Theoretical Optics

Propagating Surface Plasmon Polaritons

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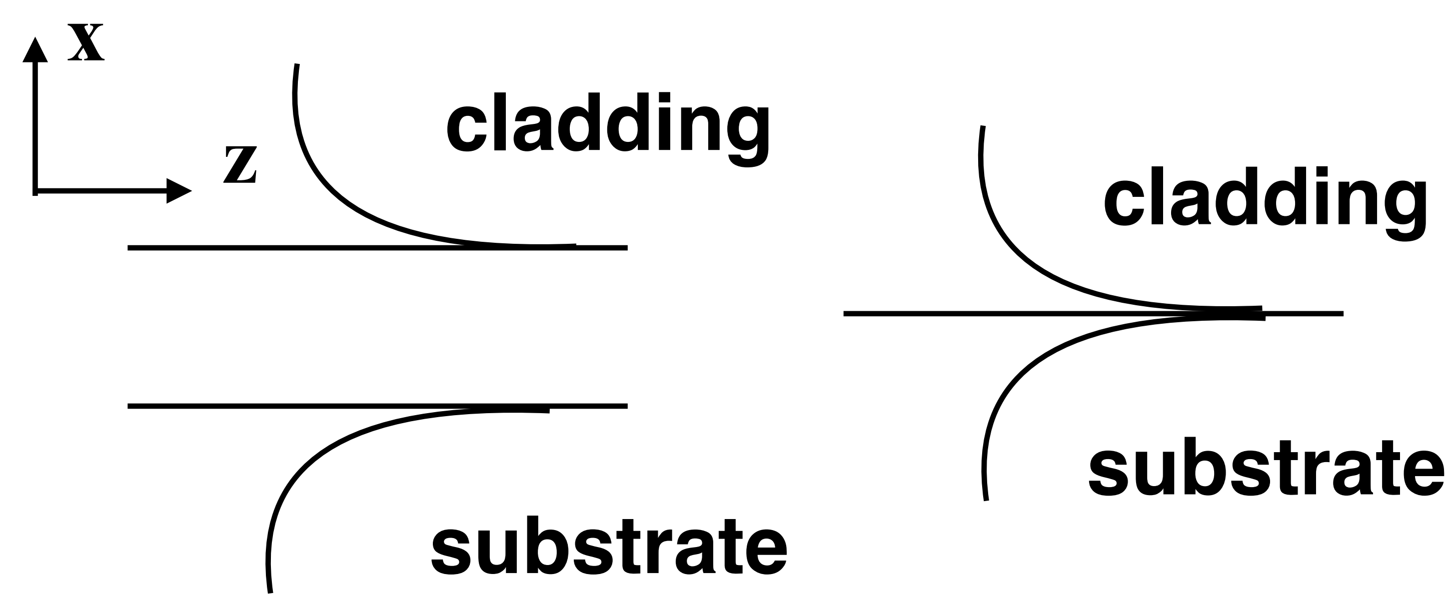
Guided modes in general

- guided modes are waves that propagate free of diffraction
- miniaturisation of optical comports in integrated circuits
- enhancement of nonlinear effects
- super-resolution using plasmonic thin films

field structure



note the modified principle propagation direction

Properties of guides modes

plane wave along the interface: $e^{ik_z z}$ **conserved in all layers**

oscillating inside the core: $\propto A \sin k_{fx} x + B \cos k_{fx} x$

$$k_{ix} = \sqrt{\frac{\omega^2}{c^2}\epsilon_i(\omega) - k_z^2}$$

Constraints on the propagation constant

at least one propagating plane wave somewhere in the core

$$k_z^2 < \frac{\omega^2}{c^2} \max_i \{\epsilon_i(\omega)\}$$

evanescent in cladding and substrate

$$k_z^2 > \frac{\omega^2}{c^2} \max \{\epsilon_{\mathrm{s,c}}(\omega)\}$$

→ field structure:

$\propto e^{-\mu_{\mathrm{c}}(x-D)}$ cladding

$\propto e^{\mu_{\mathrm{s}} x}$ substrate

$$\mu_{\mathrm{s,c}} = \sqrt{k_z^2 - \frac{\omega^2}{c^2}\epsilon_{\mathrm{s,c}}} > 0$$

$$\max\left\{\frac{\omega}{c} n_{\mathrm{s,c}}\right\} < k_z < \max_i \left\{\frac{\omega}{c} n_i\right\}$$

Existence condition for guided modes

require a reflected and transmitted field (exponentially decaying) in the absence of some illumination

$$T = \frac{\mathbf{E}_T}{\mathbf{E}_I}\ , \ R = \frac{\mathbf{E}_R}{\mathbf{E}_I} \quad \text{and} \quad \mathbf{E}_T,\ \mathbf{E}_R \neq 0 \quad \text{for} \quad \mathbf{E}_I \to 0$$

$$\longrightarrow \boxed{R,\ T \to \infty}$$

guided waves are resonances of the system

general physical principle,
i.e., compare driven harmonic oscillator

can extract dispersion relation of guided modes

Consider previous results on reflection coefficient

$$R = \frac{F_{\mathtt{R}}}{F_{\mathtt{I}}} = \frac{\left(\alpha_{\mathtt{s}} k_{\mathtt{sx}} M_{22} - \alpha_{\mathtt{c}} k_{\mathtt{cx}} M_{11}\right) - \mathtt{i}\left(M_{21} + \alpha_{\mathtt{s}} k_{\mathtt{sx}} \alpha_{\mathtt{c}} k_{\mathtt{cx}} M_{12}\right)}{\left(\alpha_{\mathtt{s}} k_{\mathtt{sx}} M_{22} + \alpha_{\mathtt{c}} k_{\mathtt{cx}} M_{11}\right) + \mathtt{i}\left(M_{21} - \alpha_{\mathtt{s}} k_{\mathtt{sx}} \alpha_{\mathtt{c}} k_{\mathtt{cx}} M_{12}\right)}$$

