Theoretische Teilchenphysik I

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Exercise Sheet 10

Due 24.06.2015

Problem 1 - Compton Scattering (100 points)

In this problem we will consider the scattering of an electron and a photon in lowest order QED, $e^-\gamma \to e^-\gamma$. Such process is called Compton Scattering.

a) (10 points) Consider the process

$$e^-(p) + \gamma(k) \to e^-(p') + \gamma(k'),$$

where external electrons and photons are on-shell, such that $p^2 = p^{'2} = m^2$, and $k^2 = k^{'2} = 0$, with m being the electron mass. Draw all tree-level Feynman diagrams contributing to the process How many diagrams are there?

b) (5 points) Using the QED Feynman rules write down the analytical expression for the diagrams drawn above.

c) (10 points) Sum the diagrams above and using Dirac equation, $(\not{p} - m)u(p) = 0$, show that the amplitude for the process can be written as

$$i\,\mathcal{M} = -i\,e^2\,\epsilon^*_\mu(k')\,\epsilon_\nu(k)\,\bar{u}(p')\left[\frac{\gamma^\mu \not{k}\gamma^\nu + 2\gamma^\mu p^\nu}{2p\cdot k} + \frac{-\gamma^\nu \not{k}'\gamma^\mu + 2\gamma^\nu p^\mu}{-2p\cdot k'}\right]u(p)\,. \tag{1}$$

d) (15 points) Compute the squared amplitude averaged over initial electron and photon polarisations and summed over final photon and electron polarisation. In order to do this recall that in QED we can simply use

$$\sum_{pol} \epsilon^*_\mu(k)\epsilon_\nu(k) = -g_{\mu\nu}\,, \quad \forall k\,. \tag{2}$$

Show that

$$\frac{1}{4}\sum_{pol}|\mathcal{M}|^2 = 2\,e^4\left[\frac{p\cdot k'}{p\cdot k} + \frac{p\cdot k}{p\cdot k'} + 2\,m^2\left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'}\right) + m^4\left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'}\right)^2\right]. \tag{3}$$

Note that this expression is Lorentz invariant and we haven't make any assumption yet on the reference frame we are working in.

e) (10 points) Consider now the process

$$e^-(p) + \gamma(k) \to e^-(p') + \gamma(k'),$$

in the reference frame where the initial electron is at rest and, for example, the initial photon moves along the positive z direction

$$p = (m, \vec{0})\,, \qquad k = (\omega, 0, 0, \omega)\,.$$

Assuming that the scattering happens in the $x - z$ plane you can parametrise the photon momentum after the scattering by the scattering angle θ as follows

$$k' = (\omega', \omega' \sin\theta, 0, \omega' \cos\theta) \,,$$

where ω' is the energy of the photon after the scattering.

Show that the energy of the outgoing photon can be written as

$$\omega' = \frac{\omega}{1 + \frac{\omega}{m}(1 - \cos\theta)} \,, \tag{4}$$

such that the process is completely parametrised in terms of the three variables m, ω and θ.

f) (10 points) Compute the differential cross-section $d\sigma$ and show that

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m^2} \left(\frac{\omega'}{\omega}\right)^2 \left[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2\theta\right] \,, \tag{5}$$

where ω' is given in Eq. (4).

g) (10 points) Study the *low-energy* limit, namely $\omega \to 0$. By integrating over the scattering angle θ show that Eq.(5) reduces to the familiar Thomson classical formula

$$\sigma_{tot} = \sigma_{Thomson} = \frac{8\pi\alpha^2}{3\,m^2} \,. \tag{6}$$

h) (30 points) Consider now the high energy limit behaviour, i.e. $\omega \to \infty$. In this limit the electron mass becomes negligible, $m \to 0$. For this reason Compton scattering in this limit is best analysed in the center-of-mass frame of the electron-photon pair. Unfortunately, we cannot simply put $m = 0$ from the beginning since this would generate a divergence in the total cross section σ_{tot}. Show that in the high-energy limit the total cross-section behaves as

$$\sigma_{tot} \approx \frac{2\pi\alpha^2}{s} \ln\left(\frac{s}{m^2}\right) , \tag{7}$$

where $s = (p + k)^2$ is the total energy in the center-of-mass frame. We see that the cross-section is logarithmically enhanced when $s \gg m^2$.