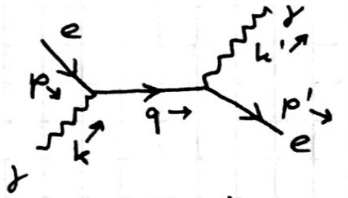
Problem 1

a) $$e^-(p) + \gamma(k) \longrightarrow e^-(p') + \gamma(k')$$

$$p^2 = p'^2 = m^2 \,, \quad k^2 = k'^2 = 0$$

① 

$$q = p + k = \sqrt{s}$$

② 

$$\tilde{q} = p - k' = \sqrt{u}$$

b) $$① = \bar{u}(p',s')\,(ie\gamma^\mu)\,\varepsilon^*_\mu(k',\lambda')\left(\frac{i}{\not{q}-m}\right) \times$$
$$\varepsilon_\nu(k,\lambda)\,(ie\gamma^\nu)\,u(p,s)$$

$$② = \bar{u}(p',s')\,(ie\gamma^\nu)\,\varepsilon_\nu(k,\lambda)\left(\frac{i}{\tilde{\not{q}}-m}\right)$$
$$\varepsilon^*_\mu(k',\lambda')\,(ie\gamma^\mu)\,u(p,s)$$

$$\Rightarrow \quad ① + ② = i\mathcal{M} = -ie^2\,\varepsilon_\nu(k,\lambda)\,\varepsilon^*_\mu(k',\lambda') \times$$
$$\bar{u}(p',s')\left[\gamma^\mu\left(\frac{1}{\not{q}-m}\right)\gamma^\nu + \gamma^\nu\left(\frac{1}{\tilde{\not{q}}-m}\right)\gamma^\mu\right]u(p,s)$$

c) $$\frac{1}{\not{q}-m} = \frac{\not{q}+m}{\not{q}^2-m^2} = \frac{\not{q}+m}{q^2-m^2} \quad \text{because}$$

$$\not{q}^2 = (\gamma^0 q^0 - \gamma^1 q^1 - \gamma^2 q^2 - \gamma^3 q^3)^2$$
$$= \gamma^0\gamma^0 q^0 q^0 + \gamma^1\gamma^1 q^1 q^1 + \gamma^2\gamma^2 q^2 q^2 + \gamma^3\gamma^3 q^3 q^3$$
$$- \gamma^0\gamma^1 q^0 q^1 - \gamma^1\gamma^0 q^0 q^1 - \gamma^0\gamma^2 q^0 q^2 - \gamma^2\gamma^0 q^0 q^2$$
$$- \gamma^0\gamma^3 q^0 q^3 - \gamma^3\gamma^0 q^0 q^3 + \gamma^1\gamma^2 q^1 q^2 + \gamma^2\gamma^1 q^1 q^2$$
$$\ldots$$
$$= \gamma^0\gamma^0 q^{0\,2} + \gamma^1\gamma^1 q^1 q^1 + \gamma^2\gamma^2 q^2 q^2 + \gamma^3\gamma^3 q^3 q^3$$
$$= q_0^2 - \vec{q}^{\,2} = q^2$$

because $\gamma^\mu\gamma^\nu = -\gamma^\nu\gamma^\mu$ for $\mu \neq \nu$.

$$\Rightarrow \quad \frac{i}{\not q - m} = \frac{\not p + \not k + m}{p^2 + k^2 + 2pk - m^2} = \frac{\not p + \not k + m}{2pk}$$

$$\frac{i}{\tilde{\not q} - m} = \frac{\not p - \not k' + m}{p^2 + k'^2 - 2pk' - m^2} = \frac{\not p - \not k' + m}{-2pk'}$$

$$\Rightarrow \quad i\mathcal{M} = -ie^2 \varepsilon_\nu(k,\lambda)\, \varepsilon^*_\mu(k',\lambda')\, \bar{u}(p',s') \times$$

$$\underbrace{\left[\gamma^\mu \left(\frac{\not p + \not k + m}{2pk}\right)\gamma^\nu + \gamma^\nu \left(\frac{\not p - \not k' + m}{-2pk'}\right)\gamma^\mu\right]}_{C^{\mu\nu}} u(p,s)$$

$$\not p \gamma^\nu = p_\mu \gamma^\mu \gamma^\nu = -p_\mu \gamma^\nu \gamma^\mu + 2 p_\mu g^{\mu\nu}$$

$$= -\gamma^\nu \not p + 2p^\nu$$

$$\Rightarrow \quad C^{\mu\nu} = \frac{1}{2pk}\left(2\gamma^\mu p^\nu - \gamma^\mu\gamma^\nu \not p + \gamma^\mu \not k \gamma^\nu + m\gamma^\mu\gamma^\nu\right)$$

$$- \frac{1}{2pk'}\left(2\gamma^\nu p^\mu - \gamma^\nu\gamma^\mu \not p - \gamma^\nu \not k' \gamma^\mu + m\gamma^\nu\gamma^\mu\right)$$

$$= \frac{1}{2pk}\left(\gamma^\mu \not k \gamma^\nu + 2\gamma^\mu p^\nu - \gamma^\mu\gamma^\nu(\not p - m)\right)$$

$$- \frac{1}{2pk'}\left(-\gamma^\nu \not k' \gamma^\mu + 2\gamma^\nu p^\mu - \gamma^\nu\gamma^\mu(\not p - m)\right)$$

$$(\not p - m)\, u(p,s) = 0 \quad \Rightarrow$$

$$i\mathcal{M} = -ie\, \varepsilon_\nu(k,\lambda)\, \varepsilon^*_\mu(k',\lambda')\, \bar{u}(p',s') \times$$

$$\underbrace{\left[\frac{\gamma^\mu \not k \gamma^\nu + 2\gamma^\mu p^\nu}{2pk} + \frac{-\gamma^\nu \not k' \gamma^\mu + 2\gamma^\nu p^\mu}{-2pk'}\right]}_{A^{\mu\nu}} u(p,s)$$

$\blacksquare$

d) $|M|^2 = M \cdot M^*$

$$M^* = e^2 \varepsilon_\rho(k',\lambda') \varepsilon^*_\sigma(k,\lambda) \times \left(\bar{u}(p',s') A^{\rho\sigma} u(p,s) \right)^*$$

$$= e^2 \varepsilon_\rho(k',\lambda') \varepsilon^*_\sigma(k,\lambda) \times u^+(p,s) (A^{\rho\sigma})^+ \gamma^{0+} \bar{u}^{+}(p',s')$$

$$= e^2 \varepsilon_\rho(k',\lambda') \varepsilon^*_\sigma(k,\lambda) \times \bar{u}^{*}(p,s) \gamma^0 (A^{\rho\sigma})^+ \gamma^0 u(p',s') \quad .$$

$$\gamma^0 \left[\frac{\gamma^\mu \not{k} \gamma^\nu + 2\gamma^\mu p^\nu}{2pk} + \frac{-\gamma^\nu \not{k}' \gamma^\mu + 2\gamma^\nu p^\mu}{-2pk'} \right]^+ \gamma^0$$

$$= \left[\frac{\gamma^0 \gamma^{\nu+} \not{k}^+ \gamma^{\mu+} \gamma^0 + 2\gamma^0 \gamma^{+\mu} p^\nu \gamma^0}{2pk} + \frac{-\gamma^0 \gamma^{\mu+} \not{k}'^+ \gamma^{\nu+} \gamma^0 + 2\gamma^0 \gamma^{\nu+} \gamma^0 p^\mu}{-2pk'} \right]$$

$$= \left[\frac{\gamma^\nu \not{k} \gamma^\mu + 2\gamma^\mu p^\nu}{2pk} + \frac{-\gamma^\mu \not{k}' \gamma^\nu + 2\gamma^\nu p^\mu}{-2pk'} \right] =: B^{\mu\nu}$$

$$\Rightarrow |M|^2 = e^4 \varepsilon_\nu(k,\lambda) \varepsilon^*_\mu(k',\lambda') \varepsilon_\rho(k',\lambda') \varepsilon^*_\sigma(k,\lambda) \times \bar{u}(p',s') A^{\mu\nu} u(p,s) \bar{u}(p,s) B^{\rho\sigma} u(p',s')$$