singularities: $\rightarrow \left(\alpha_{\mathtt{s}} k_{\mathtt{sx}} M_{22} + \alpha_{\mathtt{c}} k_{\mathtt{cx}} M_{11}\right) + \mathtt{i}\left(M_{21} - \alpha_{\mathtt{s}} k_{\mathtt{sx}} \alpha_{\mathtt{c}} k_{\mathtt{cx}} M_{12}\right) \doteq 0$

using $k_{\mathtt{sx}} = i\mu_{\mathtt{s}}, \quad k_{\mathtt{cx}} = i\mu_{\mathtt{c}}$

$$M_{11}^{\mathrm{TE,TM}} + \alpha_s \mu_s M_{12}^{\mathrm{TE,TM}} + \frac{1}{\alpha_c \mu_c} M_{21}^{\mathrm{TE,TM}} + \frac{\alpha_s \mu_s}{\alpha_c \mu_c} M_{22}^{\mathrm{TE,TM}} = 0$$

only discrete number of solutions: guides modes

Surface plasmon polaritons

Possible to sustain a guided mode at a single interface?

Transfer matrix: $\hat{\mathbf{M}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Dispersion relation: $1 + \frac{\alpha_s \mu_s}{\alpha_c \mu_c} = 0$

TE Polarisation: $\alpha = 1$

$\mu_s + \mu_c = 0$

no solution!

TM Polarisation: $\alpha = \frac{1}{\epsilon}$

$$\frac{\mu_c}{\varepsilon_c} + \frac{\mu_s}{\varepsilon_s} = 0 \xrightarrow{\mu_{c,s} > 0} \epsilon_c \epsilon_s < 0$$

guided TM-polarised surface waves exist below plasma frequency for **metals** or close to resonances of materials characterised by Lorentzian dispersion

Explicit dispersion relation

$$\frac{\mu_c}{\varepsilon_c} + \frac{\mu_s}{\varepsilon_s} = 0 \longrightarrow (\mu_c \varepsilon_s)^2 = (\mu_s \varepsilon_c)^2$$

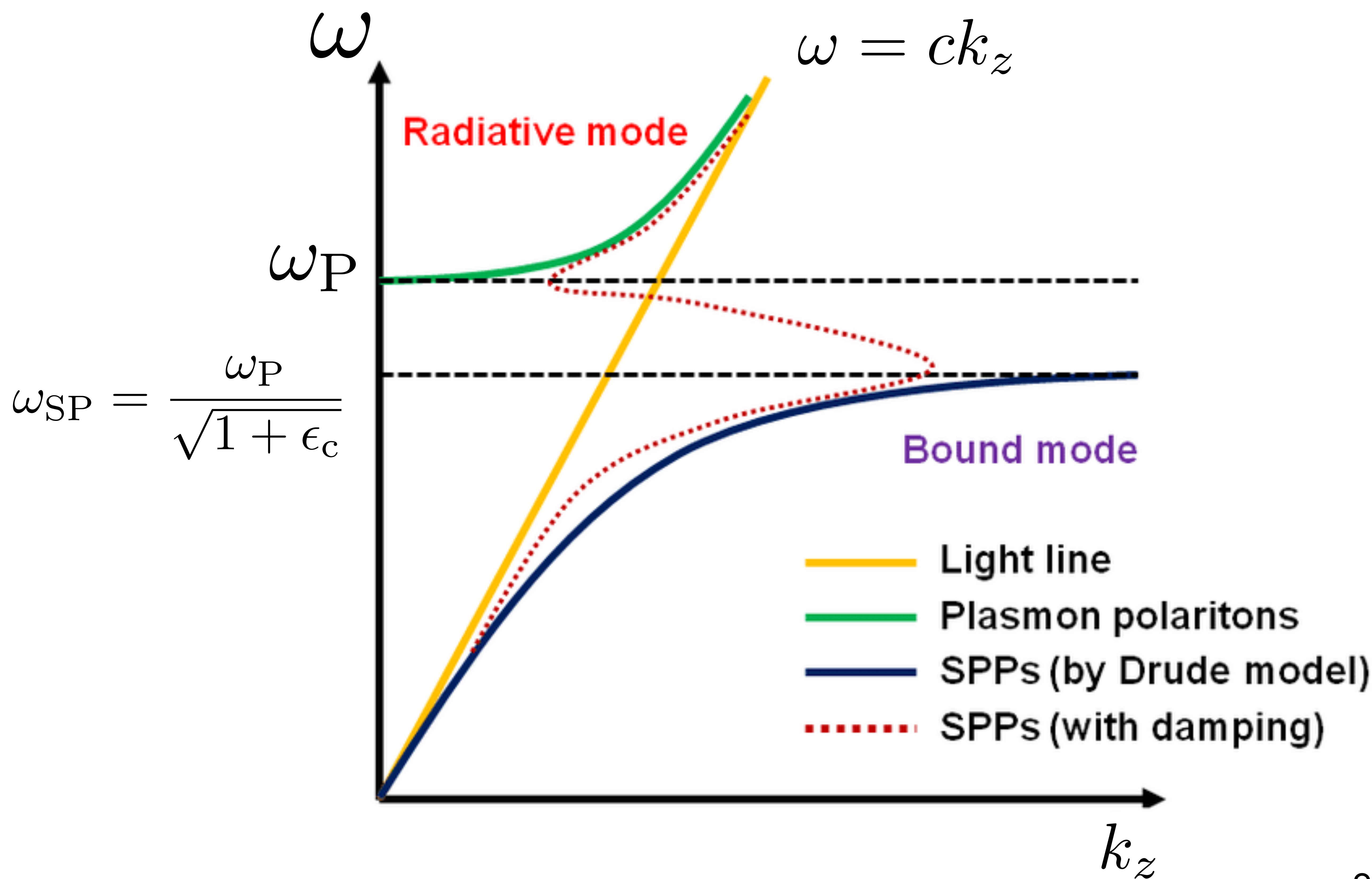
$$\longrightarrow \quad \varepsilon_s^2(\omega)\left\{k_z^2 - \frac{\omega^2}{c^2}\varepsilon_c\right\} = \varepsilon_c^2\left\{k_z^2 - \frac{\omega^2}{c^2}\varepsilon_s(\omega)\right\}$$

