Optics in anisotropic media:
uniaxial crystals

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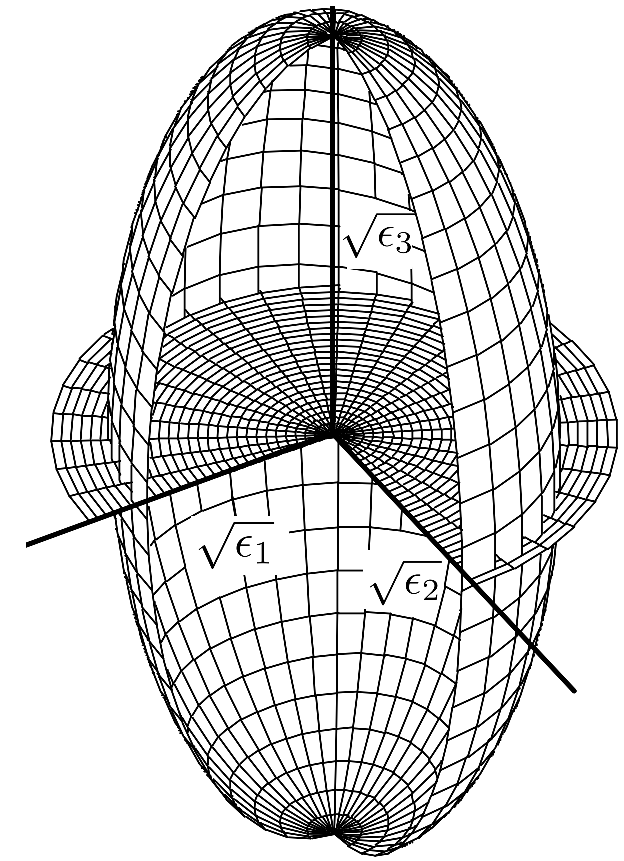
Uniaxial crystals



→ crystal structure trigonal, tetragonal, or hexagonal

→ index ellipsoid rotational symmetric around z-axes

→ characterized by an ordinary and an extraordinary index

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_{\mathrm{or}} \quad \text{and} \quad \varepsilon_3 = \varepsilon_{\mathrm{e}}$$