$$= e^4 \varepsilon^*_\sigma(k,\lambda) \varepsilon_\nu(k,\lambda) \varepsilon^*_\mu(k',\lambda') \varepsilon_\rho(k',\lambda') \times \mathrm{Tr}\left(u(p',s') \bar{u}(p',s') A^{\mu\nu} u(p,s) \bar{u}(p,s) B^{\rho\sigma} \right)$$

$$\Rightarrow \frac{1}{4} \sum_{\lambda,\lambda',s,s'} |M|^2 = \frac{1}{4} \sum_{s,s'} e^4 g_{\nu\sigma} g_{\mu\rho} \times \mathrm{Tr}\left(u(p',s') \bar{u}(p',s') A^{\mu\nu} u(p,s) \bar{u}(p,s) B^{\rho\sigma} \right)$$

$$= \frac{1}{4} \sum_{s,s'} e^4 \, \mathrm{Tr}\left(u(p',s') \bar{u}(p',s') A^{\mu\nu} u(p,s) \bar{u}(p,s) B_{\mu\nu} \right)$$

$$\sum_s u(p,s) \bar{u}(p,s) = \not{p} + m$$

$$\Rightarrow \frac{1}{4} \sum |M|^2 = \frac{e^4}{4} \mathrm{Tr}\left((\not{p}' + m) A^{\mu\nu} (\not{p} + m) B_{\mu\nu} \right)$$

both $A^{\mu\nu}$ and $B_{\mu\nu}$ contain an odd number of γ-Matrices $\Rightarrow$ mixed terms of $\not{p}$ and m are zero.

$$\Rightarrow \frac{1}{4}\sum |M|^2 = \frac{e^4}{4}\Big[\mathrm{Tr}\big(\not{p}'A^{\mu\nu}\not{p}\,B_{\mu\nu}\big) + m^2\,\mathrm{Tr}\big(A^{\mu\nu}B_{\mu\nu}\big)\Big]$$

$$\mathrm{Tr}\left(A^{\mu\nu}B_{\mu\nu}\right) =$$

$$\mathrm{Tr}\Bigg[\left(\frac{\gamma^\mu \not{k}\gamma^\nu + 2\gamma^\mu p^\nu}{2pk} + \frac{-\gamma^\nu\not{k}'\gamma^\mu + 2\gamma^\nu p^\mu}{-2pk'}\right)\cdot \left(\frac{\gamma_\nu\not{k}\gamma_\mu + 2\gamma_\mu p_\nu}{2pk} + \frac{-\gamma_\mu\not{k}'\gamma_\nu + 2\gamma_\nu p_\mu}{-2pk'}\right)\Bigg]$$

$$= \mathrm{Tr}\Bigg[\frac{1}{4(pk)^2}\Big(\gamma^\mu\not{k}\gamma^\nu\gamma_\nu\not{k}\gamma_\mu + 2\gamma^\mu p^\nu\gamma_\nu\not{k}\gamma_\mu + 2\gamma^\mu\not{k}\gamma^\nu\gamma_\mu p_\nu + 4\gamma^\mu p^\nu\gamma_\mu p_\nu\Big)$$
$$+ \frac{1}{4(pk')^2}\Big(\gamma^\nu\not{k}'\gamma^\mu\gamma_\mu\not{k}'\gamma_\nu - 2\gamma^\nu p^\mu\gamma_\mu\not{k}'\gamma_\nu - 2\gamma^\nu\not{k}'\gamma^\mu\gamma_\nu p_\mu + 4\gamma^\nu p^\mu\gamma_\nu p_\mu\Big)$$
$$- \frac{1}{4(pk)(pk')}\Big(-\gamma^\mu\not{k}\gamma^\nu\gamma_\mu\not{k}'\gamma_\nu + 2\gamma^\mu\not{k}\gamma^\nu\gamma_\nu p_\mu - 2\gamma^\mu p^\nu\gamma_\mu\not{k}'\gamma_\nu + 4\gamma^\mu p^\nu\gamma_\nu p_\mu$$
$$-\gamma^\nu\not{k}'\gamma^\mu\gamma_\nu\not{k}\gamma_\mu + 2\gamma^\nu p^\mu\gamma_\nu\not{k}\gamma_\mu - 2\gamma^\nu\not{k}'\gamma^\mu\gamma_\mu p_\nu + 4\gamma^\nu p^\mu\gamma_\mu p_\nu\Big)\Bigg]$$

$$= \frac{1}{4}\mathrm{Tr}\Bigg[\frac{1}{(pk)^2}\Big(16\not{k}^2 + 8\not{p}\not{k} + 8\not{p}\not{k} + 16p^2\Big)$$
$$+ \frac{1}{(pk')^2}\Big(16\not{k}'^2 - 8\not{p}\not{k}' - 8\not{p}\not{k}' + 16p^2\Big)$$
$$- \frac{1}{(pk)(pk')}\Big(8\not{p}\not{k} + 4\not{p}^2 + 8\not{p}\not{k} + 4\not{p}^2 - 8\not{p}\not{k}' - 8\not{p}\not{k}' - 2\gamma_\nu\gamma^\mu\not{k}\gamma^\nu\gamma_\mu\not{k}'\Big)\Bigg]$$

$$\mathrm{Tr}(\not{p}\not{k}) = p_\alpha k_\beta\,\mathrm{Tr}(\gamma^\alpha\gamma^\beta) = \frac{1}{2}p_\alpha k_\beta\,\mathrm{Tr}(2g^{\alpha\beta})$$
$$= p_\alpha k^\alpha\,\mathrm{Tr}(I_4) = 4\,p\cdot k = \mathrm{Tr}(pk)$$

$$\Rightarrow \quad \mathrm{Tr}(A^{\mu\nu} B_{\mu\nu}) =$$

$$\frac{1}{4} \mathrm{Tr}\Big[\frac{1}{(pk)^2} (16 k^2 + 16 pk + 16 p^2)\, I_4$$

$$+ \frac{1}{(pk')^2} (16 k'^2 - 16 pk' + 16 p^2)\, I_4$$

$$- \frac{1}{(pk)(pk')} \Big[(16 pk - 16 pk' + 8 p^2)\, I_4$$

$$- 2 k_\alpha k'_\beta \, \gamma_\nu \gamma_\mu \gamma^\alpha \gamma^\nu \gamma^\mu \gamma^\beta \Big] \Big]$$

$$\mathrm{Tr}[\gamma_\nu \gamma_\mu \gamma^\alpha \gamma^\nu \gamma^\mu \gamma^\beta]$$

$$= \mathrm{Tr}[2 g_{\nu\mu} \gamma^\alpha \gamma^\nu \gamma^\mu \gamma^\beta - \gamma_\mu \gamma_\nu \gamma^\alpha \gamma^\nu \gamma^\mu \gamma^\beta]$$

$$= \mathrm{Tr}[2 \gamma^\alpha \gamma^\nu \gamma_\nu \gamma^\beta - 2 \gamma_\mu g_\nu{}^\alpha \gamma^\nu \gamma^\mu \gamma^\beta$$

$$+ \gamma_\mu \gamma^\alpha \gamma_\nu \gamma^\nu \gamma^\mu \gamma^\beta]$$

$$= \mathrm{Tr}[8 \gamma^\alpha \gamma^\beta - 2 \gamma_\mu \gamma^\alpha \gamma^\mu \gamma^\beta + 4 \gamma_\mu \gamma^\alpha \gamma^\mu \gamma^\beta]$$

$$= \mathrm{Tr}[8 \gamma^\alpha \gamma^\beta + 2 \cdot 2 g_\mu{}^\alpha \gamma^\mu \gamma^\beta - 2 \gamma^\alpha \gamma_\mu \gamma^\mu \gamma^\beta]$$

$$= \mathrm{Tr}[8 \gamma^\alpha \gamma^\beta + 4 \gamma^\alpha \gamma^\beta - 8 \gamma^\alpha \gamma^\beta]$$

