Optics in anisotropic media:
classification of materials and illustration

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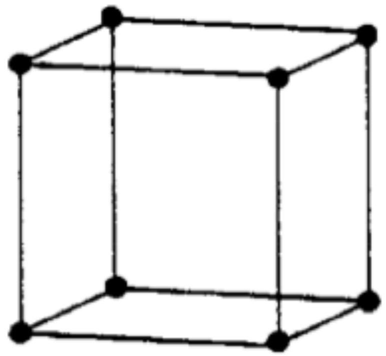
Optical classification of crystals

$$(\varepsilon_{ij}) = \begin{bmatrix} \varepsilon_1(\omega) & 0 & 0 \\ 0 & \varepsilon_2(\omega) & 0 \\ 0 & 0 & \varepsilon_3(\omega) \end{bmatrix}$$

(a) isotropic crystals

- three crystallographic identical and perpendicular axes
- cubic crystals (diamond, silicon)

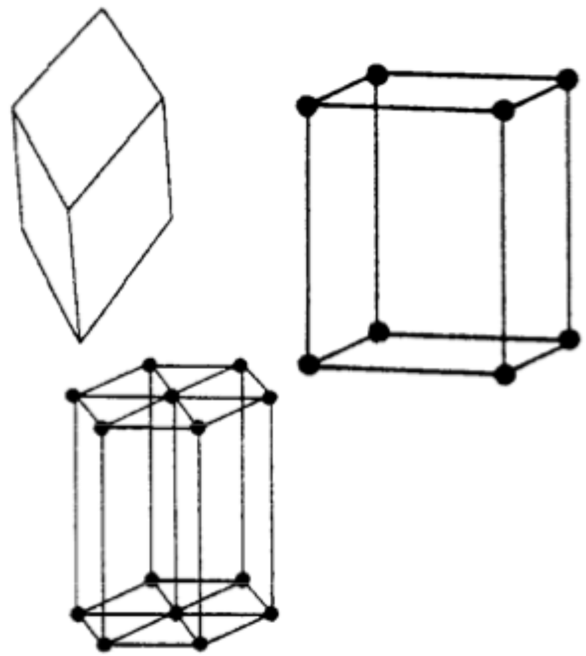


$$\varepsilon_1(\omega) = \varepsilon_2(\omega) = \varepsilon_3(\omega) \longrightarrow D_i = \varepsilon_0 \varepsilon E_i$$

- same description for gases amorphous solids, liquids

(b) uniaxial crystals

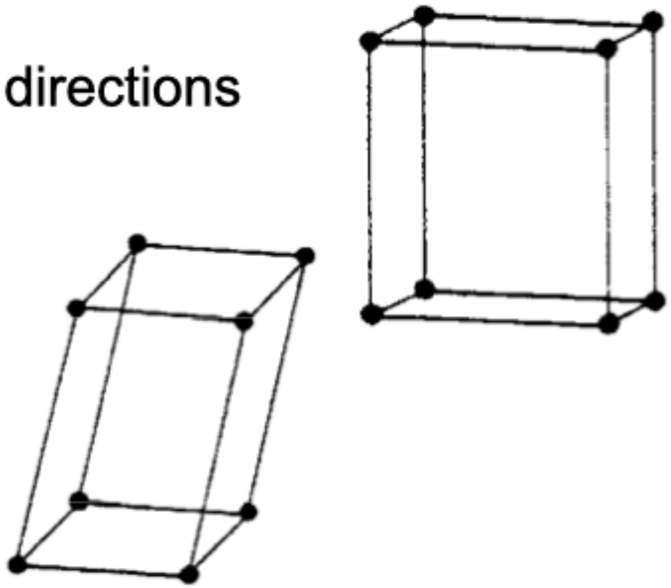
- two crystallographic equivalent directions
- trigonal, tetragonal, or hexagonal crystals



$$\varepsilon_1(\omega) = \varepsilon_2(\omega) \neq \varepsilon_3(\omega)$$

(c) biaxial crystals

- there are no two identical crystallographic directions
- orthorhombic, monocline, tricline