→ the normal modes, in general, do not see the associated indices directly

there are two different normal modes

a) ordinary waves → index is independent on the direction

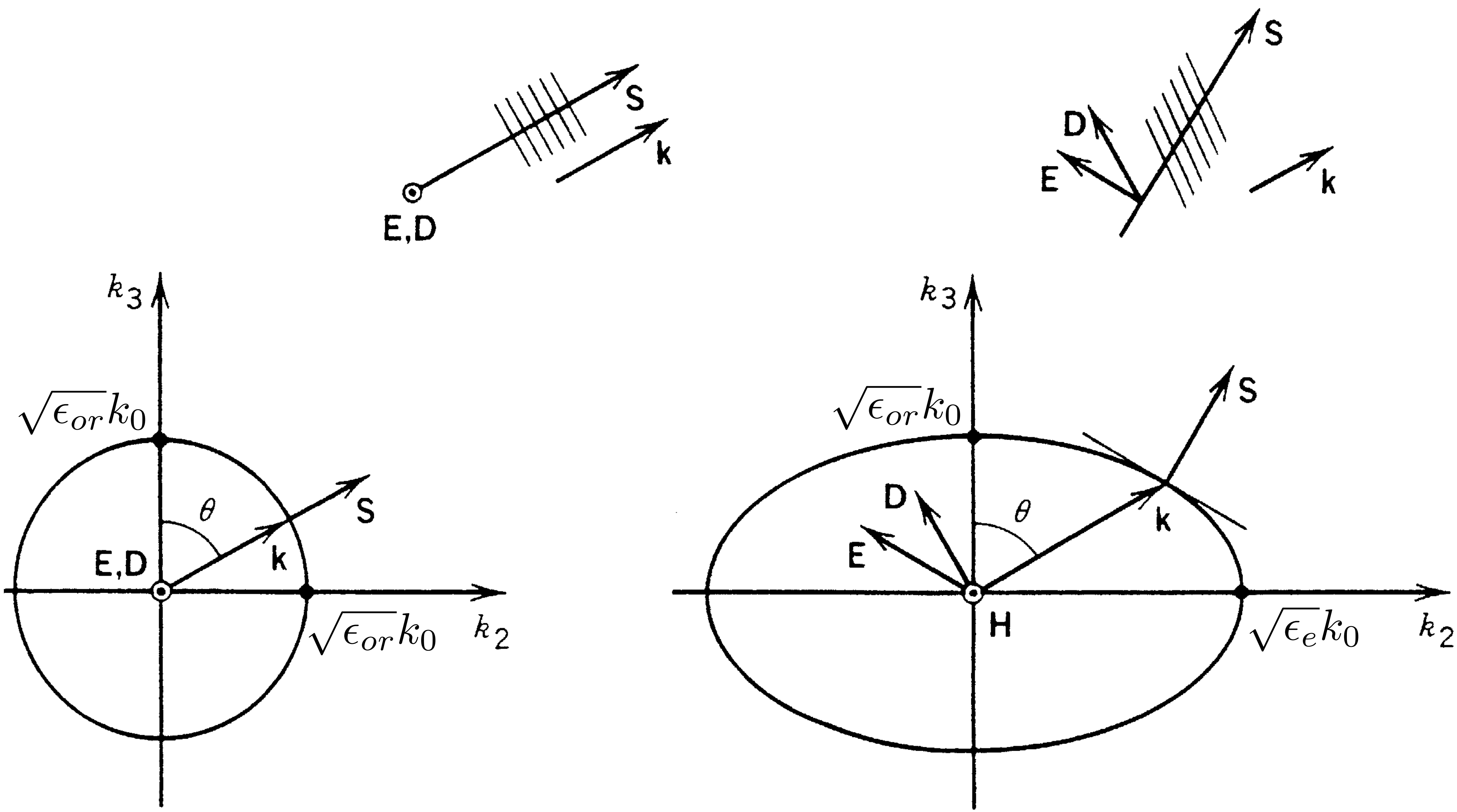
→ $D^{(\mathrm{or})}$ polarized perpendicular to the z-axis and $\mathbf{k}$

b) extraordinary waves → index is dependent on the direction

→ polarized perpendicular to $\mathbf{k}$ and in the plane spanned by the wave vector and the rotational axis

$\rightarrow D^{(e)}$ is perpendicular to $D^{(or)}$

$\rightarrow$ according to our definitions z-axes is optical axis with $n_a = n_b$



(a) Ordinary

(b) Extraordinary

Dispersion relation

$$u_1^2(n^2-\varepsilon_{\mathrm{or}})(n^2-\varepsilon_{\mathrm{e}})n^2+u_2^2(n^2-\varepsilon_{\mathrm{or}})(n^2-\varepsilon_{\mathrm{e}})n^2+u_3^2(n^2-\varepsilon_{\mathrm{or}})^2n^2$$
$$=(n^2-\varepsilon_{\mathrm{or}})^2(n^2-\varepsilon_{\mathrm{e}})$$

$$(n^2-\varepsilon_{\mathrm{or}})\left[(u_1^2+u_2^2)(n^2-\varepsilon_{\mathrm{e}})+u_3^2(n^2-\varepsilon_{\mathrm{or}})-\frac{(n^2-\varepsilon_{\mathrm{e}})(n^2-\varepsilon_{\mathrm{or}})}{n^2}\right]=0$$

two solutions

1. ordinary wave

$$n_a^2=\varepsilon_{\mathrm{or}} \quad \rightarrow \quad \boxed{k_a^2=\frac{\omega^2}{c_0^2}\varepsilon_{\mathrm{or}}=\frac{\omega^2}{c_0^2}\,n_a^2}$$