$\longrightarrow$ **final dispersion relation:**

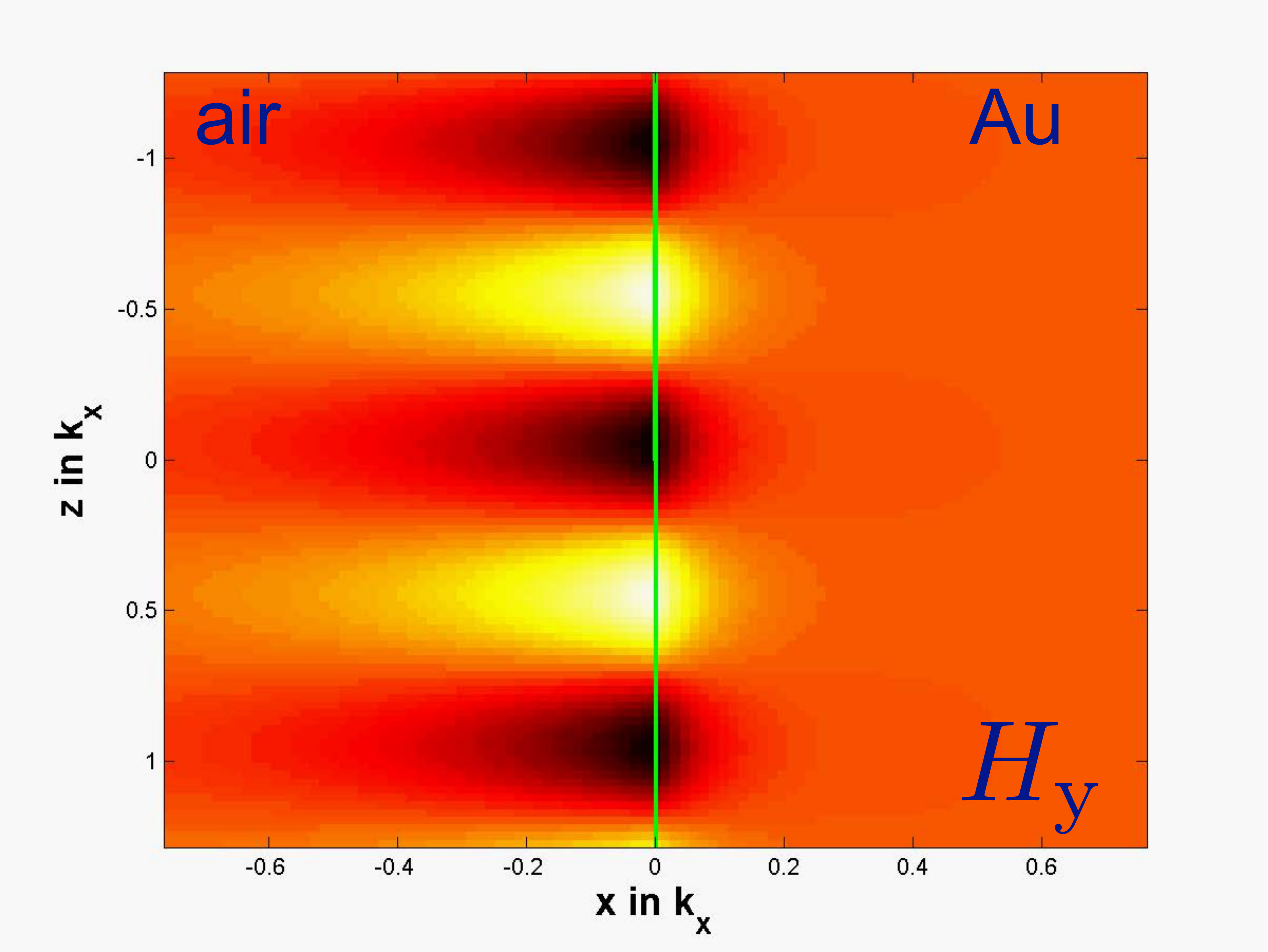
$$k_z(\omega) = \frac{\omega}{c}\sqrt{\frac{\varepsilon_s(\omega)\varepsilon_c}{\varepsilon_c + \varepsilon_s(\omega)}}$$

