Theoretische Teilchenphysik I

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Exercise Sheet 12

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Problem 1 (50 points) - The Higgs particle and the hydrogen atom

The interaction of the Higgs boson H to fermions is described by the following term in the Lagrangian

$$\mathcal{L}_{\text{int}} = \frac{m_f}{v} H \bar{f} f, \tag{1}$$

where m_f is the mass of the fermion f and $v = 250$ GeV is the vacuum expectation value of the Higgs boson field. The mass of the Higgs boson is 125 GeV.

We consider QED with electrons and protons supplemented with the Higgs boson. We can think of the proton as a fundamental fermion; its interaction with the Higgs boson is given by Eq.(1), where the mass of the proton should be used, $m_f = m_p$.

a) (20 points) Write down the amplitude for the electron-proton scattering due to the exchange of the Higgs boson in the non-relativistic approximation (consider only the leading term, explain what "leading" means). Use the relation between scattering amplitude and the interaction potential to find the interaction potential between electron and proton caused by the Higgs boson exchange. Show that the potential is proportional to a delta function $\delta(\vec{r})$ in the position space.

b) (20 points) Calculate the Higgs-boson induced correction to *every* energy level of the hydrogen atom. You may need the expression for the wave function at $\vec{r} = 0$ for nS levels. It reads

$$|\Psi_{nS}(0)|^2 = \frac{1}{\pi a^3 n^3}, \tag{2}$$

where n is the principal quantum number and a is the Bohr radius.

c) (10 points) The difference between energies of $2S$ and $1S$ levels of the hydrogen atom is measured very precisely. The error on the measurement is 2.067×10^{-13} eV. How does this error compare to the contribution of the Higgs boson to the difference between $2S$ and $1S$ energy levels? Estimate how light the Higgs boson should be to significantly impact (by significant we mean "comparable to the accuracy of the measurement") the structure of the energy levels of a hydrogen atom?

Problem 2 (50 points) - Parity violation in hydrogen atom

Z boson is one of the carriers of the weak force. It is a spin-one particle. Its interaction with fermions is described by the following term in the Lagrangian

$$\mathcal{L}_{\text{int}} = Z^\mu \bar{f} \left(g_V^{(f)} \gamma_\mu + g_A^{(f)} \gamma_\mu \gamma_5 \right) f. \tag{3}$$

Similar to the previous problem, we treat protons as fundamental fermions and describe their interaction with Z bosons using as in Eq.(3). We will in addition assume that the axial coupling for the proton is small compared to its vector coupling, $g_A^{(p)} \ll g_V^{(p)}$.

The propagator of the Z-boson

$$\langle 0|TZ_\mu Z_\nu|0\rangle \leftrightarrow \frac{-i}{q^2 - M_Z^2}\left(g_{\mu\nu} - \frac{q_\mu q_\mu}{M_Z^2}\right). \tag{4}$$

The mass of the Z-boson is $M_Z = 91$ GeV. Similar to the previous problem, we consider QED with electrons and protons supplemented with the Z-boson.

a) (15 points) QED is symmetric under parity transformations. What are the the consequences of the parity symmetry for the spectrum of hydrogen atom? Explain why the Lagrangian in Eq.(3) violates parity.

b) (25 points) To understand implications of parity violation for a hydrogen atom, we need to find parity-violating interactions caused by the existence of Z-boson. To do so, write the amplitude for electron-proton scattering in the non-relativistic approximation focusing on terms that are proportional to $g_A^{(e)} g_V^{(p)}$ (all computations should be performed to leading non-vanishing order in the non-relativistic expansion). Use the fact that protons are much heavier than electrons and simplify further the expression for the scattering amplitude.

   Use the relation between scattering amplitudes and interaction potentials to find the parity-violating potential due to Z-exchange. Explain why the interaction potential that you found is indeed parity-violating by considering parity transformations of the quantum-mechanical operators.

c) (10 points) Explain how the existence of these parity-violating interactions affects the energy level structure of the hydrogen atom. Can you think of any effect that can be used to prove the existence of such interactions?