2. extraordinary wave

$$(u_1^2+u_2^2)(n_b^2-\varepsilon_{\mathrm{e}})+u_3^2(n_b^2-\varepsilon_{\mathrm{or}})=\frac{(n_b^2-\varepsilon_{\mathrm{e}})(n_b^2-\varepsilon_{\mathrm{or}})}{n^2}$$

$$n_b^2(u_1^2+u_2^2+u_3^2)-n_b^2-(u_1^2+u_2^2)\varepsilon_{\mathrm{e}}-u_3^2\varepsilon_{\mathrm{or}}+\varepsilon_{\mathrm{e}}+\varepsilon_{\mathrm{or}}-\frac{\varepsilon_{\mathrm{e}}\varepsilon_{\mathrm{or}}}{n_b^2}=0$$

$$\varepsilon_{\mathrm{e}}(1-u_1^2-u_2^2)+\varepsilon_{\mathrm{or}}(1-u_3^2)-\frac{\varepsilon_{\mathrm{e}}\varepsilon_{\mathrm{or}}}{n_b^2}=0$$

$$\boxed{\frac{(u_1^2+u_2^2)}{\varepsilon_{\mathrm{e}}(\omega)}+\frac{u_3^2}{\varepsilon_{\mathrm{or}}(\omega)}-\frac{1}{n_b^2}=0}$$

→ calculate n_a and n_b for a given illumination direction

Geometrical representation of the normal modes

1. ordinary wave

$$k_a^2=k_1^2+k_2^2+k_3^2=k_0^2\varepsilon_{\mathrm{or}}$$

2. extraordinary wave

$$\frac{1}{\varepsilon_{\mathrm{e}}}\frac{(k_1^2+k_2^2)}{k_0^2}+\frac{1}{\varepsilon_{\mathrm{or}}}\frac{k_3^2}{k_0^2}=1$$

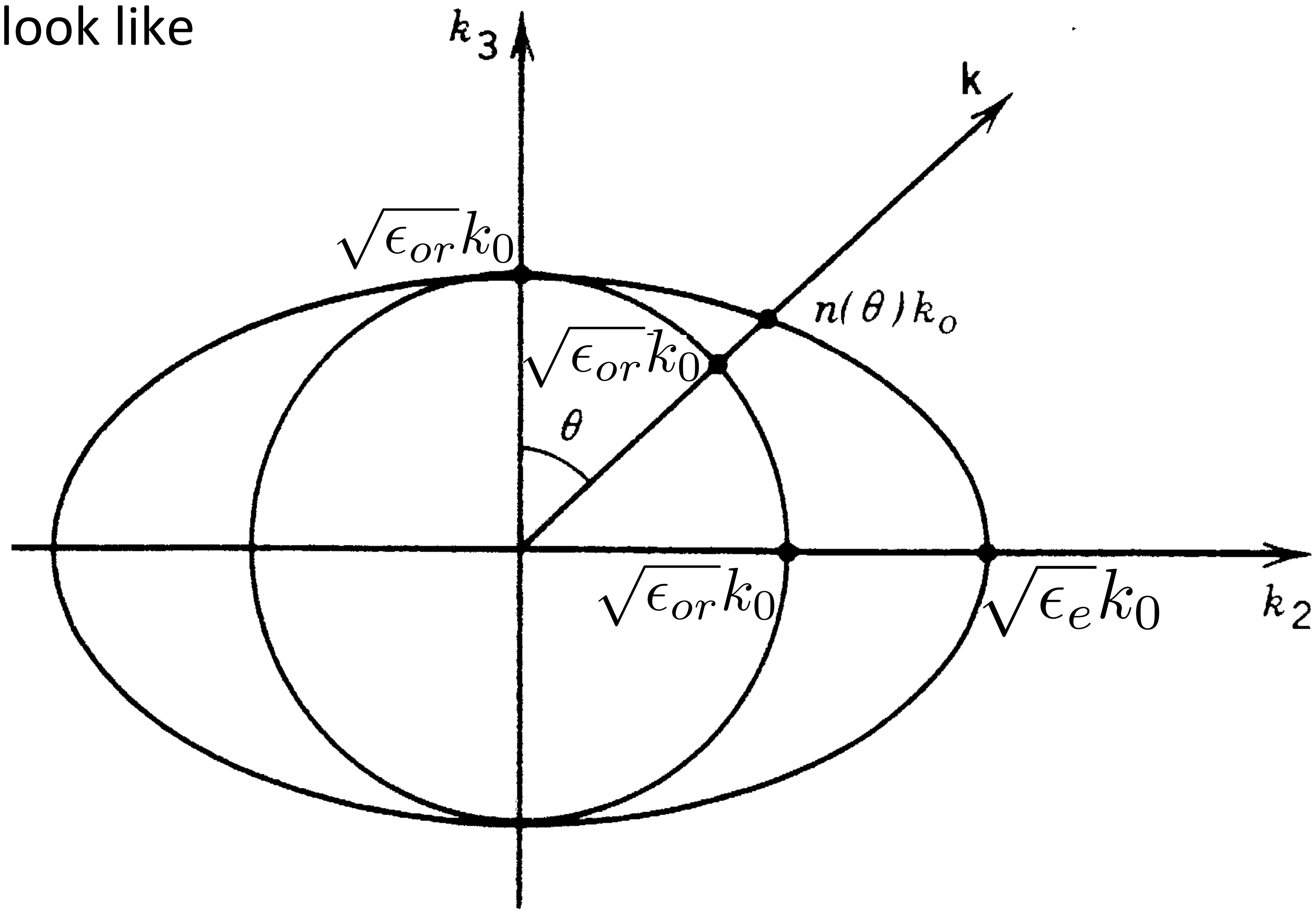
→ equation for a rotational symmetric ellipsoid

