(\xi) = t(\xi)u_1(\xi)$

thin element approximation

$$\boxed{w(\xi) \quad = a_1(1-\gamma|a_0|^2)e^{i\mathbf{k}_1\cdot\xi} - \gamma a_1 a_0^* u(\xi)e^{i(\mathbf{k}_1-\mathbf{k}_0)\cdot\xi} - \gamma a_1 a_0 u^*(\xi)e^{i(\mathbf{k}_1+\mathbf{k}_0)\cdot\xi}}$$

discussion of terms on following slide

discussion of terms with $\mathbf{k}_1 = \mathbf{k}_0$

1st term

$$\longrightarrow \quad a_1(1-\gamma|a_0|^2)e^{i\mathbf{k}_1\cdot\xi}$$

plane wave propagates into the direction of reconstruction wave

consequence of imperfection

2nd term

$$\longrightarrow \quad \gamma a_1 a_0^* u(\xi)$$

corresponds up to some amplitude to the object field

3rd term

$$\longrightarrow \quad \gamma a_1 a_0 u^*(\xi)\, e^{i2\mathbf{k}_0\cdot\xi}$$

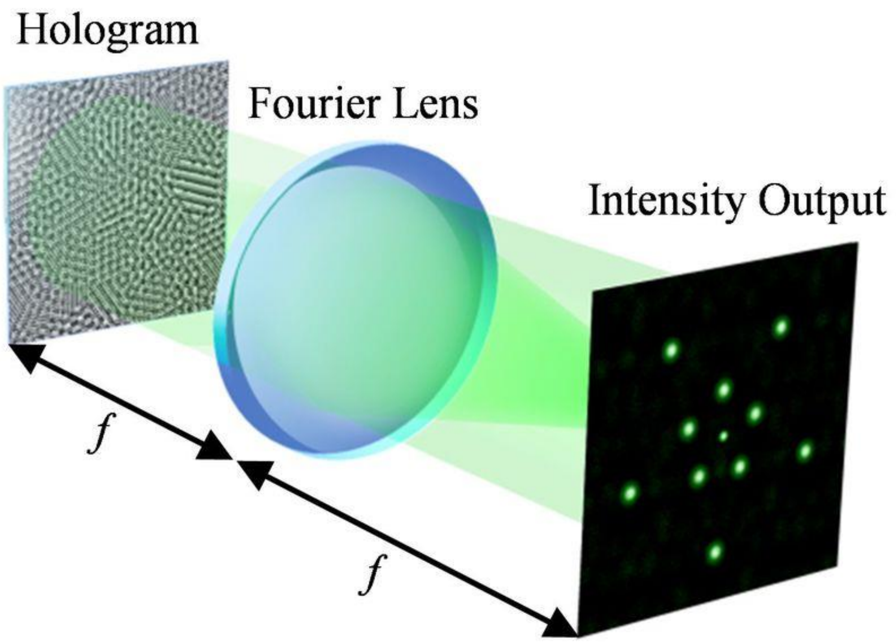
complex conjugated object field

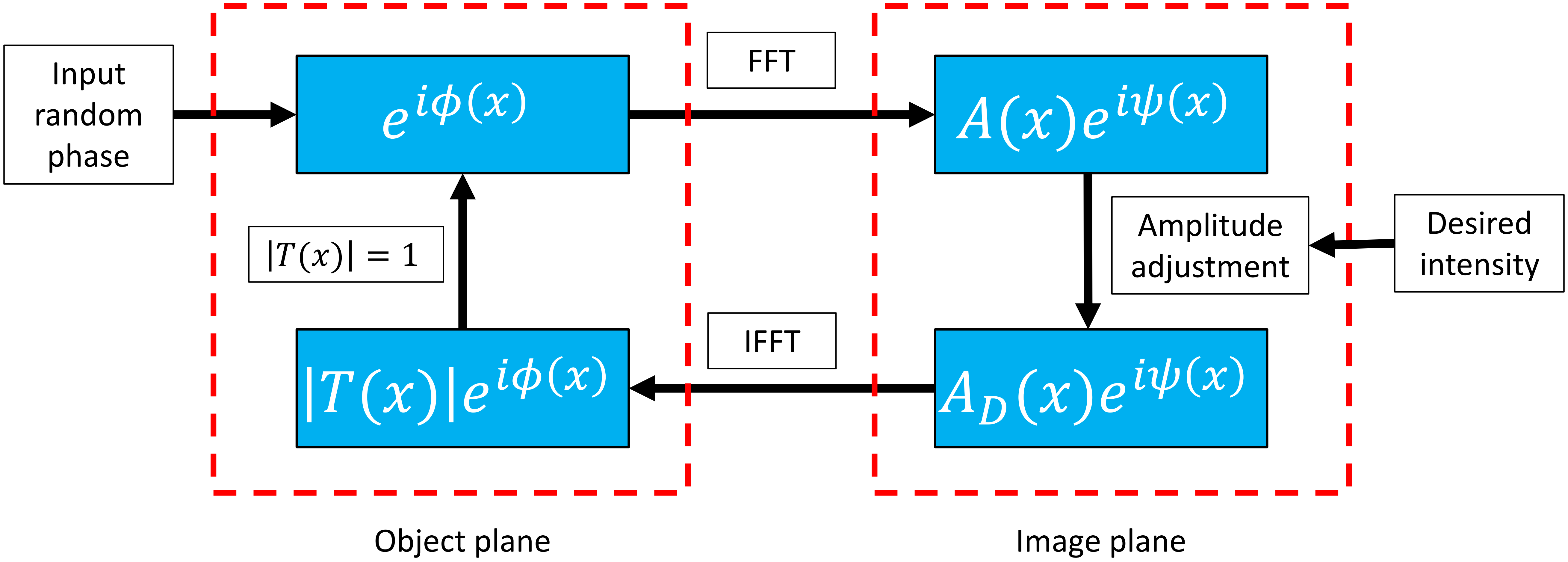
propagates into direction $e^{i2\mathbf{k}_0\cdot\xi}$