conditions: $\epsilon_c \epsilon_s < 0 \quad \epsilon_c + \epsilon_s < 0$

Dispersion relation for a Drude material

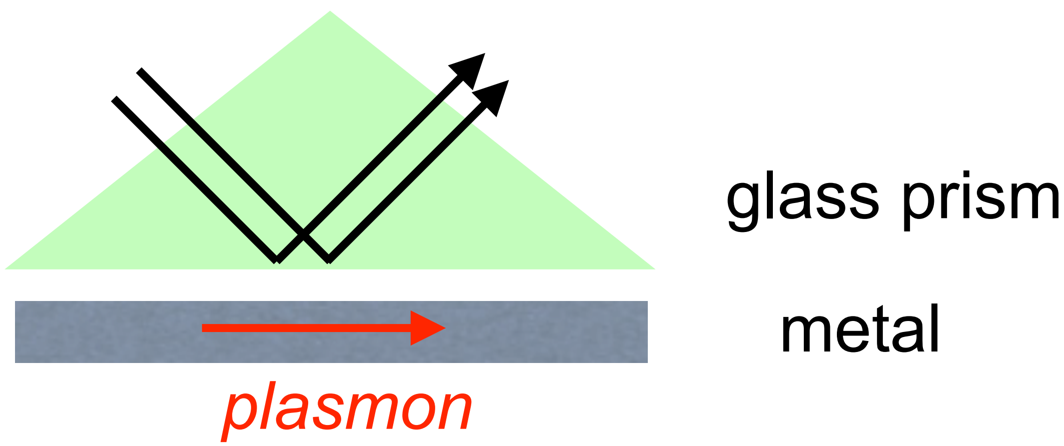


Field profile

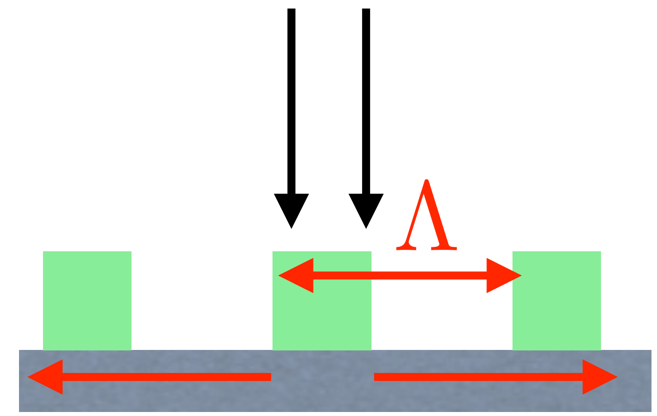


Exciting propagating surface plasmon polaritons

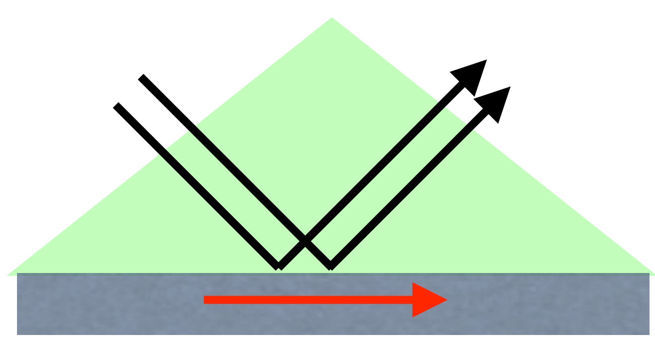
Otto configuration



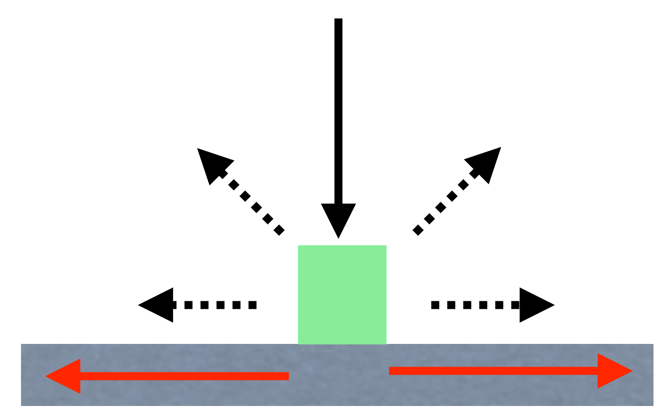
grating coupler



Kretschmann configuration



single scatterer



Propagating Surface Plasmon Polaritons

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1D Photonic Crystals

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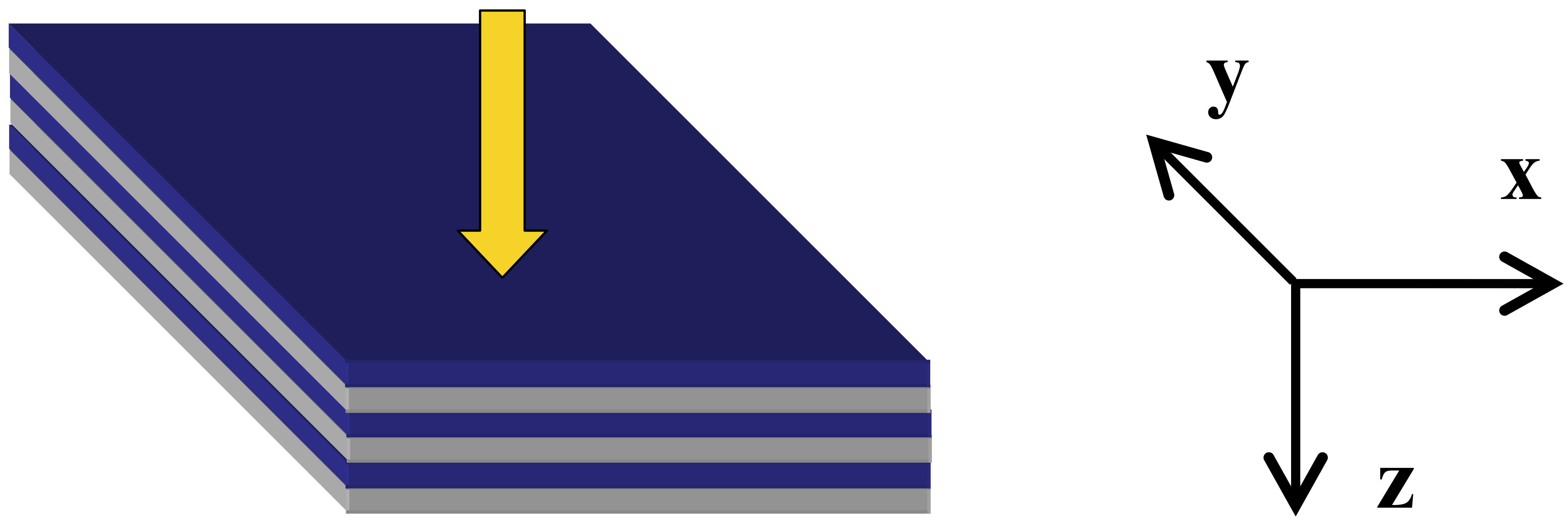
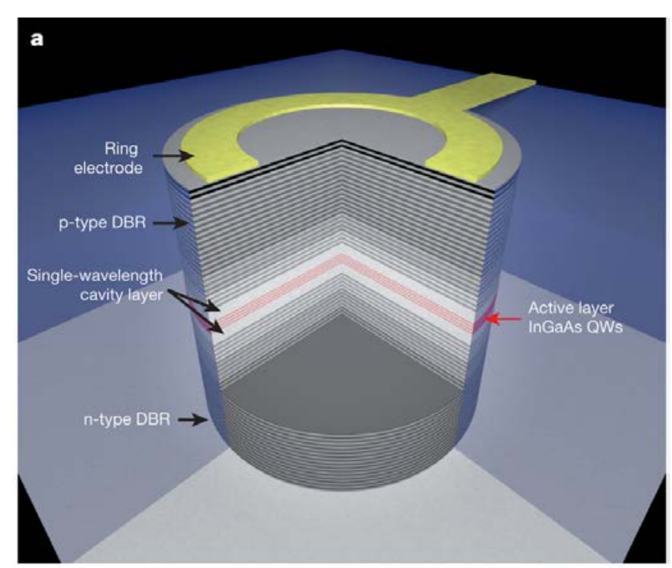
Properties of periodic systems

