Theoretische Teilchenphysik I

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Exercise Sheet 11

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Problem 1 - Spinor-helicity methods and the forward-backward asymmetry (100 points)

The goal of this exercise is to apply spinor-helicity methods to perform a realistic computation. Consider the annihilation of a e^+e^- pair to a $\mu^+\mu^-$ pair in the Standard Model (assume all fermion masses to be zero),

$$e^-(p_1) + e^+(p_2) \to \mu^-(p_3) + \bar{\mu}^+(p_4)\,. \tag{1}$$

As you know, in QED such a process can occur through the exchange of a virtual photon:

$$e^-(p_1) + e^+(p_2) \to \gamma^*(q) \to \mu^-(p_3) + \bar{\mu}^+(p_4)\,.$$

In the complete Standard Model, though, also weak interactions contribute to this process through the exchange of a virtual Z boson,

$$e^-(p_1) + e^+(p_2) \to Z^*(q) \to \mu^-(p_3) + \bar{\mu}^+(p_4)\,.$$

In QED the interaction of charged leptons with photons is given by the vertex $ie\gamma_\mu$. Their interaction with Z-bosons, instead, is more complicated and involves also a parity-violating pseudo-vector interaction proportional to $\gamma^\mu\gamma^5$. The full vertex can be written as by $ie/(2\sin(2\theta_W))\,(g_V\gamma_\mu + g_A\gamma_\mu\gamma_5)$, where $g_V = (1 - 4\sin^2\theta_W)$ and $g_A = 1$, and θ_W is the Weinberg angle $\theta_W \approx 30°$.

1. (10 points) Draw the two Feynman diagrams contributing to (1) and use the Feynman rules above to write the expression for the amplitude.

2. (15 points) Split the above vertices into vertices that describe interactions of vector bosons with fermions of definite helicities. Show, that the amplitude $e^+e^- \to \gamma + Z \to \mu^+\mu^-$ can be written by

$$\begin{aligned}\mathcal{M}_{Z+\gamma} = \frac{ie^2}{s}\Big(& A_{LL}\bar{v}_{p_2}\gamma^\mu\omega_L u(p_1)\bar{u}(p_3)\gamma_\mu\omega_L v(p_4) + A_{RL}\bar{v}_{p_2}\gamma^\mu\omega_R u(p_1)\bar{u}(p_3)\gamma_\mu\omega_L v(p_4)\\ &+ A_{LR}\bar{v}_{p_2}\gamma^\mu\omega_L u(p_1)\bar{u}(p_3)\gamma_\mu\omega_R v(p_4) + A_{RR}\bar{v}_{p_2}\gamma^\mu\omega_R u(p_1)\bar{u}(p_3)\gamma_\mu\omega_R v(p_4)\Big)\end{aligned} \tag{2}$$

   where $\omega_{L,R} = (1 \pm \gamma_5)/2$ are projectors on different helicity states. Express the coefficients A_{ij} through the couplings $g_{A,V}$.

3. (15 points) Calculate all the relevant helicity amplitudes in terms of spinor products.

4. (20 points) Calculate the sum of the helicity amplitudes squared. Show that this sum can be written as

$$\sum_{\text{hel}} |\mathcal{M}|^2 = X_1(1 + \cos^2\theta) + X_2\cos\theta, \tag{3}$$

   where θ is the μ^- production angle relative to e^- direction. Express $X_{1,2}$ in terms of A_{ij}.

5. (20 points) Since the cross-section for $e^+e^- \to \mu^-\mu^+$ is obtained from $\sum |\mathcal{M}|^2$ by integrating over the scattering angle and since

$$\int_{-1}^{1} \mathrm{d}\cos\theta \ \cos\theta = 0, \tag{4}$$

the scattering cross-section is proportional to $X_1(s)$. To study $X_2(s)$, one can define a **forward-backward asymmetry**

$$A_{FB} = \frac{\int_0^1 \mathrm{d}\cos\theta \ \mathrm{d}\sigma/\mathrm{d}\cos\theta - \int_{-1}^0 \mathrm{d}\cos\theta \ \mathrm{d}\sigma/\mathrm{d}\cos\theta}{\int_0^1 \mathrm{d}\cos\theta \ \mathrm{d}\sigma/\mathrm{d}\cos\theta + \int_{-1}^0 \mathrm{d}\cos\theta \ \mathrm{d}\sigma/\mathrm{d}\cos\theta}, \tag{5}$$

which gives the fractional difference in the number of negatively charged muons which are produced in the forward and backward hemispheres, defined w.r.t. electron direction of motion. Calculate the forward-backward asymmetry in terms of $X_{1,2}$.

6. (20 points) Find A_{FB} in the small energy limit $s \ll M_Z^2$ and in the Z-resonance limit $s \to M_Z^2$, where the photon exchange can, effectively, be neglected.