$$= 4 \mathrm{Tr}[\gamma^\alpha \gamma^\beta] = 4 \mathrm{Tr}[g^{\alpha\beta}] = g^{\alpha\beta} \cdot 16$$

$$= 4 g^{\alpha\beta} \mathrm{Tr}[I_4]$$

$$\Rightarrow \quad \mathrm{Tr}(A^{\mu\nu} B_{\mu\nu}) =$$

$$\cancel{\tfrac{1}{4}} \frac{1}{(pk)^2} (16 k^2 + 16 pk + 16 p^2)$$

$$+ \frac{1}{(pk')^2} (16 k'^2 - 16 pk' + 16 p^2)$$

$$- \frac{1}{(pk)(pk')} (16 pk - 16 pk' + 8 p^2 - 8 kk')$$

$$= \frac{16}{(pk)^2} (m^2 + pk) + \frac{16}{(pk')^2} (m^2 - pk')$$

$$- \frac{8}{(pk)(pk')} (p^2 - kk' + 2pk - 2pk')$$

$$\mathrm{Tr}\left(\not{p}' A^{\mu\nu} \not{p} B_{\mu\nu} \right)$$

$$= \mathrm{Tr}\Big[\not{p}' \Big(\frac{\gamma^\mu \not{k} \gamma^\nu + 2\gamma^\mu p^\nu}{2pk} + \frac{-\gamma^\nu \not{k}' \gamma^\mu + 2\gamma^\nu p^\mu}{-2pk'} \Big)$$

$$\times \not{p} \Big(\frac{\gamma_\nu \not{k} \gamma_\mu + 2\gamma_\mu p_\nu}{2pk} + \frac{-\gamma_\mu \not{k}' \gamma_\nu + 2\gamma_\nu p_\mu}{-2pk'} \Big) \Big]$$

$$= \mathrm{Tr}\Big[\frac{1}{4(pk)^2} \Big(\not{p}'\gamma^\mu \not{k} \gamma^\nu \not{p} \gamma_\nu \not{k} \gamma_\mu + 2 \not{p}'\gamma^\mu p^\nu \not{p} \gamma_\nu \not{k} \gamma_\mu$$
$$+ 2 \not{p}'\gamma^\mu \not{k} \gamma^\nu \not{p} \gamma_\mu p_\nu + 4 \not{p}'\gamma^\mu p^\nu \not{p} \gamma_\mu p_\nu \Big) \qquad ①$$

$$+ \frac{1}{4(pk')^2} \Big(\not{p}'\gamma^\nu \not{k}' \gamma^\mu \not{p} \gamma_\mu \not{k}' \gamma_\nu - 2 \not{p}'\gamma^\nu p^\mu \not{p} \gamma_\mu \not{k}' \gamma_\nu \qquad ②$$
$$- 2 \not{p}'\gamma^\nu \not{k} \gamma^\mu \not{p} \gamma_\nu p_\mu + 4 \not{p}'\gamma^\nu p^\mu \not{p} \gamma_\nu p_\mu \Big)$$

$$- \frac{1}{4(pk)(pk')} \Big(- \not{p}'\gamma^\mu \not{k} \gamma^\nu \not{p} \gamma_\mu \not{k}' \gamma_\nu + 2 \not{p}'\gamma^\mu \not{k} \gamma^\nu \not{p} \gamma_\nu p_\mu$$
$$- 2 \not{p}'\gamma^\mu p^\nu \not{p} \gamma_\mu \not{k}' \gamma_\nu + 4 \not{p}'\gamma^\mu p^\nu \not{p} \gamma_\nu p_\mu \qquad ③$$
$$- \not{p}'\gamma^\nu \not{k}' \gamma^\mu \not{p} \gamma_\nu \not{k} \gamma_\mu + 2 \not{p}'\gamma^\nu p^\mu \not{p} \gamma_\nu \not{k} \gamma_\mu$$
$$- 2 \not{p}'\gamma^\nu \not{k}' \gamma^\mu \not{p} \gamma_\mu p_\nu + 4 \not{p}'\gamma^\nu p^\mu \not{p} \gamma_\mu p_\nu \Big) \Big]$$

$$① = \frac{1}{4(pk)^2} \mathrm{Tr}\Big(p'_\alpha \gamma^\alpha \gamma^\mu k_\beta \gamma^\beta \gamma^\nu p_\delta \gamma^\delta \gamma_\nu k_\varepsilon \gamma^\varepsilon \gamma_\mu$$
$$+ 2 p'_\alpha \gamma^\alpha \gamma^\mu p^\nu p_\beta \gamma^\beta \gamma_\nu k_\delta \gamma^\delta \gamma_\mu$$
$$+ 2 p'_\alpha \gamma^\alpha \gamma^\mu k_\beta \gamma^\beta \gamma^\nu p_\delta \gamma^\delta \gamma_\mu p_\nu$$
$$+ 4 p'_\alpha \gamma^\alpha \gamma^\mu p^\nu p_\nu p_\beta \gamma^\beta \gamma_\mu \Big)$$

$$= \frac{1}{4(pk)^2} \Big(p'_\alpha k_\beta p_\delta k_\varepsilon \mathrm{Tr}(\gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu \gamma^\delta \gamma_\nu \gamma^\varepsilon \gamma_\mu)$$
$$+ 2 p'_\alpha p_\nu p_\beta k_\delta \mathrm{Tr}(\gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu \gamma^\delta \gamma_\mu)$$
$$+ 2 p'_\alpha k_\beta p_\delta p_\nu \mathrm{Tr}(\gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu \gamma^\delta \gamma_\mu)$$
$$+ 4 p'_\alpha p_\beta p^2 \mathrm{Tr}(\gamma^\alpha \gamma^\mu \gamma^\beta \gamma_\mu) \Big)$$

* $\mathrm{Tr}(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\delta\gamma_\nu\gamma^\epsilon\gamma_\mu)$

$= \mathrm{Tr}(\gamma_\mu\gamma^\alpha\gamma^\mu\gamma^\beta \underbrace{2g^{\nu\delta}\gamma_\nu}_{2\gamma^\delta}\gamma^\epsilon - \gamma_\mu\gamma^\alpha\gamma^\mu\gamma^\beta\underbrace{\gamma^\delta\gamma^\nu\gamma_\nu}_{4\gamma^\delta}\gamma^\epsilon)$

$= \mathrm{Tr}(-2\gamma_\mu\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\delta\gamma^\epsilon)$

$= \mathrm{Tr}(-4\underbrace{\gamma_\mu g^{\alpha\mu}}_{\gamma^\alpha}\gamma^\beta\gamma^\delta\gamma^\epsilon + 2\underbrace{\gamma_\mu\gamma^\mu}_{8}\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\epsilon)$

$= 4\,\mathrm{Tr}(\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\epsilon)$

$= 16\,(\eta^{\alpha\beta}\eta^{\delta\epsilon} - \eta^{\alpha\delta}\eta^{\beta\epsilon} + \eta^{\alpha\epsilon}\eta^{\beta\delta})$ //

$\mathrm{Tr}(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\delta\gamma_\mu)$

$= \mathrm{Tr}(\underbrace{\gamma_\mu 2g^{\alpha\mu}}_{2\gamma^\alpha}\gamma^\beta\gamma^\nu\gamma^\delta - \underbrace{\gamma_\mu\gamma^\mu\gamma^\alpha}_{4\gamma^\alpha}\gamma^\beta\gamma^\nu\gamma^\delta)$

$= -2\,\mathrm{Tr}(\gamma^\alpha\gamma^\beta\gamma^\nu\gamma^\delta)$

$= -8\,(\eta^{\alpha\beta}\eta^{\nu\delta} - \eta^{\alpha\nu}\eta^{\beta\delta} + \eta^{\alpha\delta}\eta^{\beta\nu})$

$\mathrm{Tr}(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma_\mu)$

$= \mathrm{Tr}(\underbrace{\gamma_\mu 2g^{\alpha\mu}}_{2\gamma^\alpha}\gamma^\beta - \underbrace{\gamma_\mu\gamma^\mu\gamma^\alpha}_{4\gamma^\alpha}\gamma^\beta)$