→ for simplicity we assume that the propagation takes place in the $y - z$-plane

→ $u_1 = 0$

→ size of the ellipse changes with frequency

→ isosurfaces look like



Statements about the field

$$D_1 : D_2 : D_3 = \frac{\varepsilon_{\mathrm{or}} k_1}{\left(\frac{\omega^2}{c_0^2}\varepsilon_{\mathrm{or}} - k^2\right)} : \frac{\varepsilon_{\mathrm{or}} k_2}{\left(\frac{\omega^2}{c_0^2}\varepsilon_{\mathrm{or}} - k^2\right)} : \frac{\varepsilon_{\mathrm{e}} k_3}{\left(\frac{\omega^2}{c_0^2}\varepsilon_{\mathrm{e}} - k^2\right)}$$

→for extraordinary wave the denominators are finite and with $k_1 = 0 \rightarrow D_1^{(e)} = 0$

→field is polarized in the y-z-plane

→ since ordinary wave are polarized perpendicular to the extraordinary wave

→ ordinary wave must be polarized in the x-direction

Effective index of the extraordinary wave

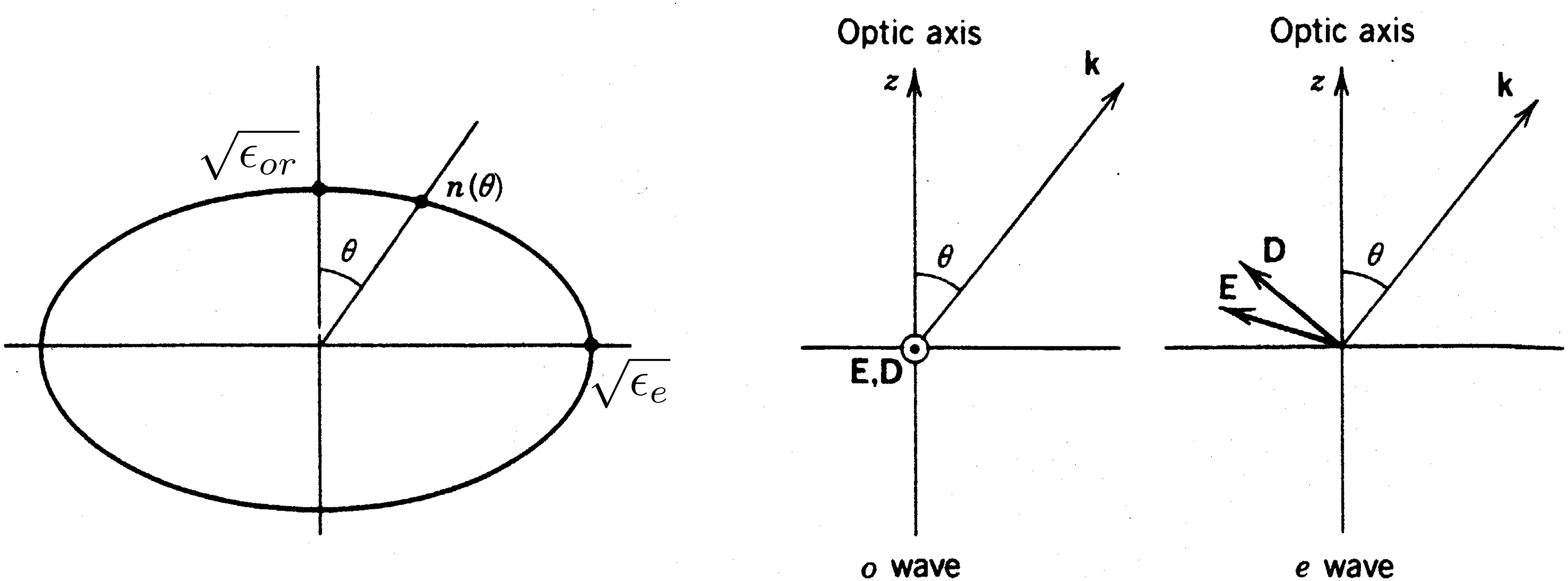
depends on the angle of propagation in the y-z-plane

