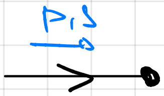
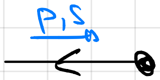
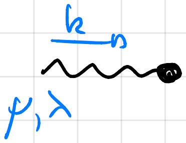
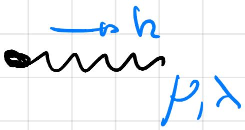
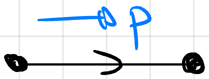
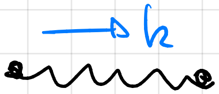
8. FEYNMAN rules for QED

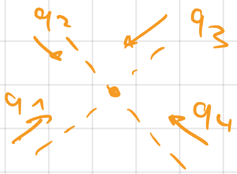
(a) From the considerations in 7. we can derive the following FEYNMAN rules:

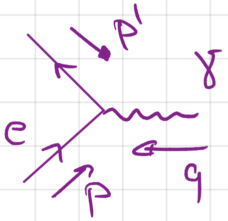
				Origin
incoming	e^-		$u_s(p)$	$\underbracket{\psi\ a_s^+}$
	e^+		$\bar{v}_s(p)$	$\underbracket{\bar{\psi}\ b_s^+}$
	γ		$\varepsilon_\mu^{(\lambda)}(\vec{k})$	$\underbracket{A_\mu\ c_\lambda^+}$
outgoing	e^-		$\bar{u}_s(p)$	$\underbracket{a_s\ \bar{\psi}}$
	e^+		$v_s(p)$	$\underbracket{b_s\ \psi}$
	γ		$\varepsilon_\mu^{(\lambda)*}(\vec{k})$	$\underbracket{c_\lambda\ A_\mu}$
propagators	e		$\dfrac{i}{\not{p}-m+i\varepsilon}$	$\underbracket{\bar{\psi}\ \psi}$
	γ		$\dfrac{-ig_{\mu\nu}}{k^2+i\varepsilon}$	$\underbracket{A_\mu\ A_\nu}$
vertex	$ee\gamma$		$-ie\gamma^\mu$	$-e\bar{\psi}\gamma^\mu\psi A_\mu$

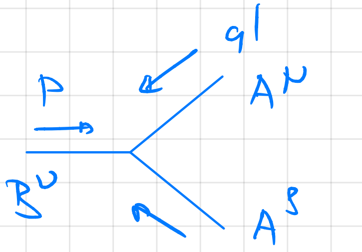
9. From $\mathcal{L}$ to FEYNMAN rules

[short cut; a recipe]

Vertices







① Look for all terms in $\mathcal{L}$, which contain exactly the fields of the vertex.	$-\frac{\lambda}{24}\phi^4$	$-e\bar{\Psi}\gamma^\mu\psi A_\mu$	$ig(\partial_\mu \overset{q}{A_\nu})A^\mu B^\nu$ $= ig(\partial_\mu A^\rho)A^\mu B^\nu g_{\rho\nu}$
② Replace all derivatives by (-i) * < incoming momentum > of the fields they are acting on.	$-\frac{\lambda}{24}\phi^4$	$-e\bar{\Psi}\gamma^\mu\psi A_\mu$	$ig(-iq_\mu)A^\rho A^\mu B^\nu g_{\rho\nu}$
③ Sum over all permutations of indices and momenta of the same external fields	$-\lambda\phi^4$	$-e\bar{\Psi}\gamma^\mu\psi A_\mu$	$g\, q_\mu g_{\rho\nu} A^\rho A^\mu B^\nu$ $+ g\, q'_\rho g_{\mu\nu} A^\rho A^\mu B^\nu$
④ Eliminate the external fields and multiply by i.	$-i\lambda$	$-ie\gamma^\mu$	$ig(q_\mu g_{\rho\nu} + q'_\rho g_{\mu\nu})$

Propagators

	 p $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2$	 p $\mathcal{L} = \bar{\Psi}(i\not{\partial} - m)\psi$
①-④ Determine 2-vertex	$i(p^2 - m^2)$	$i(\not{p} - m)$
⑤ Construct negative inverse	$\frac{i}{p^2 - m^2}$	$\frac{i(\not{p}+m)}{p^2-m^2} = \frac{i}{\not{p}-m}$
⑥ Add $i\varepsilon$ to denominator	$\frac{i}{p^2 - m^2 + i\varepsilon}$	$\frac{i}{\not{p} - m + i\varepsilon}$