$= -2\,\mathrm{Tr}(\gamma^\alpha\gamma^\beta) = \cancel{-8\eta^{\alpha\beta}}\ -8\eta^{\alpha\beta}.$

$\Rightarrow (1) = \frac{1}{4(pk)^2}\Big(p'_\alpha k_\beta p_\delta k_\epsilon \cdot 16(\eta^{\alpha\beta}\eta^{\delta\epsilon} - \eta^{\alpha\delta}\eta^{\beta\epsilon} + \eta^{\alpha\epsilon}\eta^{\beta\delta})$

$+ 2p'_\alpha p_\nu (k_\delta p_\beta + k_\beta p_\delta)(-8)(\eta^{\alpha\beta}\eta^{\nu\delta} - \eta^{\alpha\nu}\eta^{\beta\delta} + \eta^{\alpha\delta}\eta^{\beta\nu})$

$+ 4p^2 p'_\alpha p_\beta (-8)\,\eta^{\alpha\beta}\Big)$

$= \frac{1}{(pk)^2}\Big(4(\cancel{\ldots}\,(p'k)(pk) - (p'p)k^2 + (p'k)(pk))$

$-4\,((p'p)(pk) - (p'p)(pk) + (p'k)p^2$

$+ (p'k)(pp) - (p'p)(pk) + (p'p)(pk))$

$-8p^4\Big)$

$$= \frac{1}{(pk)^2} \left(8(p'k)(pk) - 4k^2(p'p) - 8p^2(p'k) - 8p^4 \right)$$

$$(2) = \frac{1}{4(pk')^2} \mathrm{Tr}\Big[p'_\alpha \gamma^\alpha \gamma^\nu k'_\beta \gamma^\beta \gamma^\mu p_\delta \gamma^\delta \gamma_\mu k'_\varepsilon \gamma^\varepsilon \gamma_\nu$$
$$- 2 p'_\alpha \gamma^\alpha \gamma^\nu p^\mu p_\beta \gamma^\beta \gamma_\mu k'_\delta \gamma^\delta \gamma_\nu$$
$$- 2 p'_\alpha \gamma^\alpha \gamma^\nu k_\beta \gamma^\beta \gamma^\mu p_\delta \gamma^\delta \gamma_\nu p_\mu$$
$$+ 4 p'_\alpha \gamma^\alpha \gamma^\nu p^\mu p_\beta \gamma^\beta \gamma_\nu p_\mu \Big]$$

$$= \frac{1}{4(pk')^2} \Big[p'_\alpha k'_\beta p_\delta k'_\varepsilon \, \mathrm{Tr}(\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\mu \gamma^\delta \gamma_\mu \gamma^\varepsilon \gamma_\nu)$$
$$- 2 p'_\alpha p_\mu p_\beta k'_\delta \, \mathrm{Tr}(\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\mu \gamma^\delta \gamma_\nu)$$
$$- 2 p'_\alpha k_\beta p_\delta p_\mu \, \mathrm{Tr}(\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\mu \gamma^\delta \gamma_\nu)$$
$$+ 4 p'_\alpha p^2 p_\beta \, \mathrm{Tr}(\gamma^\alpha \gamma^\nu \gamma^\beta \gamma_\nu) \Big]$$

$$= \frac{1}{(pk')^2} \Big[p'_\alpha k'_\beta p_\delta k'_\varepsilon \cdot 4 \left(\eta^{\alpha\beta} \eta^{\delta\varepsilon} - \eta^{\alpha\delta} \eta^{\beta\varepsilon} + \eta^{\alpha\varepsilon} \eta^{\beta\delta} \right)$$
$$- 2 p'_\alpha p_\mu (p_\beta k'_\delta + p_\delta k'_\beta)(-2)\left(\eta^{\alpha\beta} \eta^{\mu\delta} - \eta^{\alpha\mu} \eta^{\beta\delta} + \eta^{\alpha\delta} \eta^{\beta\mu} \right)$$
$$+ 4 p'_\alpha p_\beta p^2 \, (-2) \, \eta^{\alpha\beta} \Big]$$

$$= \frac{1}{(pk')^2} \Big[4 \left((p'k')(pk') - (p'p) k'^2 + (p'k')(pk') \right)$$
$$+ 4 \big((p'p)(pk') - (p'p)(pk') + (p'k')p^2 + (p'k')p^2$$
$$- (p'p)(pk') + (p'p)(pk') \big)$$
$$- 8 p^4 \Big]$$

$$= \frac{1}{(pk')^2} \left[8(p'k')(pk') - 4k'^2(p'p) + 8p^2(p'k') - 8p^4 \right]$$

$$③ = -\frac{1}{4(pk)(pk')} \mathrm{Tr}\Big(2\, p'_\alpha \gamma^\alpha \gamma^\mu k_\beta \gamma^\beta \gamma^\nu p_\delta \gamma^\delta \gamma_\nu p_\mu$$
$$+4\, p'_\alpha \gamma^\alpha \gamma^\mu p_\nu p_\beta \gamma^\beta \gamma^\nu p_\mu$$
$$+2\, p'_\alpha \gamma^\alpha \gamma^\nu p_\mu p_\beta \gamma^\beta \gamma_\nu k_\delta \gamma^\delta \gamma^\mu$$
$$+4\, p'_\alpha \gamma^\alpha \gamma^\nu p_\mu p_\beta \gamma^\beta \gamma^\mu p_\nu$$
$$-\, p'_\alpha \gamma^\alpha \gamma^\mu k_\beta \gamma^\beta \gamma^\nu p_\delta \gamma^\delta \gamma_\mu k'_\varepsilon \gamma^\varepsilon \gamma_\nu$$
$$-2\, p'_\alpha \gamma^\alpha \gamma^\mu p_\nu p_\beta \gamma^\beta \gamma_\mu k'_\delta \gamma^\delta \gamma^\nu$$
$$-\, p'_\alpha \gamma^\alpha \gamma^\nu k'_\beta \gamma^\beta \gamma^\mu p_\delta \gamma^\delta \gamma_\nu k_\varepsilon \gamma^\varepsilon \gamma_\mu$$
$$-2\, p'_\alpha \gamma^\alpha \gamma^\nu k'_\beta \gamma^\beta \gamma^\mu p_\delta \gamma^\delta \gamma_\mu p_\nu \Big)$$

$$= -\frac{1}{4(pk)(pk')} \Big(2\, p'_\alpha k_\beta p_\delta p_\mu \,\mathrm{Tr}(\gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu \gamma^\delta \gamma_\nu)$$
$$+4\, p'_\alpha p_\nu p_\beta p_\mu \,\mathrm{Tr}(\gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu)$$
$$+2\, p'_\alpha p_\mu p_\beta k_\delta \,\mathrm{Tr}(\gamma^\alpha \gamma^\nu \gamma^\beta \gamma_\nu \gamma^\delta \gamma^\mu)$$
$$+4\, p'_\alpha p_\mu p_\beta p_\nu \,\mathrm{Tr}(\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\mu)$$
$$-\, p'_\alpha k_\beta p_\delta k'_\varepsilon \,\mathrm{Tr}(\gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu \gamma^\delta \gamma_\mu \gamma^\varepsilon \gamma_\nu)$$
$$-2\, p'_\alpha p_\nu p_\beta k'_\delta \,\mathrm{Tr}(\gamma^\alpha \gamma^\mu \gamma^\beta \gamma_\mu \gamma^\delta \gamma^\nu)$$
$$-\, p'_\alpha k'_\beta p_\delta k_\varepsilon \,\mathrm{Tr}(\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\mu \gamma^\delta \gamma_\nu \gamma^\varepsilon \gamma_\mu)$$
$$-2\, p'_\alpha k'_\beta p_\delta p_\nu \,\mathrm{Tr}(\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\mu \gamma^\delta \gamma_\mu) \Big)$$