$$\frac{(u_1^2 + u_2^2)}{\varepsilon_{\mathrm{e}}(\omega)} + \frac{u_3^2}{\varepsilon_{\mathrm{or}}(\omega)} - \frac{1}{n_b^2(\omega)} = 0 \xrightarrow[u_1 = 0]{} \boxed{\frac{u_2^2}{\varepsilon_{\mathrm{e}}(\omega)} + \frac{u_3^2}{\varepsilon_{\mathrm{or}}(\omega)} - \frac{1}{n_b^2(\omega)} = 0}$$

$$u_2 = \sin\Theta \quad \text{and} \quad u_3 = \cos\Theta$$

$$\frac{\sin^2\Theta}{\varepsilon_\mathrm{e}(\omega)} + \frac{\cos^2\Theta}{\varepsilon_\mathrm{or}(\omega)} - \frac{1}{n_b^2(\omega,\Theta)} = 0$$

$$\boxed{n_b^2(\omega,\Theta) = \frac{\varepsilon_\mathrm{e}(\omega)\varepsilon_\mathrm{or}(\omega)}{\varepsilon_\mathrm{or}(\omega)\sin^2\Theta + \varepsilon_\mathrm{e}(\omega)\cos^2\Theta}}$$



classification: $\varepsilon_\mathrm{or}(\omega) > \varepsilon_\mathrm{e}(\omega) \quad \rightarrow \quad$ negatively uniaxal

$\varepsilon_\mathrm{or}(\omega) < \varepsilon_\mathrm{e}(\omega) \quad \rightarrow \quad$ positively uniaxal

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Coherence Optics

Basics

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Basics

- part of statistical optics
- Emil Wolf 1950s and later others

> *"One day I received a letter from Born in which he asked me why the manuscript was not yet finished. I wrote back saying that the manuscript is almost completed, except for a chapter on partial coherence on which I was still working. Born replied at once saying,* ***„Wolf, who apart from you is interested in coherence? Leave the chapter out and send the manuscript to the printers.*** *"*

WOLF, E.. Early days of coherence theory and the first Rochester conference on coherence.
Journal of the European Optical Society - Rapid publications, Europe, v. 5, sep. 2010. ISSN 1990-2573.
Available at: <https://www.jeos.org/index.php/jeos_rp/article/view/10044s>.

- current research: partially coherent light (solar cells)