principle propagation direction



- Bragg mirrors
- chirped mirrors for dispersion compensation
- interferometers



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Properties of periodic systems

assumption: periodic repetition of two layers

requirement ⇒ $\epsilon(z) = \epsilon(z + \Lambda) \qquad \Lambda = d_1 + d_2$

Bloch theorem ⇒ $E(x, z; \omega) = e^{i[k_x x + k_z(k_x,\omega) z]} E_{k_z}(z)$

$$E_{k_z}(z + \Lambda) = E_{k_z}(z)$$

k_z = unknown Bloch vector

$k_z(k_x,\omega)$ = dispersion relation for Bloch wave

use $K \doteq k_z$ for simplification

Properties of periodic systems

Bloch theorem connecting layer N with N+1

$$\begin{pmatrix} F \\ G \end{pmatrix}_{(N+1)d} = \begin{pmatrix} E \\ E' \end{pmatrix}_{(N+1)d} = e^{iK\Lambda} \begin{pmatrix} E \\ E' \end{pmatrix}_{Nd}$$

known from matrix equation

$$\begin{pmatrix} E \\ E' \end{pmatrix}_{(N+1)d} = \hat{\mathbf{M}} \begin{pmatrix} E \\ E' \end{pmatrix}_{Nd}$$

$$\hat{\mathbf{M}} = \hat{\mathbf{m}}^{(2)}(d_2)\,\hat{\mathbf{m}}^{(1)}(d_1) \Longrightarrow M_{ij} = \sum_k m_{ik}^{(2)} m_{kj}^{(1)}$$

equating both expressions

$$\left\{\hat{\mathbf{M}} - e^{iK\Lambda}\hat{\mathbf{I}}\right\} \begin{pmatrix} E \\ E' \end{pmatrix}_{Nd} = 0$$

Properties of periodic systems

explicit solution using

$$\mu = e^{iK\Lambda} \qquad \det\left\{\hat{\mathbf{M}} - \mu\hat{\mathbf{I}}\right\} = 0$$

$$\mu = e^{iK\Lambda} = \frac{M_{11} + M_{22}}{2} \pm \sqrt{\left\{\frac{M_{11} + M_{22}}{2}\right\}^2 - 1}$$

evanescent solution ⟹ K purely imaginary

μ = real valued ⟹ $\left|\frac{M_{11} + M_{22}}{2}\right| \geq 1$

$$M_{11} = \cos(k_{1z}d_1)\cos(k_{2z}d_2) - \frac{k_{2z}}{k_{1z}}\sin(k_{1z}d_1)\sin(k_{2z}d_2)$$

$$M_{22} = \cos(k_{1z}d_1)\cos(k_{2z}d_2) - \frac{k_{1z}}{k_{2z}}\sin(k_{1z}d_1)\sin(k_{2z}d_2)$$

⟹ defines optical band gap with no propagating solutions

Properties of periodic systems

more explicit dispersion relation

$$\mu = e^{iK\Lambda} = \cos(K\Lambda) + i\sin(K\lambda) = \cos(K\Lambda) - \sqrt{\{\cos(K\Lambda)\}^2 - 1}$$

$$\cos(K(k_x, \omega)\Lambda) = \frac{M_{11} + M_{22}}{2}$$

consequences for evanescent waves

$$\Re\{K(k_x, \omega)\Lambda\} = n\pi$$

$$\Im\{K(k_x, \omega)\Lambda\} = \ln\left((-1)^n \left\{\frac{M_{11} + M_{22}}{2} \pm \sqrt{\left\{\frac{M_{11} + M_{22}}{2}\right\}^2 - 1}\right\}\right)$$

Properties of periodic systems

- komplex wave vector ⟹ attenuation
- no propagation in this specific parameter region
- evanescently decaying field solution
- infinite number of such regions $(n = 1....\infty)$
- called photonic band gaps
- bounds given by