$$= -\frac{1}{4(pk)(pk')} \Big(2\, p_\delta p_\mu (p'_\alpha k_\beta - p'_\alpha k'_\beta) \,\mathrm{Tr}(\gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu \gamma^\delta \gamma_\nu)$$
$$+\, 8\, p'_\alpha p_\nu p_\beta p_\mu \,\mathrm{Tr}(\gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu)$$
$$+2\, p'_\alpha p_\beta (p_\mu k_\delta - p_\mu k'_\delta) \,\mathrm{Tr}(\gamma^\alpha \gamma^\nu \gamma^\beta \gamma_\nu \gamma^\delta \gamma^\mu)$$
$$-\, p'_\alpha p_\delta (k_\beta k'_\varepsilon + k'_\beta k_\varepsilon) \,\mathrm{Tr}(\gamma^\alpha \gamma^\nu \gamma^\beta \gamma^\mu \gamma^\delta \gamma_\nu \gamma^\varepsilon \gamma_\mu) \Big)$$

$$\mathrm{Tr}(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\delta\gamma_\nu)$$

$$= \mathrm{Tr}(\gamma^\alpha\gamma^\mu\gamma^\beta\, \underbrace{2g^{\nu\delta}\gamma_\nu}_{2\gamma^\delta} - \gamma^\alpha\gamma^\mu\gamma^\beta \underbrace{\gamma^\delta\gamma^\nu\gamma_\nu}_{4\gamma^\delta})$$

$$= \mathrm{Tr}(-2\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\delta)$$

$$= -2\ \mathrm{Tr}(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\delta)$$

$$= -8\ (\eta^{\alpha\mu}\eta^{\beta\delta} - \eta^{\alpha\beta}\eta^{\mu\delta} + \eta^{\alpha\delta}\eta^{\mu\beta})$$

$$\mathrm{Tr}(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma_\nu\gamma^\delta\gamma^\mu) = -2\,\mathrm{Tr}(\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\mu)$$

$$= -8\,(\eta^{\alpha\beta}\eta^{\delta\mu} - \eta^{\alpha\delta}\eta^{\beta\mu} + \eta^{\alpha\mu}\eta^{\beta\delta})$$

$$\mathrm{Tr}(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu) = 4\ (\eta^{\alpha\mu}\eta^{\beta\nu} - \eta^{\alpha\beta}\eta^{\mu\nu} + \eta^{\alpha\nu}\eta^{\beta\mu})$$

$$\mathrm{Tr}(\gamma^\alpha\gamma^\nu\gamma^\beta\gamma^\mu\gamma^\delta\gamma_\nu\gamma^\varepsilon\gamma_\mu)$$

$$= \mathrm{Tr}\ (\gamma_\mu\gamma^\alpha\, 2g^{\nu\beta}\gamma^\mu\gamma^\delta\gamma_\nu\gamma^\varepsilon - \gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\nu\gamma^\mu\gamma^\delta\gamma_\nu\gamma^\varepsilon)$$

$$= \mathrm{Tr}\,(2\gamma_\mu\gamma^\alpha\gamma^\mu\gamma^\delta\gamma^\beta\gamma^\varepsilon - \gamma_\mu\gamma^\alpha\gamma^\beta\, 2g^{\mu\nu}\gamma^\delta\gamma_\nu\gamma^\varepsilon$$
$$+ \gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu\gamma^\delta\gamma_\nu\gamma^\varepsilon)$$

$$= \mathrm{Tr}(\ 4\gamma_\mu g^{\alpha\mu}\gamma^\delta\gamma^\beta\gamma^\varepsilon - 2\gamma_\mu\gamma^\mu\gamma^\alpha\gamma^\delta\gamma^\beta\gamma^\varepsilon$$
$$-2\gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\mu\gamma^\varepsilon + \gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\mu 2g^{\nu\delta}\gamma_\nu\gamma^\varepsilon$$
$$- \gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\delta\gamma^\nu\gamma_\nu\gamma^\varepsilon)$$

$$= \mathrm{Tr}(\ 4\gamma^\alpha\gamma^\delta\gamma^\beta\gamma^\varepsilon - 8\gamma^\alpha\gamma^\delta\gamma^\beta\gamma^\varepsilon - 2\gamma^\mu\gamma^\varepsilon\gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\delta$$
$$+ 2\ \gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\delta\gamma^\varepsilon - 4\gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\delta\gamma^\varepsilon)$$

$$= \mathrm{Tr}\ (-4\gamma^\alpha\gamma^\delta\gamma^\beta\gamma^\varepsilon - 4g^{\mu\varepsilon}\gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\delta + 2\gamma^\varepsilon\gamma^\mu\gamma_\mu\gamma^\alpha\gamma^\beta\gamma^\delta$$
$$- 2\ \gamma_\mu\gamma^\alpha 2g^{\mu\beta}\gamma^\delta\gamma^\varepsilon + 2\ \gamma_\mu\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\delta\gamma^\varepsilon)$$

$$= \mathrm{Tr}(-4\gamma^\alpha\gamma^\delta\gamma^\beta\gamma^\epsilon - 4\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\epsilon + 8\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\epsilon$$
$$- 4\gamma^\beta\gamma^\alpha\gamma^\delta\gamma^\epsilon + 2\gamma_\mu 2g^{\alpha\mu}\gamma^\beta\gamma^\delta\gamma^\epsilon$$
$$- 2\gamma_\mu\gamma^\mu\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\epsilon)$$

$$= \mathrm{Tr}(-4\gamma^\beta\gamma^\alpha\gamma^\delta\gamma^\epsilon + 4\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\epsilon$$
$$- 8\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\epsilon)$$

$$= -4\,\mathrm{Tr}(\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\epsilon + \gamma^\beta\gamma^\alpha\gamma^\delta\gamma^\epsilon)$$

$$= -4\,\mathrm{Tr}(2g^{\alpha\beta}\gamma^\delta\gamma^\epsilon)$$

$$= -8\,\mathrm{Tr}(\gamma^\delta\gamma^\epsilon)g^{\alpha\beta} = -8g^{\alpha\beta}g^{\delta\epsilon}\cdot 4$$

$$= -32\,g^{\alpha\beta}g^{\delta\epsilon}$$

$$\Rightarrow (3) = \frac{1}{-4(pk)(pk')}\Big(2p_\delta p_\mu(p_\alpha' k_\beta - p_\alpha' k_\beta')(-8)(\eta^{\alpha\mu}\eta^{\beta\delta} - \eta^{\alpha\beta}\eta^{\mu\delta} + \eta^{\alpha\delta}\eta^{\mu\beta})$$
$$+ 8p_\alpha' p_\nu p_\beta p_\mu \cdot 4\cdot(\eta^{\alpha\mu}\eta^{\beta\nu} - \eta^{\alpha\beta}\eta^{\mu\nu} + \eta^{\alpha\nu}\eta^{\beta\mu})$$
$$+2p_\mu p_\beta(p_\alpha' k_\delta - p_\alpha' k_\delta')(-8)(\eta^{\alpha\beta}\eta^{\delta\mu} - \eta^{\alpha\delta}\eta^{\beta\mu} + \eta^{\alpha\mu}\eta^{\beta\delta})$$
$$- p_\alpha' p_\delta(k_\beta k_\epsilon' + k_\beta' k_\epsilon)(-32)g^{\alpha\beta}g^{\delta\epsilon}\Big)$$

$$= \frac{1}{-4(pk)(pk')}\Big(-16\big[(p'p)(kp) - (p'k)p^2 + (p'p)(pk)$$
$$-(pp')(k'p) + (p'k')p^2 - \cancel{(pp)}(p'p)(pk')\big]$$
$$+32\big[(p'p)p^2 - (p'p)p^2 + (p'p)p^2\big]$$
$$-16\big[(p'p)(kp) - (p'k)p^2 + (p'p)(pk)$$
$$-(p'p)(k'p) + (p'k')p^2 - (p'p)(k'p)\big]$$
$$+32\big[(p'k)(pk') + (p'k')(pk)\big]\Big)$$