$$\varepsilon_1(\omega) \neq \varepsilon_2(\omega) \neq \varepsilon_3(\omega)$$

Index ellipsoid

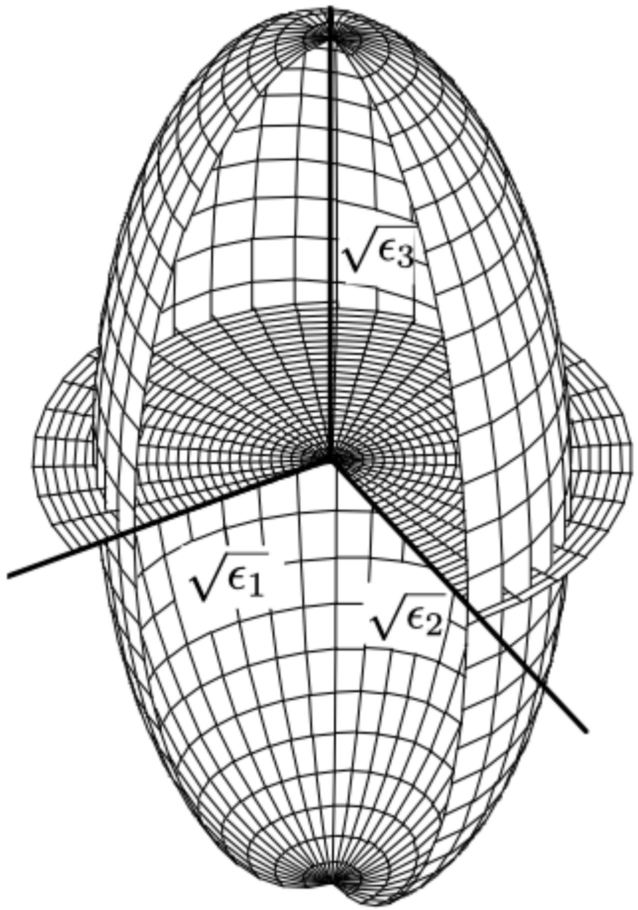
→ geometrical representation of the inverse dielectric tensor

→ defining equation for the index ellipsoid is a surface of second order

→the representation is invariance against coordinate transformations

$$\sum_{i,j=1}^{3} \sigma_{ij}\, x_i x_j = 1$$

(surface 2nd order)



→ defines a surface of constant electric energy density since

$$\sum_{i,j=1}^{3} \sigma_{ij}\, D_i D_j = \varepsilon_0 \sum_{i=1}^{3} E_i D_i = 2w_{el}$$

→ in principle coordinate system index ellipsoid:

$$\sigma_1 x_1^2 + \sigma_2 x_2^2 + \sigma_3 x_3^2 = \frac{x_1^2}{\varepsilon_1} + \frac{x_2^2}{\varepsilon_2} + \frac{x_3^2}{\varepsilon_3} = 1$$

degeneracy: cubical geometry → sphere; uniaxial → rotational symmetric

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Theoretical Optics

Optics in anisotropic media:
normal modes

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Normal modes in anisotropic media

→ solution to the source free wave equation

→ experiences upon propagation only a spatial-temporal phase variation

→ amplitude and polarization are preserved and constant.

→ spatial-temporal phase variation is connected via the dispersion relation

Reminder isotropic material:

elliptically polarised
monochromatic plane wave

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_\omega e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \qquad \mathbf{k}^2(\omega) = k^2(\omega) = \frac{\omega^2}{c_0^2}\varepsilon(\omega)$$

with $\varepsilon(\omega) > 0 \longrightarrow \mathbf{k}\cdot\mathbf{E}_\omega = \mathbf{k}\cdot\mathbf{D}_\omega = 0$

NOW: identifying now eigenmodes in anisotropic materials

Approach 1: modes for a propagation along an axis of the principal coordinate system

→the principal axes are in the x, y, z directions

→require that $\mathbf{k} \cdot \mathbf{E}_\omega = \mathbf{k} \cdot \mathbf{D}_\omega = 0.$

→assume that z-axis is principal direction ($\mathbf{k} \to k_z$)

→ $D_x, D_y \neq 0$

→ field can be arbitrary in the $x - y -$ plane.