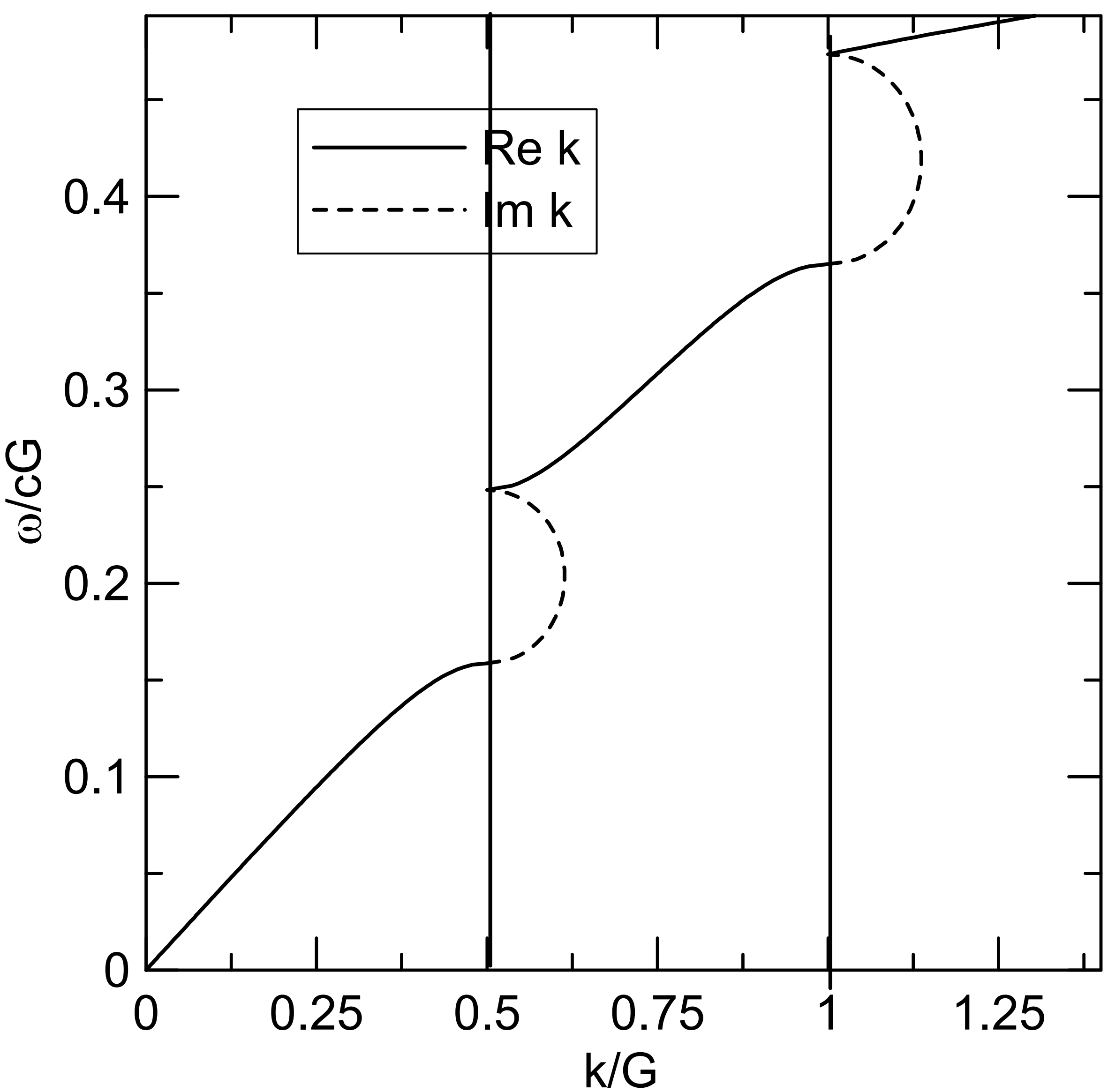
$$\Re\{K(k_x,\omega)\Lambda\} = n\pi \quad \rightarrow \quad K(k_x,\omega) = n\pi/\Lambda$$

$$\Im\{K(k_x,\omega)\Lambda\} = 0$$

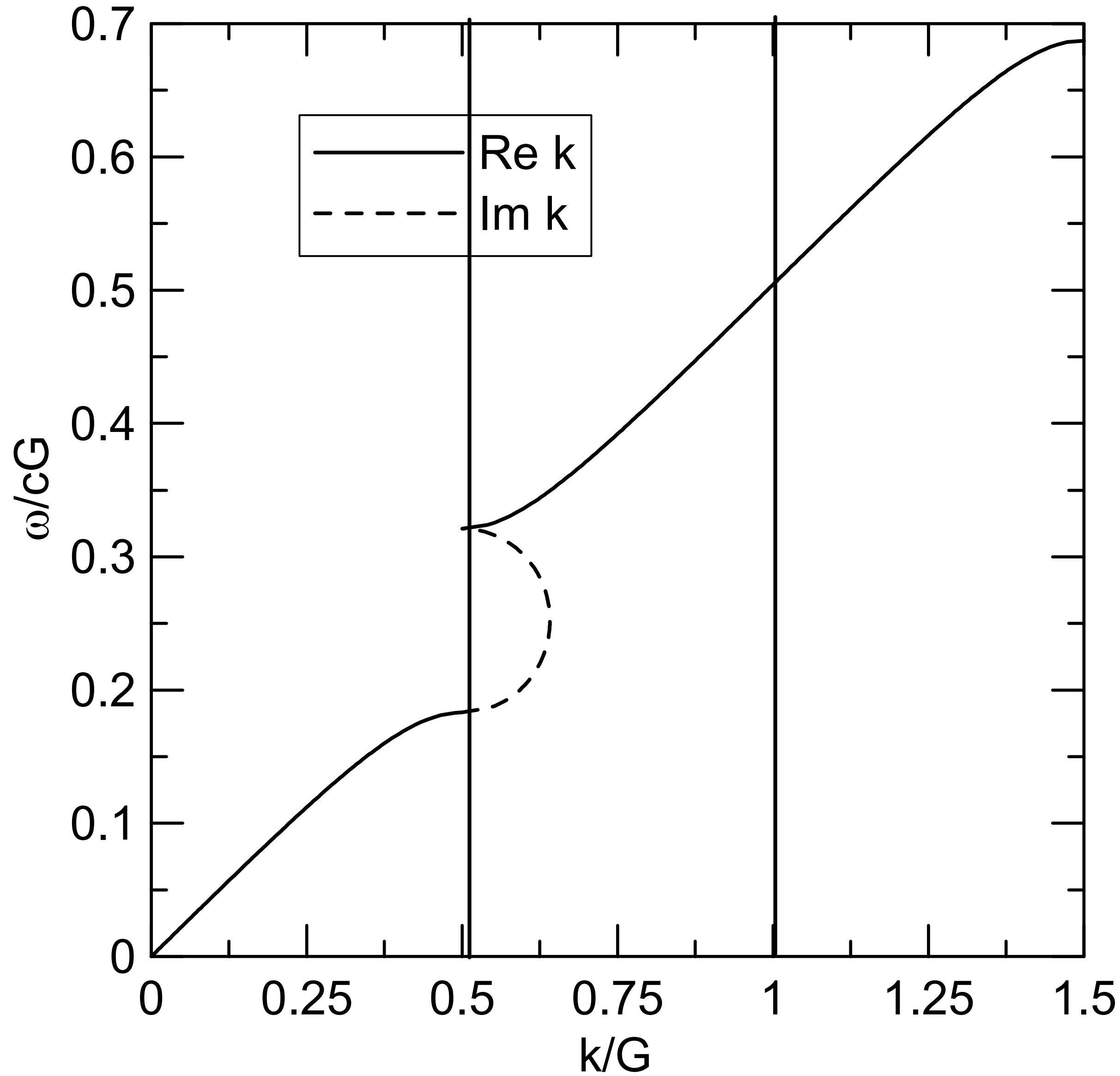
- outside the band gaps propagating exist solutions
- differ largely in their properties from free space modes