$$= \frac{4}{(pk)(pk')}\Big[2(kp)(p'p) + p^2(p'k' - p'k) - 2(p'p)(pk')$$
$$-2(p'p)p^2 + 2(p'p)(pk) + p^2(p'k' - p'k)$$
$$-2(p'p)(k'p) - 2(p'k)(pk')$$
$$-2(p'k')(pk)\Big]$$

$$= \frac{4}{(pk)(pk')}\Big[\,4(p'p)(pk) - 4(p'p)(pk') + 2p^2(p'k' - p'k) - 2p^2(p'p) - 2(p'k)(pk') - 2(pk)(p'k')\,\Big]$$

$$= \frac{8}{(pk)(pk')}\Big[\,2(p'p)(pk) - 2(p'p)(pk') + m^2(p'k' - p'k) - m^2(p'p) - (p'k)(pk') - (pk)(p'k')\,\Big]$$

$$\Rightarrow \quad \mathrm{Tr}\left(\not{p}'\, A^{\mu\nu}\, \not{p}\, B_{\mu\nu}\right) = ① + ② + ③$$

$$\Rightarrow \frac{1}{(pk)^2}\Big(8(p'k)(pk) - 4k^2(p'p) - 8p^2(p'k) - 8p^4\Big)$$
$$+ \frac{1}{(pk')^2}\Big(8(p'k')(pk') - 4k'^2(p'p) + 8p^2(p'k') - 8p^4\Big)$$
$$+ \frac{8}{(pk)(pk')}\Big(2(p'p)(pk - pk') - (p'k)(pk') - (pk)(p'k') + m^2(p'k' - p'k - p'p)\Big)$$

$$= \frac{1}{(pk)^2}\Big(8(p'k)(pk) - 8m^2(p'k) - 8m^4\Big)$$
$$+ \frac{1}{(pk')^2}\Big(8(p'k')(pk') + 8m^2(p'k') - 8m^4\Big)$$
$$+ \frac{8}{(pk)(pk')}\Big(2(p'p)(pk - pk') - (p'k)(pk') - (pk)(p'k') + m^2(p'k' - p'k - p'p)\Big)$$

$$= \frac{8p'k}{pk} - \frac{8m^2 p'k}{(pk)^2} - \frac{8m^4}{(pk)^2} + \frac{8p'k'}{pk'} + \frac{8m^2 p'k'}{(pk')^2}$$
$$- \frac{8m^4}{(pk')^2} + \frac{16p'p}{(pk')} - \frac{16p'p}{(pk)} - \frac{8(p'k)}{(pk)}$$
$$- \frac{8p'k'}{pk'} + \frac{8m^2}{(pk)(pk')}\left(p'k' - p'k - p'p\right)$$

$$= 16\, p'p \left(\frac{1}{(pk')} - \frac{1}{(pk)} \right) + 8m^2 \left(\frac{p'k'}{(pk')^2} - \frac{p'k}{(pk)^2} \right)$$

$$+ \frac{8m^2}{(pk)(pk')} \left(p'k' - p'k - p'p \right) - 8m^4 \left(\frac{1}{(pk)^2} + \frac{1}{(pk')^2} \right)$$

$$\Rightarrow \frac{1}{4} \sum |M|^2 = \frac{e^4}{4} \Big[16\, p'p \left(\frac{1}{(pk')} - \frac{1}{(pk)} \right) + 8m^2 \left(\frac{p'k'}{(pk')^2} - \frac{p'k}{(pk)^2} \right)$$

$$+ \frac{8m^2}{(pk)(pk')} \left(p'k' - p'k - p'p \right) - 8m^4 \left(\frac{1}{(pk)^2} + \frac{1}{(pk')^2} \right)$$

$$+ \frac{16m^4}{(pk)^2} + \frac{16m^2}{pk} + \frac{16m^4}{(pk')^2} - \frac{16m^2}{(pk')}$$

$$- \frac{8m^4}{(pk)(pk')} - \frac{8m^2}{(pk)(pk')} \left(2pk - 2pk' - kk' \right) \Big]$$

$$= \frac{e^4}{4} \Big[m^4 \left(\frac{8}{(pk)^2} + \frac{8}{(pk')^2} - \frac{8}{(pk)(pk')} \right)$$

$$+ 8m^2 \Big(\frac{p'k'}{(pk')^2} - \frac{p'k}{(pk)^2} + \frac{p'k'}{(pk)(pk')} - \frac{p'k}{(pk)(pk')} - \frac{p'p}{(pk)(pk')}$$

$$+ \frac{2}{pk} - \frac{2}{(pk')} - \frac{2}{(pk')} + \frac{2}{pk} + \frac{kk'}{(pk)(pk')} \Big)$$

$$+ 16\, p'p \left(\frac{1}{(pk')} - \frac{1}{(pk)} \right) \Big]$$

$$= \frac{e^4}{4} \Big[8m^4 \left(\frac{1}{(pk)^2} + \frac{1}{(pk')^2} - \frac{1}{(pk)(pk')} \right)$$

$$+ 8m^2 \Big(\frac{p'k'}{(pk')^2} - \frac{p'k}{(pk)^2} + \frac{p'k'}{(pk)(pk')} - \frac{p'k}{(pk)(pk')} - \frac{p'p}{(pk)(pk')}$$

$$+ \frac{4}{(pk)} - \frac{4}{(pk')} + \frac{kk'}{(pk)(pk')} \Big)$$

$$+ 16\, p'p \left(\frac{1}{(pk')} - \frac{1}{(pk)} \right) \Big].$$

$$p' = p + k - k'$$

$$\Rightarrow \frac{1}{4}\sum_i |M|^2 =$$

$$2e^4 \Big[m^4 \left(\frac{1}{(pk')^2} + \frac{1}{(pk)^2} - \frac{1}{(pk)(pk')} \right)$$

$$+ m^2 \Big(\frac{pk' + kk' - k'^2}{(pk')^2} - \frac{pk + k^2 - k'k}{(pk)^2}$$

$$+ \frac{pk' + kk' - k'^2}{(pk)(pk')} - \frac{(pk + k^2 - k'k)}{(pk)(pk')}$$

$$- \frac{p^2 + pk - pk'}{(pk)(pk')} + \frac{4}{(pk)} - \frac{4}{(pk')}$$

$$+ \frac{kk'}{(pk)(pk')} \Big)$$

$$+ 16 (p^2 + pk + pk') \left(\frac{1}{(pk')} - \frac{1}{(pk)} \right) \Big]$$

$$= 2e^4 \Big[m^4 \left(\frac{1}{(pk')^2} + \frac{1}{(pk)^2} - \frac{1}{(pk)(pk')} \right)$$

$$+ m^2 \Big(\cancel{\frac{1}{(pk')}} + \frac{kk'}{(pk')^2} - \cancel{\frac{1}{(pk)}} + \frac{k'k}{(pk)^2}$$

$$+ \cancel{\frac{1}{(pk)}} + \frac{kk'}{(pk)(pk')} - \cancel{\frac{1}{(pk')}} + \frac{k'k}{(pk)(pk')}$$

$$- \frac{m^2}{(pk)(pk')} - \frac{1}{(pk')} + \frac{1}{(pk)} + \frac{4}{(pk)} - \frac{4}{(pk')}$$

$$+ \frac{kk'}{(pk)(pk')} \Big)$$

$$+ 16 m^2 \left(\frac{1}{(pk')} - \frac{1}{(pk)} \right) + 16 \frac{pk}{pk'} - \cancel{16}$$

$$+ \cancel{16} - 16 \frac{pk'}{pk} \Big) \Big]$$

$$= 2e^4 \Big[m^4 \left(\frac{1}{(pk')^2} + \frac{1}{(pk)^2} - \frac{2}{(pk)(pk')} \right)$$

$$+ m^2 \Big(kk' \left(\frac{1}{(pk)^2} + \frac{1}{(pk')^2} + \frac{3}{(pk)(pk')} \right)$$

$$+ \frac{5}{(pk)} - \frac{5}{(pk')} + \frac{16}{(pk')} - \frac{16}{(pk)} \Big)$$

$$+ 16 \left(\frac{pk}{pk'} - \frac{pk'}{pk} \right) \Big]$$

$$= 2e^4 \Big[m^4 \left(\frac{1}{pk} - \frac{1}{pk'} \right)^2$$

$$+ m^2 \left(\frac{11}{pk'} - \frac{11}{pk} \right)$$

$$\vdots$$

e) $p = (m, \vec{0})$, $k = (\omega, 0, 0, \omega)$

$k' = (\omega', \omega' \sin\theta, 0, \omega' \cos\theta)$ $\quad p' = (p_0', \vec{p}')$