→ electric displacement linked to electric field by $D_i = \varepsilon_0 \varepsilon_i E_i$ (no summation here)

unique situation that two possible polarization directions are decouple

$$D_1, \varepsilon_1 \;\to D_1 e^{i(\mathbf{k}_1 \cdot \mathbf{r} - \omega t)} = D_1 e^{i\varphi_1} e^{-i\omega t} \quad \text{with} \quad \mathbf{k}_1^2 = \frac{\omega^2}{c_0^2} \varepsilon_1(\omega)$$

$$D_2, \varepsilon_2 \;\to D_2 e^{i(\mathbf{k}_2 \cdot \mathbf{r} - \omega t)} = D_2 e^{i\varphi_2} e^{-i\omega t} \quad \text{with} \quad \mathbf{k}_2^2 = \frac{\omega^2}{c_0^2} \varepsilon_2(\omega)$$

→ linearly polarized propgating along principal axes are normal modes

→ $\mathbf{D} \nparallel \mathbf{E}$ → elliptical polarization is not a normal mode

→ polarization would change upon propagation

phase change: $$\delta = \varphi_2 - \varphi_1 = (k_2 - k_1) z$$

→ if field polarized along a principal axis: normal mode

$$\mathbf{D}^{(\mathtt{a})} = \{ D_1 \exp[\mathtt{i}(\mathbf{k}_{\mathtt{a}}\mathbf{r} - \omega t)] \} \mathbf{e}_1 \quad \rightarrow \mathbf{k}_{\mathtt{a}}^2 = \frac{\omega^2}{c^2} n_{\mathtt{a}}^2 = \mathbf{k}_1^2$$

$$\mathbf{D}^{(\mathtt{b})} = \{ D_2 \exp[\mathtt{i}(\mathbf{k}_{\mathtt{b}}\mathbf{r} - \omega t)] \} \mathbf{e}_2 \quad \rightarrow \mathbf{k}_{\mathtt{b}}^2 = \frac{\omega^2}{c^2} n_{\mathtt{bk}}^2 = \mathbf{k}_2^2$$