subject: **properties of "random" or "chaotic" light**

cause: **fluctuations of sources or optical properties of medium**

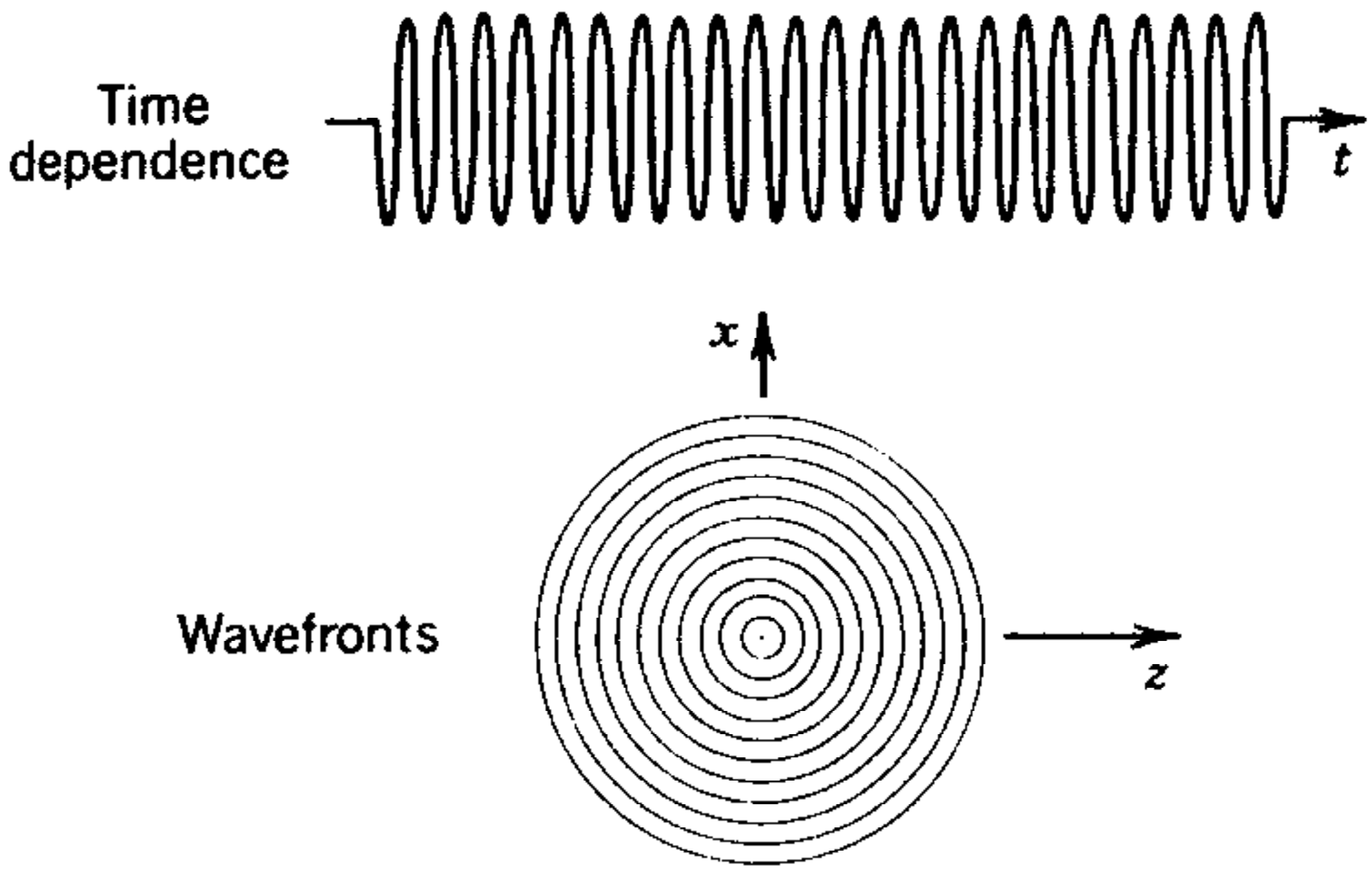
example: **light emitted from thermal source**
light reflected from random surface