application in phase conjugation

Design of holograms

propagation into the far-field corresponds to a Fourier transformation





Input random phase

$e^{i\phi(x)}$

FFT

$A(x)e^{i\psi(x)}$

Amplitude adjustment

Desired intensity

$A_D(x)e^{i\psi(x)}$

IFFT

$|T(x)|e^{i\phi(x)}$

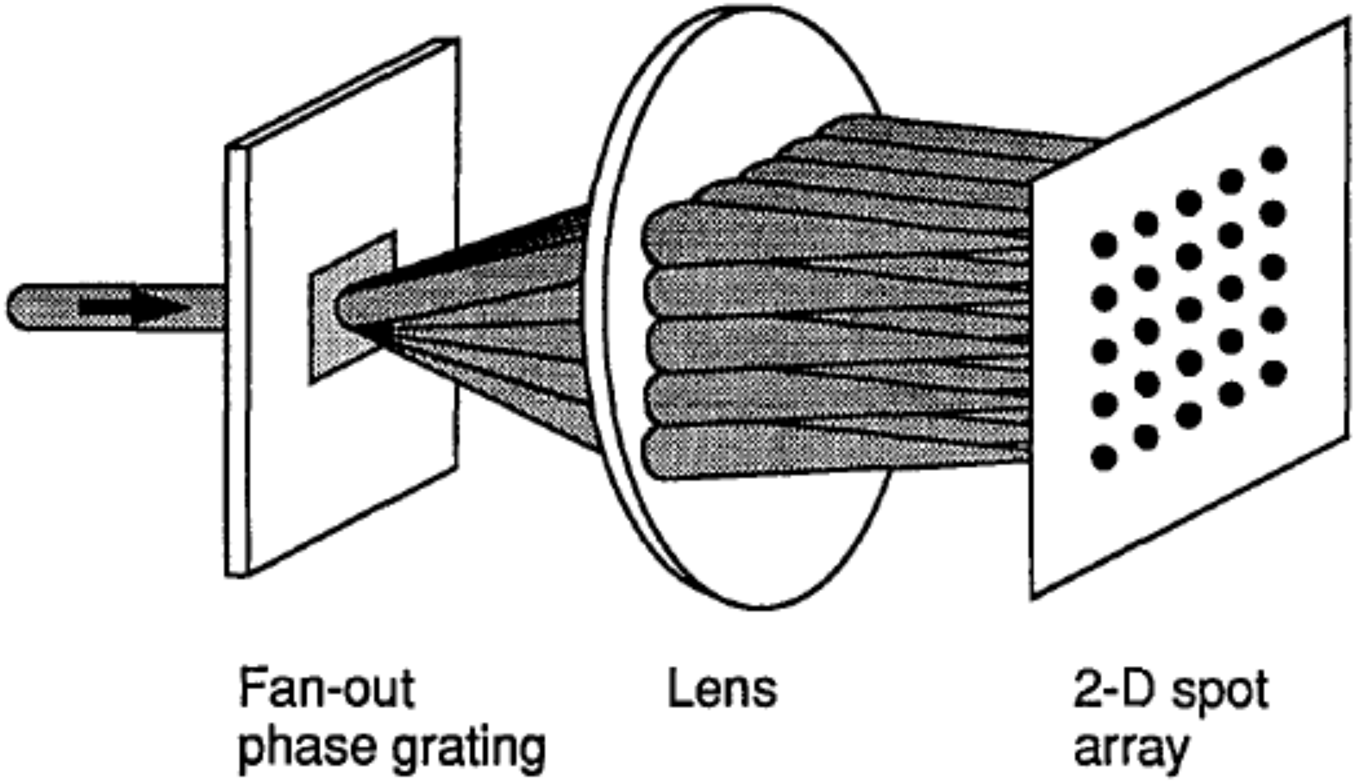
$|T(x)| = 1$

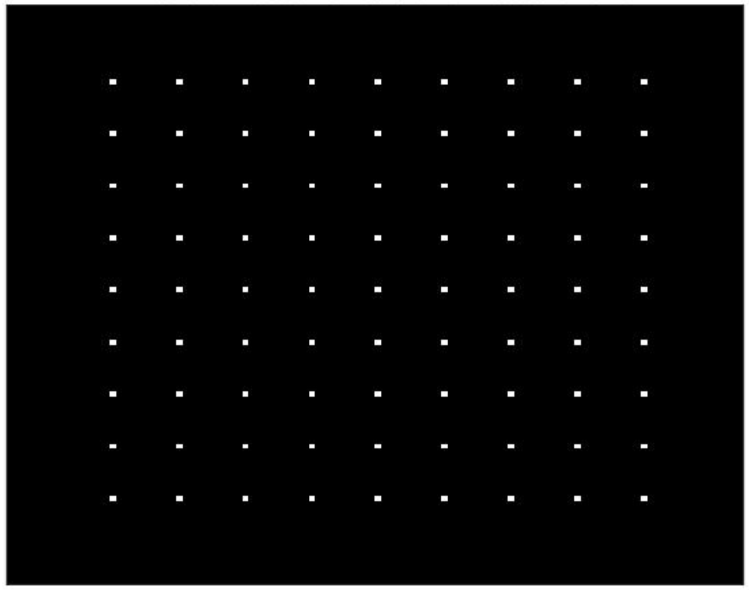
Object plane

Image plane

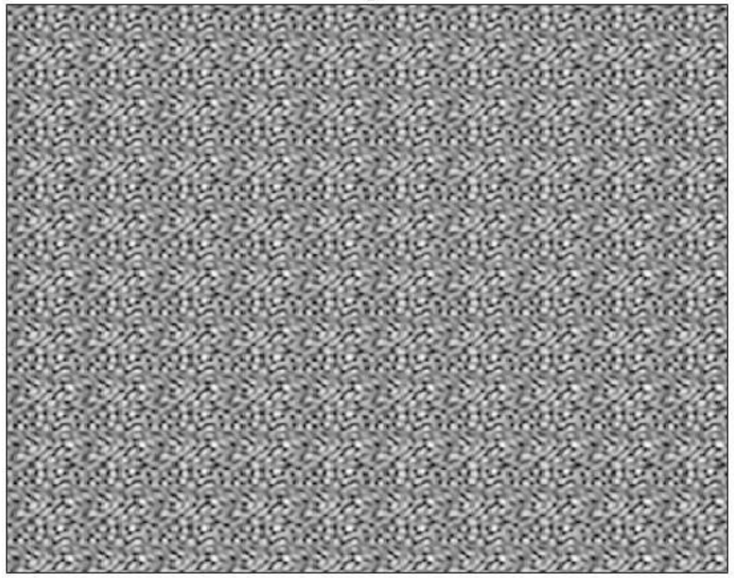
Gerchberg-Saxton / Ping-Pong / Iterative Fourier Transform Algorithm / Method of Generalizd Projection / …..