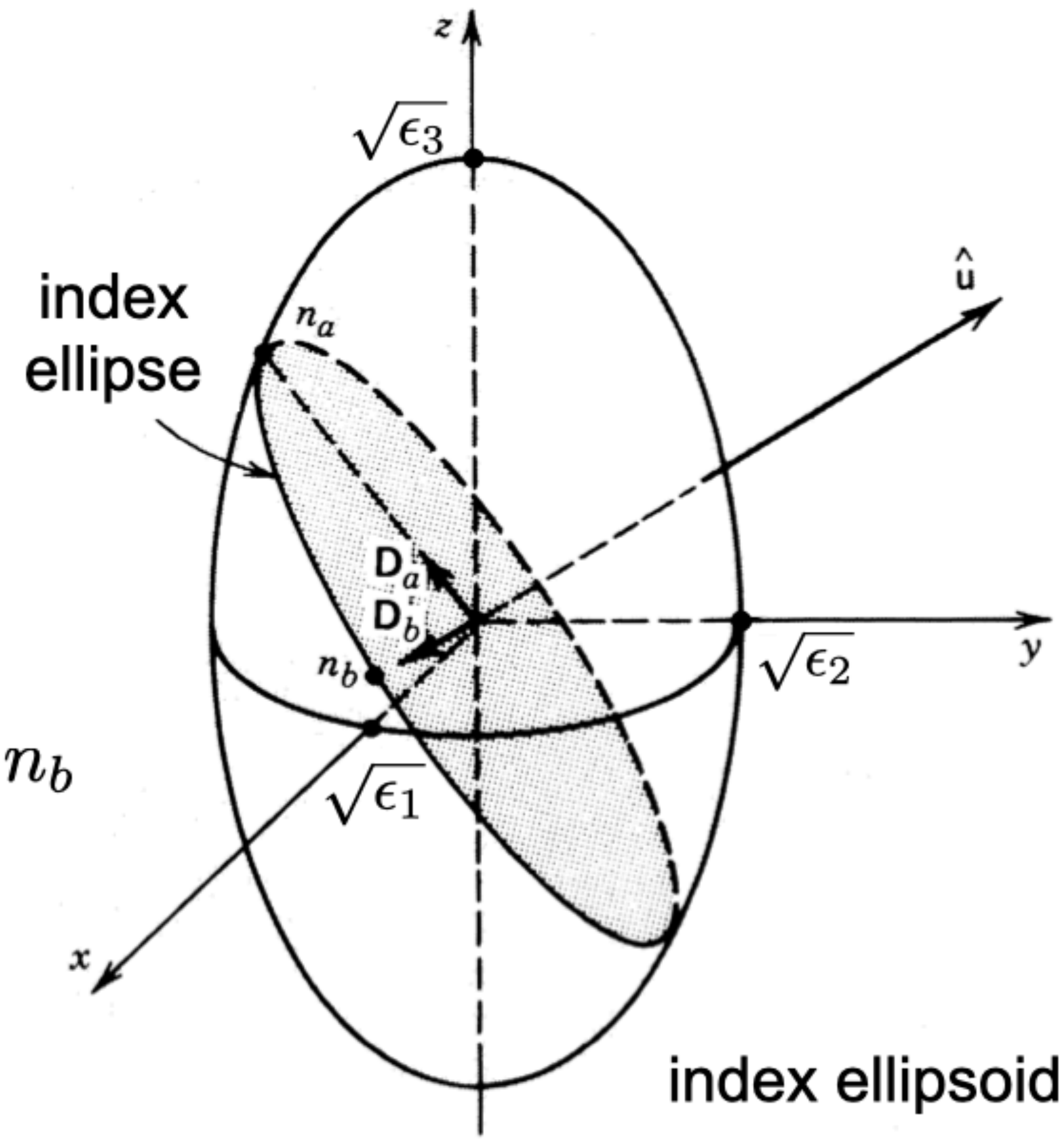
there are two orthonormal linearly polarized modes

Approach 2: geometrical construction

- for a given frequency and a crystal the permittivity tensor is known
- plot the index ellipsoid
- define a direction of the propagation direction we are interested in

$$\rightarrow \mathbf{k}/k$$

- sketch the plane perpendicular to $\mathbf{k}$ and which goes through the origin
- cross-section is an ellipse that is the index ellipse
- principal axes correspond to n_a and n_b of the normal modes in this direction