Properties of periodic systems

➡ visualisation of dispersion relation for $k_x = 0$ using dimensionless quantities



n_1=1.4, d_1=0.5Λ
n_2=3.4, d_2=0.5Λ

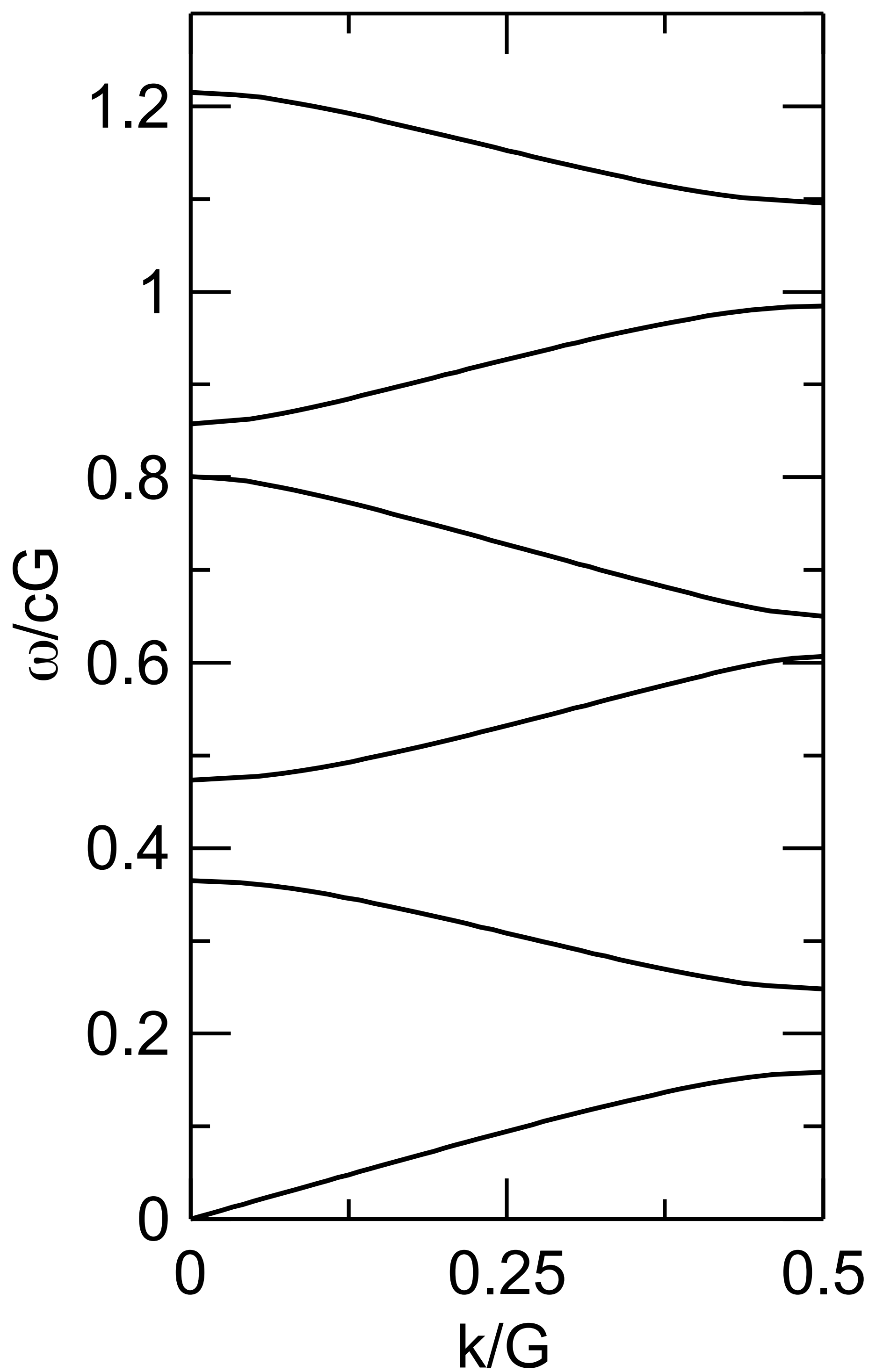


n_1=1.4, d_1 = 17/24 Λ
n_2=3.4 , d_2 =7/24 Λ

Properties of periodic systems

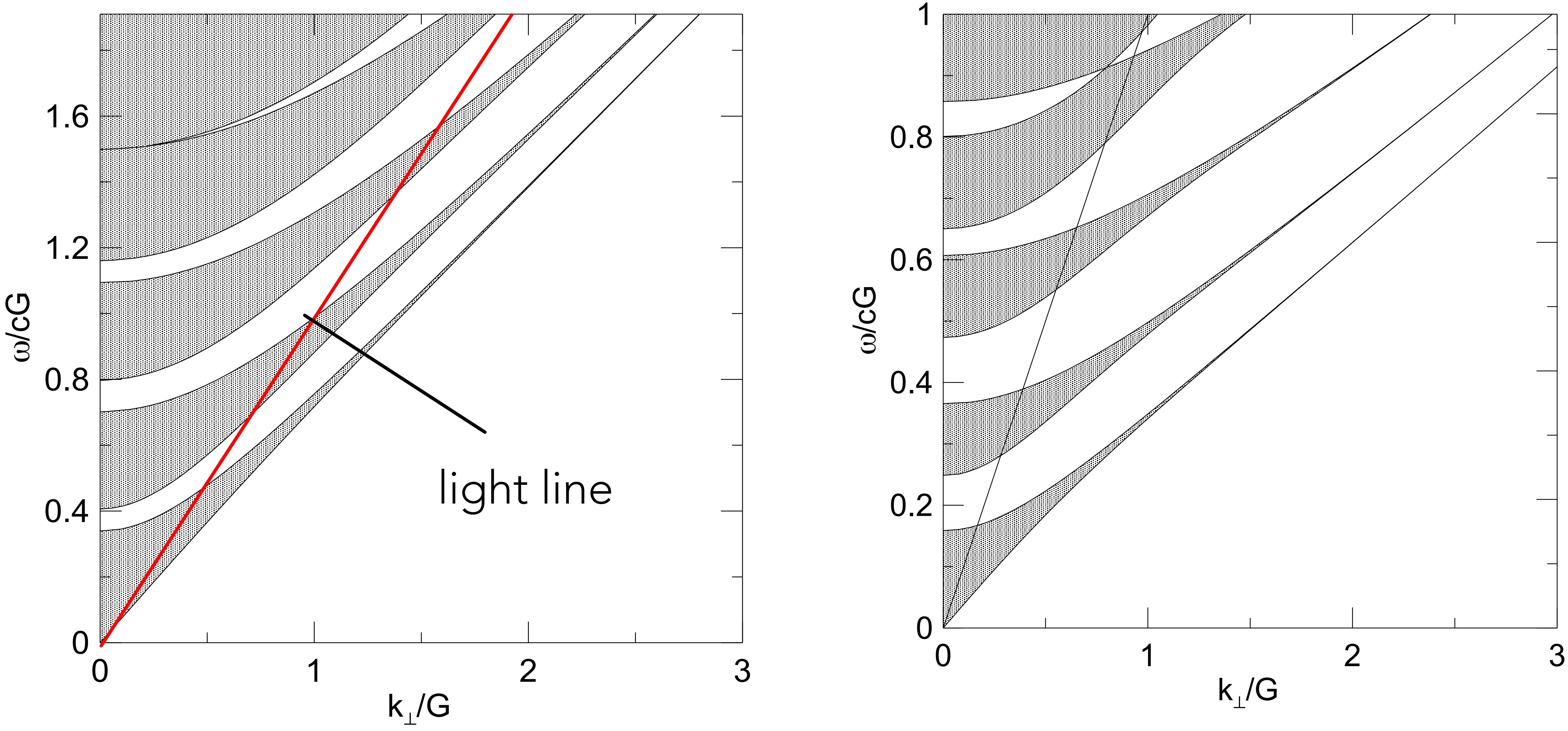
- visualisation of dispersion relation for $k_x = 0$ using dimensionless quantities

- reduced band structure (Brillouin zone)



Properties of periodic systems

projection of the bands for oblique incidence



n_1=1.0, d_1=0.33Λ
n_2=1.5, d_2=0.67Λ

n_1=1.4, d_1=0.5Λ
n_2=3.4, d_2=0.5Λ

Properties of periodic systems

attenuation for a a binary systems

$$\cos(K(k_x,\omega)\Lambda) = \frac{M_{11}+M_{22}}{2}$$