$p + k = p' + k'$

$$\Rightarrow \quad m + \omega = p_0' + \omega'$$
$$0 = p_x' + \omega' \sin\theta$$
$$0 = p_y'$$
$$\omega = p_z' + \omega' \cos\theta$$

$$p_0'^2 = m^2 + p_x'^2 + p_z'^2$$
$$= m^2 + \omega'^2 + \omega^2 - 2\omega\omega' \cos\theta$$

$$\Rightarrow (m + \omega - \omega')^2 = m^2 + \omega'^2 + \omega^2 - 2\omega\omega' \cos\theta$$

$$\Rightarrow \quad 2m\omega - 2m\omega' - 2\omega\omega' = -2\omega\omega' \cos\theta$$

$$\Rightarrow \quad \omega'(\omega - \omega\cos\theta + m) = m\omega$$

$$\Rightarrow \omega' = \frac{m\omega}{\omega(1-\cos\theta) + m}$$

$$\Rightarrow \omega' = \frac{\omega}{1 + \frac{\omega}{m}(1-\cos\theta)}$$

f) $$\frac{1}{4}\sum_i |M|^2 = 2e^4 \Big[\frac{m\omega'}{m\omega} + \frac{m\omega}{m\omega'} + \dots$$
$$+ m^4 \Big(\frac{1}{m\omega} - \frac{1}{m\omega'}\Big)^2 \Big]$$

$$= 2e^4 \Big[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} + 2\Big(\frac{m}{\omega} - \frac{m}{\omega'}\Big) + \Big(\frac{m}{\omega} - \frac{m}{\omega'}\Big)^2 \Big]$$

$$\cancel{= 2e^4 \Big[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} + 2\Big(\frac{m\omega' - m\omega}{\omega\omega'}\Big) + \Big(\frac{m\omega' - m\omega}{\omega\omega'}\Big)^2 \Big]}$$

$$\cancel{= 2e^4 \Big[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} + 2\frac{m}{\omega} - \frac{2m}{\omega'}}$$

$$\frac{1}{\omega'} = \frac{1 + \frac{\omega}{m}(1-\cos\theta)}{\omega}$$

$$\rightarrow \frac{1}{4}\sum_i |M|^2 = 2e^4 \Big[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} + 2\Big(\frac{m}{\omega}\Big(1 - 1 - \frac{\omega}{m}(1-\cos\theta)\Big)\Big)$$
$$+ \frac{m^2}{\omega^2}\Big(1 - 1 - \frac{\omega}{m}(1-\cos\theta)\Big)^2 \Big]$$

$$= 2e^4 \Big[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} + 2(\cos\theta - 1)$$
$$+ (1-\cos\theta)^2 \Big]$$

$$= 2e^4 \Big[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} + 2\cos\theta - 2 + 1 + \cos^2\theta - 2\cos\theta \Big]$$

$$= 2e^4 \Big[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - 1 + \cos^2\theta \Big]$$

$$= 2e^4 \Big[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2\theta \Big]$$

~~noch zz~~:

$$d\sigma = \frac{d^3p'\, d^3k'}{(2\pi)^6\, 4\, E_{p'} E_{k'}}\, \delta^{(4)}(p' + k' - p - k)\, \frac{\frac{1}{4}\sum_i |M|^2}{4\sqrt{(p\cdot k)^2}}$$

still to show:

$$dLIPS_2 = \frac{d^3p'\, d^3k'}{(2\pi)^2\, 4\, E_{p'} E_{k'}}\, \delta^{(4)}(p' + k' - p - k)\, \frac{1}{4\sqrt{(p\cdot k)^2}}$$

$$= d\cos\theta\, \frac{\pi\alpha^2}{m^2}\Big(\frac{\omega'}{\omega}\Big)^2$$

$$dLIPS_2 = \frac{d^3p'\, d^3k'}{(2\pi)^2\, 4 E_{p'} E_{k'}}\, \delta(p' + k' - \cancel{p} - k) \cdot \frac{1}{4m\omega}$$

$$= \frac{d^3k'}{(2\pi)^2 4 E_{p'} E_{k'}}\, \delta(p_0' + k_0' - m - \omega)\, \frac{1}{4m\omega} \Bigg|_{\vec{p}' = \vec{p} + \vec{k} - \vec{k}' = \vec{k} - \vec{k}'}$$

for $\vec{k}' = |\vec{k}'|\, \hat{e}_{k'} =: \rho\, \hat{e}_k$

$k_0 = |\vec{k}'| = \rho$, $E_{k'} = \sqrt{\vec{k}'^2} = \rho$

$$dLIPS_2 = \frac{d\rho\, \rho^2\, d\varphi\, d\cos\theta \quad \delta\left(\sqrt{(\vec{k} - \vec{k}')^2 + m^2} + \rho - m - \omega\right)}{(2\pi)^2\, 16\, m\omega\rho \sqrt{(\vec{k} - \vec{k}')^2 + m^2}}$$

$$= \frac{d\rho\, \rho^2\, d\varphi\, d\cos\theta\, \delta\left(\sqrt{\omega^2 + \rho^2 - 2\omega\rho\cos\theta + m^2} + \rho - m - \omega\right)}{(2\pi)^2\, 16\, m\omega\rho \sqrt{\omega^2 + \rho^2 - 2\omega\rho\cos\theta + m^2}}$$

$$\delta\left(\sqrt{\omega^2 + \rho^2 - 2\omega\rho\cos\theta + m^2} + \rho - m - \omega\right)$$

$$= \delta(f(\rho))$$

$$f'(\rho) = \frac{\rho - \omega\cos\theta}{\sqrt{\omega^2 + \rho^2 - 2\omega\rho\cos\theta + m^2}} + 1$$

$$= \frac{\rho - \omega\cos\theta}{f(\rho) + m + \omega - \rho} + 1$$

$$f(\rho) = 0 \Leftrightarrow \omega^2 + \rho^2 - 2\omega\rho\cos\theta + m^2 = m^2 + \omega^2 + \rho^2 + 2m\omega - 2m\rho - 2\omega\rho$$

$$\Leftrightarrow -2\omega\rho\cos\theta = 2m\omega - 2m\rho - 2\omega\rho$$

$$\Leftrightarrow \rho = \frac{\omega}{1 + \frac{\omega}{m}(1 - \cos\theta)} =: \omega'$$

$$f'(\omega') = \frac{\omega' - \omega\cos\theta}{m + \omega - \omega'} + 1$$

$$= \frac{\omega' - \omega\cos\theta + m + \omega - \omega'}{m + \omega - \omega'} = \frac{m + \omega(1 - \cos\theta)}{m + \omega - \omega'}$$

$$\Rightarrow dLIPS_2 = \frac{d\varrho\, \varrho^2\, d\varphi\, d\cos\theta \quad \delta(\varrho-\omega')}{(2\pi)^2 16\, m \varrho \omega \left(f(\varrho) + m+\omega-\varrho \right)} \cdot \frac{m+\omega-\omega'}{m+\omega(1-\cos\theta)}$$

$$= \frac{\omega'^2\, d\varphi\, d\cos\theta \quad (m+\omega-\omega')}{16(2\pi)^2\, m\omega'\omega\, (m+\omega-\omega')(m+\omega(1-\cos\theta))}$$

$$= \frac{\omega'\, d\varphi\, d\cos\theta}{16(2\pi)^2\, \omega m\, (m+\omega(1-\cos\theta))}$$

$$= \frac{\omega'\, d\varphi\, d\cos\theta}{16(2\pi)^2\, \omega m^2 \left(1+\frac{\omega}{m}(1-\cos\theta)\right)}$$