wave number

$$k_a = \frac{\omega}{c} n_a, k_b = \frac{\omega}{c} n_b$$

Approach 2: geometrical construction

- direction of principal axes correspond to direction of displacement vector

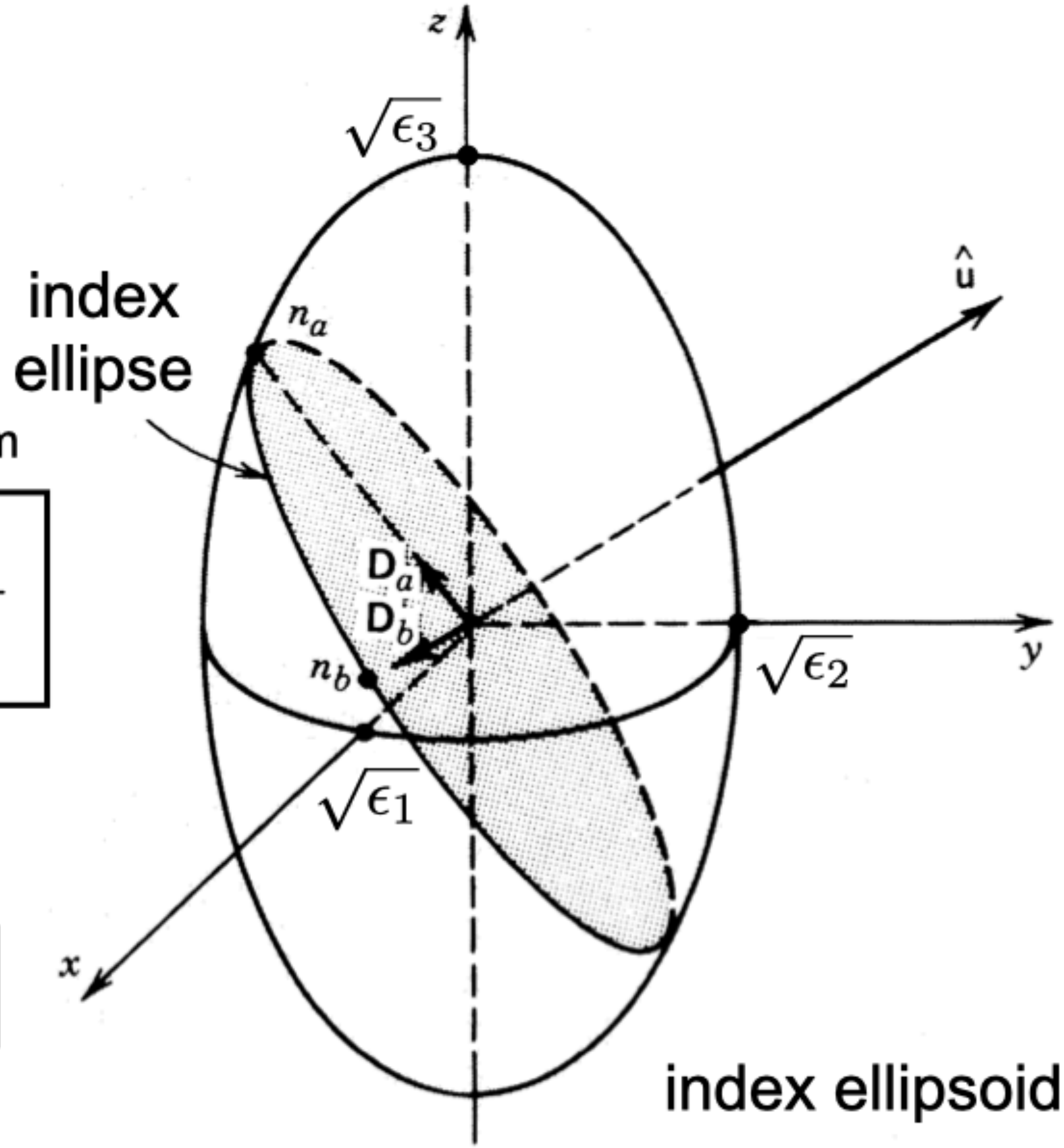
$$\mathbf{D}^{(\mathtt{a})} \text{ and } \mathbf{D}^{(\mathtt{b})}$$

- orientation of electric field follows from

$$E_i^{(\mathtt{a})} = \frac{D_i^{(\mathtt{a})}}{\varepsilon_0 \varepsilon_{\mathtt{i}}}, \qquad E_i^{(\mathtt{b})} = \frac{D_i^{(\mathtt{b})}}{\varepsilon_0 \varepsilon_{\mathtt{i}}}$$

- it follows that $\mathbf{D}^{(\mathtt{a,b})} \nparallel \mathbf{E}^{(\mathtt{a,b})}$

