Theoretische Teilchenphysik I

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Exercise Sheet 7 | SS-2023 | Due date: 13.06.23

Exercise 7.1: *(10 points)* Investigate $\mathcal{P}, \mathcal{C}$ and $\mathcal{T}$ transformations of a complex scalar field studied in the previous exercise sheet.

Following discussion in the lecture, postulate the action of $\mathcal{P}, \mathcal{C}$ and $\mathcal{T}$ operators on the creation and annihiltion operators of particles and anti-particles, and use these results to derive the following transformation properties of the complex scalar field

$$\mathcal{P}\phi(t,\vec{x})\mathcal{P}^{-1} = \phi(t,-\vec{x}), \quad \mathcal{C}\phi(t,\vec{x})\mathcal{C}^{-1} = \phi^{+}(t,\vec{x}), \quad \mathcal{T}\phi(t,\vec{x})\mathcal{T}^{-1} = \phi^{+}(-t,\vec{x}). \tag{1}$$

Exercise 7.2: *(25 points)* Derive $\mathcal{C}, \mathcal{P}$ and $\mathcal{T}$ transformations of the following spinor bilinears:

$$\bar{\psi}\gamma_5\psi, \quad \bar{\psi}\gamma_\mu\gamma_5\psi, \quad \bar{\psi}\sigma_{\mu\nu}\psi = \frac{1}{2}\bar{\psi}\left[\gamma_\mu, \gamma_\nu\right]\psi. \tag{2}$$

For each of these operators, write explicitely the result of the $\mathcal{CPT}$ transformation.

Exercise 7.3: *(25 points)* In quantum electrodynamics, electormagnetic fields are described using a vector potential $A_\mu(t,\vec{x})$ familiar to you from classical electordynamics. Although we have not discussed its quantization, we may assume that $A_\mu(t,\vec{x})$ transforms as a four-vector under $\mathcal{C}, \mathcal{P}$ and $\mathcal{T}$ transformations, i.e. in exactly the same way as the vector current $J^\mu = \bar{\psi}(t,\vec{x})\gamma^\mu\psi(t,\vec{x})$.

Consider the following action $S = \int d^4x\mathcal{L}$, with

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}A_\rho R_\sigma\,, \tag{3}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and R_μ is a constant vector which *does not change* under $\mathcal{C}, \mathcal{P}$ and $\mathcal{T}$ transformations.

(a) *(10 points)* Check that under this assumption the contribution of the first term in (3) to the action is $\mathcal{CPT}$ invariant.

(b) *(15 points)* Study the action of $\mathcal{C}, \mathcal{P}$ and $\mathcal{T}$ on the second term in (3). Derive its transformation rules under $\mathcal{CP}$ and $\mathcal{CPT}$ transformations.

Exercise 7.4: *(40 points)* Discrete symmetries of the Lagrangian.

(a) *(5 points)* Suppose we want to write a Lagrangian for a scalar field that is invariant under the transformation $\varphi \to -\varphi$, known as Z_2 symmetry. Start with a general Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\varphi\right)^2 + c_1\varphi + c_2\varphi^2 + c_3\varphi^3 + c_4\varphi^4 \tag{4}$$

and derive constraints on the parameters c_i under the assumption that $\mathcal{L}$ is Z_2 symmetric.

(b) *(20 points)* For the Lagrangian describing photon interaction with fermions,

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + g_V\bar{\psi}\gamma_\mu\psi A_\mu + g_A\bar{\psi}\gamma_\mu\gamma_5\psi A_\mu + g_T\bar{\psi}\sigma_{\mu\nu}\psi F_{\mu\nu}\,, \tag{5}$$

derive constraints on parameters m, g_V, g_A, g_T (i.e. can they be complex, should they vanish, etc.) assuming that A_μ transforms as a four-vector under $\mathcal{C}, \mathcal{P}$ and $\mathcal{T}$ transformations. Study under which conditions the action $S = \int d^4\mathcal{L}$ is $\mathcal{P}$ symmetric, $\mathcal{CP}$ symmetric and $\mathcal{CPT}$ symmetric.

(c) *(15 points)* Write down at least two possible Lagrangian terms describing interaction of the *charged* scalar field with fermions and investigate its properties under $\mathcal{C}, \mathcal{P}, \mathcal{T}$ transformations. Start with the simplest case of scalar currents $\bar{\psi}\psi|\phi|^2$ and further consider more complicated types of interactions, e.g. vector current for charged scalar field $J_\mu = \phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi$.