$$= \frac{\omega'\, d\varphi\, d\cos\theta}{(2\pi)^2 16\, \omega m^2 \left(\frac{\omega}{\omega'}\right)} = \frac{d\varphi\, d\cos\theta}{64\pi^2 m^2}\left(\frac{\omega'}{\omega}\right)^2$$

$$\rightarrow d\sigma = \frac{d\cos\theta}{32\pi\, m^2}\left(\frac{\omega'}{\omega}\right)^2 2e^4\left[\frac{\omega'}{\omega}+\frac{\omega}{\omega'}-\sin^2\theta\right]$$

$$\frac{d\sigma}{d\cos\theta} = \frac{e^4}{16\pi m^2}\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\omega'}{\omega}+\frac{\omega}{\omega'}-\sin^2\theta\right]$$

$$= \frac{\pi}{m^2}\left(\frac{e^2}{4\pi}\right)^2\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\omega'}{\omega}+\frac{\omega}{\omega'}-\sin^2\theta\right]$$

$$= \frac{\pi\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\omega'}{\omega}+\frac{\omega}{\omega'}-\sin^2\theta\right]$$

g) $$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m^2}\left[\frac{\omega'^3}{\omega^3} + \frac{\omega'}{\omega} - \frac{\omega'^2}{\omega^2}\sin^2\theta\right]$$

$$\frac{\omega'}{\omega} = \frac{1}{1+\frac{\omega}{m}(1-\cos\theta)} \longrightarrow 1 \quad (\omega \to 0)$$

$$\Rightarrow \frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m^2}\left[2-\sin^2\theta\right]$$

$$\sigma = \int_{-1}^{1} \frac{\pi\alpha^2}{m^2}(1+x^2)\,dx$$

$$= \frac{\pi\alpha^2}{m^2}\left[1-(-1)+\tfrac{1}{3}1-(-\tfrac{1}{3})\right]$$

$$= \frac{8\pi\alpha^2}{3m^2}$$

h) $$d\sigma = \frac{d^3p'\,d^3k'}{(2\pi)^2\,4\,E_{p'}E_{k'}}\;\frac{\delta^{(4)}(p'+k'-p-k)}{4\sqrt{(p\cdot k)^2}}\;\frac{1}{4}\sum|M|^2$$

c.o.m: $\vec{p}+\vec{k} = 0 = \vec{p}'+\vec{k}' \quad \vec{k} = (\omega,0,0,\omega)$

$$\Rightarrow \omega + E_p = \omega + \sqrt{\vec{k}^2+m^2}$$
$$= \omega + \sqrt{\omega^2+m^2} = W$$
$$E_p = W-\omega = \sqrt{\omega^2+m^2}$$

$$p\cdot k = E_p\cdot\omega + \vec{k}^2 = (E_p+\omega)\omega = W\omega$$

$$d\sigma = \frac{d^3k'}{(2\pi)^2\,4\sqrt{\omega'^2+m^2}\,\omega'}\;\frac{\delta(p_0'+k_0'-W)}{4W\omega}\;\frac{1}{4}\sum|M|^2$$

$$= \frac{d\rho\,\rho^2\,d\Omega\;\delta\left(\sqrt{\rho^2+m^2}+\rho-W\right)}{(2\pi)^2\,16\,W\omega\rho\sqrt{\rho^2+m^2}} \times \frac{1}{4}\sum|M|^2$$

$$f(\rho) = \sqrt{\rho^2+m^2}+\rho-W \qquad \rho_0 = (W-\rho)^2-m^2$$

$$f'(\rho) = \frac{\rho}{\sqrt{\rho^2+m^2}}+1 = \frac{\rho}{f(\rho_0)+W-\rho}+1$$

$$f'(\rho_0) = \frac{\rho_0}{W-\rho_0} + 1 = \frac{\rho_0 + W - \rho_0}{W-\rho} = \frac{W}{W-\rho_0}$$

$$\Rightarrow d\sigma = \frac{d\rho\, \rho^2\, d\Omega \quad \delta(\rho-\rho_0)\,(W-\rho_0)}{(2\pi)^2 16\, W\omega\rho \sqrt{\rho^2+m^2} \cdot W} \times \frac{1}{4}\sum_i |M|^2$$

$$= \frac{\rho_0\, d\Omega}{(2\pi)^2 16\, W\omega} \frac{(W-\rho_0)}{(W-\rho_0)\, W} \times \frac{1}{4}\sum |M|^2$$

$$= \frac{\rho_0}{\omega} \frac{d\Omega}{64\pi^2 W^2} \times \frac{1}{4}\sum |M|^2$$

$$= \frac{\rho_0}{\omega} \frac{d\Omega}{s} \times \frac{1}{4}\sum |M|^2 \qquad \rho_0 = \omega$$

$$\frac{1}{4}\sum |M|^2 = 2e^4\left[\frac{p\cdot k'}{W\omega} + \frac{W\omega}{pk'} + 2m^2\left(\frac{1}{W\omega} - \frac{1}{pk'}\right) + m^4\left(\frac{1}{W\omega} - \frac{1}{(pk')}\right)^2\right]$$

$$p\cdot k' = \cancel{p\cdot k \ldots} \; E_p\omega' - p\omega'\cos\theta$$

$$E_p = W-\omega\,, \quad p = |\vec{p}| = \omega$$

$$p\cdot k' = (W-\omega)\omega' - \omega\omega'\cos\theta$$

$$= [W - (1+\cos\theta)\omega]\,\omega'$$

$$\Rightarrow \cancel{\frac{1}{4}\sum |M|^2 = 2e^4\{} = \frac{[W-(1+\cos\theta)\omega]\,\omega}{1+\frac{\omega}{m}(1-\cos\theta)}$$

$$\frac{p\cdot k'}{W\omega} = \frac{[W-(1+\cos\theta)\omega]}{W + W\frac{\omega}{m}(1-\cos\theta)} = \frac{\frac{W}{\omega} - (1+\cos\theta)}{\frac{W}{\omega} + \frac{W}{m}(1-\cos\theta)}$$

$$d\sigma = d\Omega \, \frac{\omega'}{\omega s} \frac{1}{4} \sum_i |M|^2$$

$$= \frac{d\Omega}{s} 2e^4 \Big[\frac{\omega'^2 [W-(1+\cos\theta)\omega]}{W\omega^2}$$

$$+ \frac{W}{[W-(1+\cos\theta)\omega]}$$

$$+ 2m^2 \left(\frac{\omega'}{W\omega^2} - \frac{1}{\omega[W-(1+\cos\theta)\omega]} \right)$$

$$+ \mathcal{O}(m^4) \Big]$$

$$\approx \frac{d\Omega}{s} 2e^4 \Big[\left(\frac{\omega'}{\omega}\right)^2 \left(1 - (1+\cos\theta)\frac{\omega}{W}\right)$$

$$+ \frac{1}{1-(1+\cos\theta)\frac{\omega}{W}}$$

$$+ 2m^2 \left(\left(\frac{\omega'}{\omega}\right) \frac{1}{W\omega} - \frac{1}{W\omega} \frac{1}{[1-(1+\cos\theta)\frac{\omega}{W}]} \right)$$

$$+ \mathcal{O}(m^4) \Big]$$

$$\frac{\omega'}{\omega} \to 0 \quad (\omega \to \infty) \qquad \frac{\omega}{W} \to \frac{1}{2} \quad (\omega \to \infty)$$

$$\rightarrow d\sigma = \frac{d\Omega}{s} 2e^4 \Big[+ \mathcal{O}(1) + \mathcal{O}(m^4)$$

$$- \frac{2m^2}{W\omega} \left(\frac{1}{1-(1+\cos\theta)\frac{\omega}{W}} \right) \Big]$$

$$\approx \frac{d\Omega}{s} 2e^4 \left[- \frac{4m^2}{s} \left(\frac{2}{2-(1+\cos\theta)} \right) \right]$$

$$\approx \frac{d\Omega}{s^2} 2e^4 \left[- 4m^2 \left(\frac{2}{1-\cos\theta} \right) \right]$$