Theoretische Teilchenphysik I

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Exercise Sheet 9 | SS-2023 | Due date: 27.06.23

Matrix elements (100 Points)

Exercise 9.1: *(30 points)* Consider a theory of two interacting fields ϕ_1 and ϕ_2, described by the following Lagrangian:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi_1\right)^2 + \frac{1}{2}\left(\partial_\mu \phi_2\right)^2 - \frac{1}{2} m_1^2\, \phi_1^2 - \frac{1}{2} m_2^2\, \phi_2^2 - \frac{\lambda}{4}\, \phi_1^2 \phi_2^2 \,. \tag{1}$$

Derive the Feynman rules (propagators, interaction vertices) that should be used to construct the Green's functions. Use the derived Feynman rules to compute the matrix elements for the following scattering process up to the first and second order in the perturbative expansion in λ

$$\phi_1 + \phi_1 \to \phi_1 + \phi_1 \,, \quad \phi_1 + \phi_1 \to \phi_2 + \phi_2 \,, \quad \phi_1 + \phi_2 \to \phi_1 + \phi_2 \,. \tag{2}$$

During the derivation, write explicitly all the delta-functions in the vertices and show how you remove them by integrating over (some) momenta. Compare the expression for $\mathcal{M}_{\phi_1\phi_1\to\phi_1\phi_1}$ with the one derived in lectures for $\lambda\phi^4$ theory.

Exercise 9.2: *(30 points)* In the lectures the matrix element for the process $\bar{\psi}\psi \to \phi\phi$ in the Yukawa theory was derived. Follow the steps described in the lecture to draw Feynman diagrams and derive the expression for the scattering amplitude for the related processes $\bar{\psi}\phi \to \bar{\psi}\phi$, $\psi\phi \to \psi\phi$ and $\phi\phi \to \bar{\psi}\psi$ at the first non-vanishing order in the coupling constant. *Note:* Remove all δ-functions from vertices by integrating over momenta that are assigned to internal lines.

Exercise 9.3: *(40 points)* In this exercise, we consider the generalisation of the Yukawa theory, discussed during the lectures. To make the Dirac structure of the interacting vertex non-trivial, we consider a theory with the following interaction:

$$\mathcal{L}_I = \phi\, \bar{\psi}\left(g_y \hat{1} + g_5 \gamma_5\right) \psi, \tag{3}$$

where $\hat{1}$ is the identity matrix.

(a) *(10 points)* Use CPT theorem to derive constraints on the coupling g_5.

(b) *(10 points)* Summarise the Feynman rules (vertices, propagators).

(c) *(10 points)* Draw diagrams for processes $\bar{\psi}\psi \to \phi\phi$, $\bar{\psi}\phi \to \bar{\psi}\phi$, $\bar{\psi}\psi \to \bar{\psi}\psi$ and $\psi\psi \to \psi\psi$ in the theory with the interaction Lagrangian (3) at the second order in the coupling constants.

(d) *(10 points)* Derive the matrix elements for the process $\bar{\psi}\psi \to \bar{\psi}\psi$ and $\psi\psi \to \psi\psi$.

Note: assign momenta to internal and external lines, show explicitly directions of the fermion flow and make clear how Dirac matrices that appear in the amplitude are multiplied.