Example fan-out element

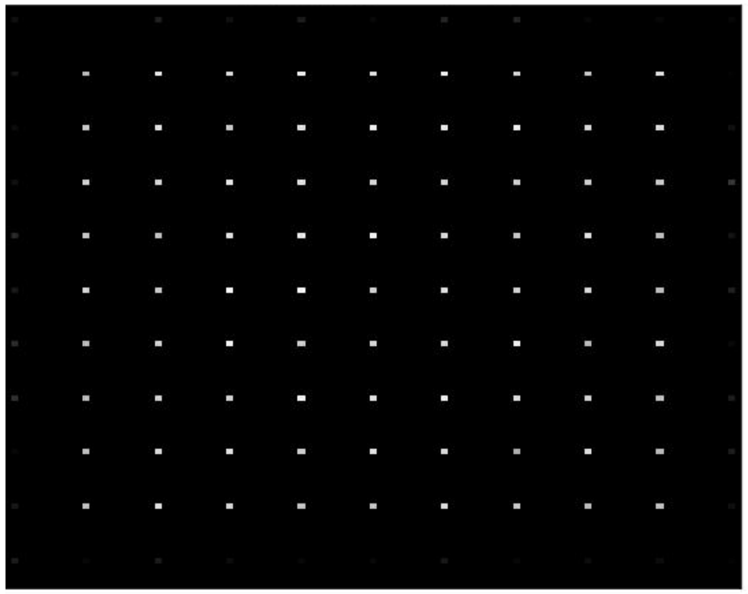




desired image



hologram



actual image

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Theoretical Optics

Optics in anisotropic media:

susceptibility and permittivity tensor

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Karlsruhe Institute of Technology

Susceptibility and permittivity tensor

so far

isotropic materials: properties independent on light propagation direction

now

anisotropic materials: properties depend on light propagation direction

polarisability of atoms depends on the orientation of the electric field

examples

Lithiumniobate ($LiNbO_3$) → electro-optical material

Quartz → polarizers

Liquid crystals → display elements, nonlinear optics

Multiple quantum wells → optoelectronics

Assumptions:

- monochromatic field and plane wave
  (one temporal frequency and one spatial frequency)
- no absorption

Isotropic materials:

normal modes are elliptically polarized, monochromatic waves

$$\bar{\mathbf{P}}(\mathbf{r},\omega) = \varepsilon_0\chi(\omega)\bar{\mathbf{E}}(\mathbf{r},\omega)$$

$$\bar{\mathbf{D}}(\mathbf{r},\omega) = \varepsilon_0\varepsilon(\omega)\bar{\mathbf{E}}(\mathbf{r},\omega)$$

Anisotropic materials:

$$\boxed{P_i(\mathbf{r},\omega) = \varepsilon_0\sum_{j=1}^{3}\chi_{ij}(\omega)\,E_j(\mathbf{r},\omega) \overset{\text{def}}{=} \varepsilon_0\chi_{ij}(\omega)E_j(\mathbf{r},\omega)}$$

task: need to find normal modes in anisotropic materials

material property: 2^{nd} rank tensor $\hat{\boldsymbol{\chi}}(\omega)$ and $\chi_{ij}(\omega)$

Consequences:
- $\mathbf{P} \nparallel \mathbf{E}$
- susceptibility tensor reflects the crystal symmetry

Anisotropic materials (1st alternativ description):

$$\boxed{\begin{gathered} D_i(\mathbf{r},\omega) = \varepsilon_0 \varepsilon_{ij}(\omega) E_j(\mathbf{r},\omega) \\ \mathbf{D}(\mathbf{r},\omega) = \varepsilon_0 \hat{\boldsymbol{\varepsilon}}(\omega) \mathbf{E}(\mathbf{r},\omega) \end{gathered}}$$

Consequences:
- $\mathbf{D} \nparallel \mathbf{E}$
- permittivity tensor reflects the crystal symmetry

Anisotropic materials (2nd alternativ description):

$$\boxed{\sigma_{ij}(\omega) D_j(\mathbf{r},\omega) = \varepsilon_0 E_i(\mathbf{r},\omega) \qquad \hat{\boldsymbol{\sigma}} = (\hat{\boldsymbol{\varepsilon}})^{-\mathbf{1}} = \left(\sigma_{ij}\right)}$$

inverse permittivity tensor

Properties of tensors: → real valued in transparency region

→ tensors are symmetric

→ six independent components $\sigma_{ij} = \sigma_{ji}$ and $\varepsilon_{ij} = \varepsilon_{ji}$

→ matrices are Hermitian ($\varepsilon_{ij}^{H} = \varepsilon_{ji}^{*}$)