assumptions: scalar approximation $u(\mathbf{r},t)$

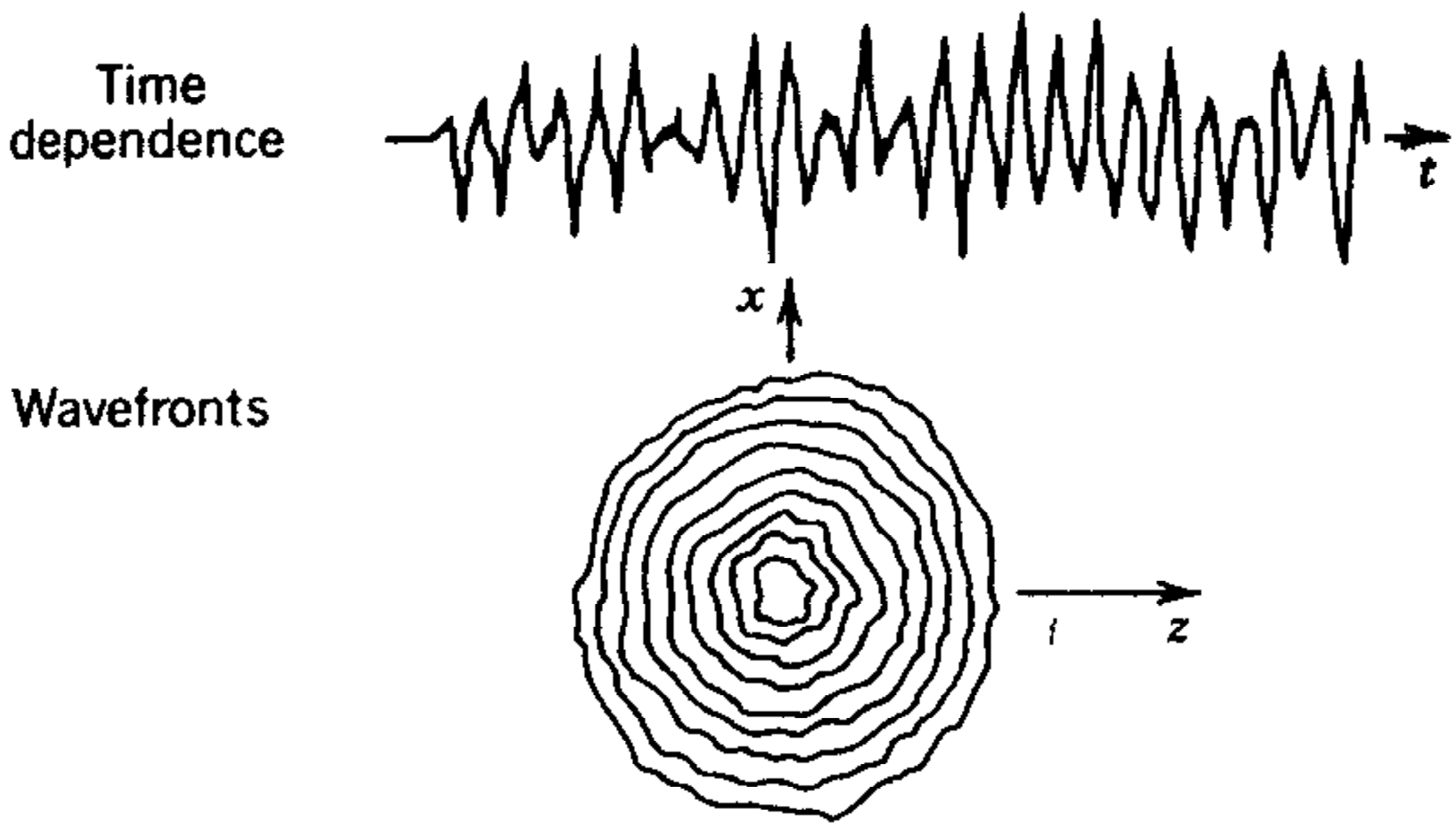
thus far: $u(\mathbf{r},t) = U(\mathbf{r},\omega)e^{-i\omega t} + c.c.$

with e.g. $U(\mathbf{r},\omega) = e^{i\mathbf{k}(\omega)\cdot\mathbf{r}}$

coherent light, determined in all its properties



Now: field is random to some extent ⟶ obeys wave equation



Classification:

(according to the degree of correlation in the phase)

- Coherent light
- Partially coherent light
- Incoherent light

this is <u>not a polychromatic</u> source ⟶ superposition frequency, amplitude, phase of harmonics of a source are not known

$\longrightarrow$ only statistical properties

$\longrightarrow$ statistical features carried over to field

$\longrightarrow$ cannot predict all experimental results: but statistics

mean: first order coherence functions

wave equation cannot be used to describe the evolution of fields but it can be used to describe the evolution of the coherence functions

distinguish time, space, and mutual coherence

Coherence Optics

Basics

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