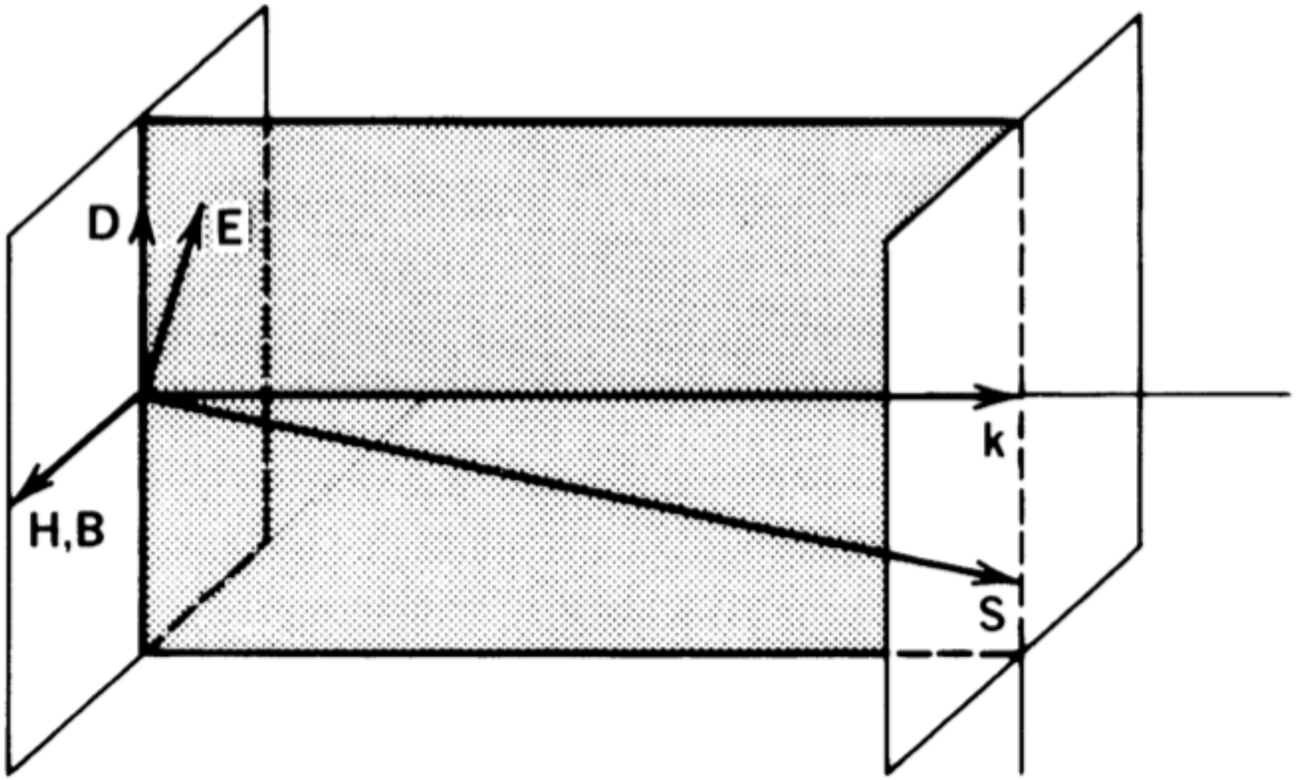
$$\mathbf{E}^{(a,b)} \not\perp \mathbf{k}^{(a,b)}$$



- Poynting vector no longer parallel to wave vector

$$\langle \mathbf{S} \rangle = \frac{1}{2} \Re(\mathbf{E} \times \mathbf{H}^*) \quad \rightarrow \quad \mathbf{k} \nparallel \langle \mathbf{S} \rangle \text{ because } \mathbf{E} \perp \langle \mathbf{S} \rangle$$

a picture to memorise



$$\mathbf{D}^{(a,b)} \perp \mathbf{k}^{(a,b)}$$

$$\mathbf{D}^{(a,b)} \nparallel \mathbf{E}^{(a,b)}$$

$$\mathbf{k} \nparallel \langle \mathbf{S} \rangle$$

$$\mathbf{E}^{(a,b)} \not\perp \mathbf{k}^{(a,b)}$$

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