Proof of hermiticity: Maxwell equations in Fourier space

$$\mathbf{div}\left(\mathbf{E}\times\mathbf{H}^*\right)+\mathtt{i}\omega\left(\varepsilon_0\mathbf{E}\cdot\left(\hat{\boldsymbol{\varepsilon}}^*\mathbf{E}^*\right)-\mu_0\mathbf{H}^*\cdot\mathbf{H}\right)+\mathbf{j}^*\cdot\mathbf{E}=\mathbf{0}$$

lossless medium: $\mathbf{j} = 0$ ⟶ $\mathbf{div}\ \langle\mathbf{S}\rangle = \frac{1}{2}\Re\left[\mathbf{div}\left(\mathbf{E}\times\mathbf{H}^*\right)\right]=0$

$$\longrightarrow \quad \Re\left(\mathtt{i}\mathbf{E}\cdot\left(\hat{\boldsymbol{\varepsilon}}^*\ \mathbf{E}^*\right)\right)=-\Im\left(\mathbf{E}\cdot\left(\hat{\boldsymbol{\varepsilon}}^*\ \mathbf{E}^*\right)\right)=0$$

has to hold for an arbitrary electric field consider a field where $E_{\mathbf{x}} \neq 0$ $\Im\left[E_{\mathbf{x}}\left(\varepsilon^*_{\mathbf{xx}}\right)E_{\mathbf{x}}{}^*\right] = \left|E_{\mathbf{x}}\right|^2 \Im\left(\varepsilon^*_{\mathbf{xx}}\right) = 0$

analog for $\varepsilon_{\mathbf{yy}}$ and $\varepsilon_{\mathbf{zz}}$ ⟶

diagonal elements of tensor
must be real valued

remaining components

$$
\begin{aligned}
\Im\left(\varepsilon^*_{\mathbf{xy}} E_{\mathbf{x}} E^*_{\mathbf{y}} + \varepsilon^*_{\mathbf{yx}} E_{\mathbf{y}} E^*_{\mathbf{x}}\right) &= \Im\left(\varepsilon^*_{\mathbf{xy}} E_{\mathbf{x}} E^*_{\mathbf{y}} - \varepsilon_{\mathbf{yx}} E^*_y E_{\mathbf{x}}\right) \\
\rightarrow \qquad &= \Im\left[\left(\varepsilon^*_{\mathbf{xy}} - \varepsilon_{\mathbf{yx}}\right) E_{\mathbf{x}} E^*_{\mathbf{y}}\right] \\
&= 0
\end{aligned}
$$

$$\rightarrow \boxed{\varepsilon^*_{\mathbf{xy}} = \varepsilon_{\mathbf{yx}}}$$

can be generalised to all components

transformation into principal coordinate system possible

→ tensors are diagonal

→ example of the inverse permittivity tensor

→ only in this coordinate system $\mathbf{D} \parallel \mathbf{E}$

→ mathematically this can be expressed in terms of an eigenvalue equation

$$\varepsilon_0 E_i = \sigma_{ij} D_j \doteq \lambda D_i$$

non-trivial solution: vanishing determinant

$$\det\left[\sigma_{ij} - \lambda I_{ij}\right] = 0$$

equation of 3rd order with three roots as solutions → called $\lambda^{(\alpha)}$

→ associated eigenvectors

$$\sigma_{ij} D_j^{(\alpha)} = \lambda^{(\alpha)} D_i^{(\alpha)}$$

eigenvectors are orthogonal

consider: $\sigma_{ij} D_j^{(\alpha)} = \lambda^{(\alpha)} D_i^{(\alpha)}$ and $\sigma_{ij} D_j^{(\beta)} = \lambda^{(\beta)} D_i^{(\beta)}$

multiplication and subtraction: $D_i^{(\beta)} \sigma_{ij} D_j^{(\alpha)} - D_i^{(\alpha)} \sigma_{ij} D_j^{(\beta)} = \left(\lambda^{(\alpha)} - \lambda^{(\beta)}\right) D_i^{(\beta)} D_i^{(\alpha)}$

but $D_j^{(\beta)} \sigma_{ji} D_i^{(\alpha)} - D_i^{(\alpha)} \sigma_{ij} D_j^{(\beta)} = 0$ because $\sigma_{ji} = \sigma_{ij}$

$$\rightarrow \boxed{D_i^{(\beta)} D_i^{(\alpha)} = 0} \quad \text{for} \quad \lambda^{(\alpha)} \neq \lambda^{(\beta)}$$

principal axes reflect crystal symmetry: $\varepsilon_{ij} = \varepsilon_i \delta_{ij},\ \sigma_{ij} = \sigma_i \delta_{ij} = \frac{1}{\varepsilon_i} \delta_{ij}$

$$\boxed{(\varepsilon_{ij}) = \begin{bmatrix} \varepsilon_1(\omega) & 0 & 0 \\ 0 & \varepsilon_2(\omega) & 0 \\ 0 & 0 & \varepsilon_3(\omega) \end{bmatrix}}$$

anisotropic material fully described using three different dielectric functions

calculations mostly done in principal coordinate system and results are transferred later to the lab coordinate system

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