$$= \cos\left(\frac{\omega}{c}n_1 d_1\right)\cos\left(\frac{\omega}{c}n_2 d_2\right) - \frac{1}{2}\left(\frac{n_2}{n_1}+\frac{n_1}{n_2}\right)\sin\left(\frac{\omega}{c}n_1 d_1\right)\sin\left(\frac{\omega}{c}n_2 d_2\right)$$

in the first forbidden band with $K\Lambda = \pi + ix$

$$\frac{\omega_B}{c}n_1 d_1 = \frac{\omega_B}{c}n_2 d_2 = \frac{\pi}{2}$$

lambda-quarter plates

$$\cosh x_{\mathrm{ma}} = \frac{1}{2}\left(\frac{n_2}{n_1}+\frac{n_1}{n_2}\right)$$

$$x_{\mathrm{ma}} = \Lambda\Im\{K\}_{\mathrm{ma}} \approx \frac{n_2-n_1}{n_2+n_1}$$

attenuation proportional to index contrast

width of the gap

$$\Delta\omega_{\mathrm{gap}} \approx \frac{2\omega_B}{\pi}x_{\mathrm{ma}} = \frac{2\omega_B}{\pi}\Lambda\Im(K)_{\mathrm{ma}}$$

spectral width proportional to index contrast

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